CONFIDENTIAL

SET A

# UNIVERSITI KUALA LUMPUR

## FINAL EXAMINATION
## JANUARY 2011 SESSION

| | | |
|---|---|---|
| SUBJECT CODE | : | WQD 10202 |
| SUBJECT TITLE | : | TECHNICAL MATHEMATICS II |
| LEVEL | : | DIPLOMA |
| TIME / DURATION | : | 2.00 pm – 4.30 pm<br>( 2.5 HOURS) |
| DATE | : | 03 MAY 2011 |

### INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of THREE (3) parts. Part A, B and C. Answer all questions in Part A and B. For Part C, answer two (2) questions only.
6. Answer all questions in English.
7. Formula Sheet is appended.

THERE ARE 8 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

**PART A (Total: 15 marks)**

**MULTIPLE CHOICE QUESTIONS**

**INSTRUCTION: Answer ALL questions.**

**Please use the answer booklet provided.**

1. Determine the amplitude of $y = -3\cos(x+2)$.

   A. 2

   B. $-2$

   C. 3

   D. $-3$

2. Simplify the trigonometric expression: $\csc\theta \tan\theta$

   A. $\sec\theta$

   B. $\cos\theta$

   C. $\sin\theta$

   D. $\cot\theta$

3. Solve $2\sin\theta + 1 = 0$

   A. $30^\circ$, $150^\circ$

   B. $150^\circ$, $210^\circ$

   C. $30^\circ$, $330^\circ$

   D. $210^\circ$, $330^\circ$

4. $(f \circ g)(x)$ means

   A. $f(x) \cdot g(x)$

   B. $f[g(x)]$

   C. $g[f(x)]$

   D. $g(x) \cdot f(x)$

5. $(g \circ g)(x)$ means

   A. $g(x) \cdot g(x)$

   B. $[g(x)]^2$

   C. $g[g(x)]$

   D. $g^2(x)$

6. Compute the limit of $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$.

   A. undefined

   B. 0

   C. 1

   D. 10

7. The differentiation of $2\ln(x+1)$ is

   A. $\frac{1}{2(x+1)}$

   B. $\frac{1}{x+1}$

   C. $\frac{2}{2x+1}$

   D. $\frac{2}{x+1}$

8. Given that $g(x) = (8x-3)^{\frac{1}{2}}$. Determine $g'(x)$.

   A. $4(8x-3)^{-\frac{1}{2}}$

   B. $\frac{1}{2}(8x-3)^{\frac{3}{2}}$

   C. $4(8x-3)^{\frac{3}{2}}$

   D. $\frac{1}{2}(8x-3)^{-\frac{1}{2}}$

9. The second derivative of $y=-x^3+2x^2-5x+3$ is

A. $-3x^2+4x-5$

B. $3x^2+4x$

C. $-6x+4$

D. $6x+4$

10. The differentiation of $\sin 2x$ with respect to $x$ is

A. $2\sin 2x$

B. $2\cos 2x$

C. $\sin 4x$

D. $\cos 4x$

11. Given $y=\dfrac{f(x)}{g(x)}$, choose the correct formula for *quotient rule.*

A. $\dfrac{g(x)f'(x)-g'(x)f(x)}{(g(x))^2}$

B. $f(x)g'(x)+g(x)f'(x)$

C. $\dfrac{g'(x)f(x)+g(x)f'(x)}{(g(x))^2}$

D. $f(x)g'(x)-g(x)f'(x)$

12. The integration of $y=\dfrac{x^{-\frac{1}{2}}}{2}$ is

A. $-\dfrac{x^{-\frac{3}{2}}}{3}+C$

B. $x^{\frac{1}{2}}+C$

C. $\dfrac{x^{\frac{1}{2}}}{2}+C$

D. $\dfrac{x^{-\frac{3}{2}}}{2}+C$

13. If $\int_a^b f(x)\,dx = c$ and $\int_a^b g(x)\,dx = e$, evaluate $\int_a^b \left(3f(x) + \frac{1}{2}g(x) - 1\right)dx$ terms of $a$, $b$, $c$ and $e$.

A. $3c + \frac{1}{2}e - b - a$

B. $3e + \frac{1}{2}c - b - a$

C. $3c + \frac{1}{2}e - b + a$

D. $3c + \frac{1}{2}e + b - a$

14. $\int_{-1}^{1} (5 - 2x)dx =$

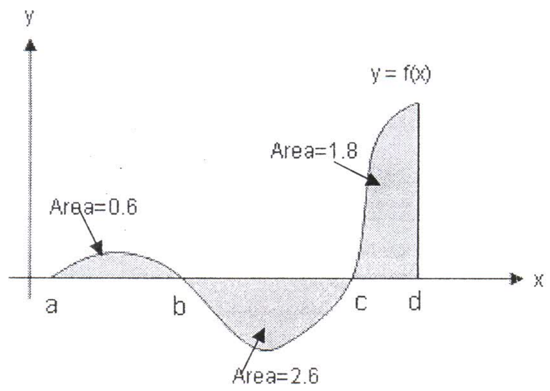
A. 10

B. 8

C. 12

D. 2

15. Use the areas in the shaded regions shown in the figure below to determine $\int_a^d f(x)\,dx$.




A. $-0.2\ unit^2$

B. $2.4\ unit^2$

C. $2.6\ unit^2$

D. $5\ unit^2$

## PART B (Total: 45 marks)

**INSTRUCTION: Answer ALL questions.**

**Please use the answer booklet provided.**

### Question 1

Given that $\cos\alpha = \frac{4}{5}$ with $\alpha$ is an acute angle ($\alpha$ in the first quadrant) and $\tan\beta = -\frac{12}{5}$ with $\beta$ is an obtuse angle ($\beta$ in the second quadrant). Determine:

a) $\sin(\beta - \alpha)$

b) $\tan 2\beta$

[7 marks]

### Question 2

Given $f(x) = 2x^2 + 5x$ and $g(x) = x + 5$, determine each of the following:

a) $f(-2)$

b) $(f \circ g)(x)$

[5 marks]

### Question 3

Evaluate the limits of the following function;

a) $\lim_{x \to 3} \frac{x^2 - 9}{x + 3}$

b) $\lim_{t \to 1} \frac{t^2 + t - 2}{t^2 - 1}$

[5 marks]

**Question 4**

Given $f(x) = \frac{x^4 + 4x + 4}{1 + x^3}$.

a) Determine $f'(x)$

b) Hence, calculate the value of $f'(2)$.

[7 marks]

**Question 5**

Determine $\frac{dy}{dx}$ of $xy^2 - 3x^2 = xy + 1$

[7 marks]

**Question 6**

Sketch the graphs of $y = 2x^2$ and the line $y = 8$. Hence determine the area of the region enclosed by the graphs.

[8 marks]

**Question 7**

Evaluate $\int_0^1 \frac{x^2}{x^3 + 1}\, dx$ by using substitution method.

[6 marks]

## PART C (Total: 40 marks)

**INSTRUCTION: Answer TWO questions.**

**Please use the answer booklet provided.**

### Question 1

Given $y = 4\sin 2x$ .

a) State the period and the amplitude.

b) Sketch the graph for two cycles beginning with $x = 0$.

c) On the same axes, sketch $y = 8\sin 2x$

[20 marks]

### Question 2

a) Given $x^3y + xy^3 = 2$, determine $\frac{dy}{dx}$ at the point $(1,1)$.

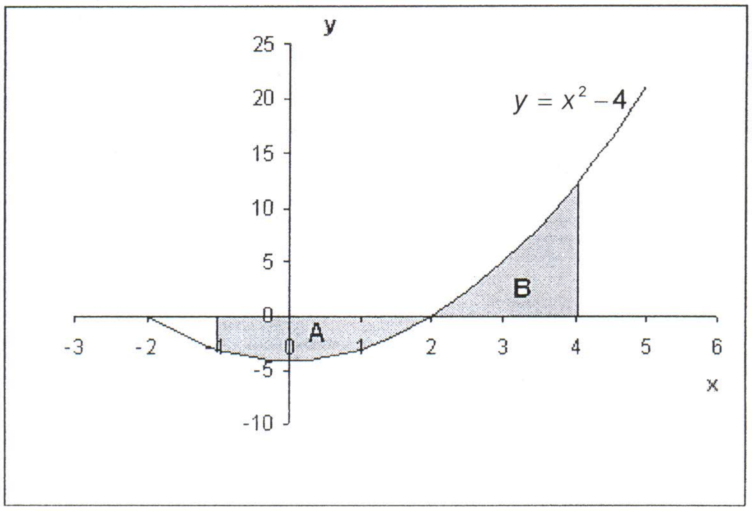
b) The volume of a water in a hemispherical bowl of radius 30 cm is given by $V = \frac{1}{3}\pi\left(36x^2 - x^3\right)$ where $x$ cm is the depth of the water. If the water is poured in at a constant rate of 10 $cm^3s^{-1}$, at what rate is the level rising when the depth is $8\,cm$?

[20 marks]

**Question 3**

a) Determine $\int \frac{x+4}{2x^2+x-1}dx$.

b) Calculate the area of the region bounded by the curve $y = x^2 - 4$, the x-axis, $x = -1$ and $x = 4$.




[20 marks]

**END OF QUESTION**

## FORMULA SHEET

### TRIGONOMETRY IDENTITIES

| FUNDAMENTAL IDENTITIES | FORMULAS FOR NEGATIVES |
|---|---|
| $\csc\theta = \dfrac{1}{\sin\theta}$ | $\sin(-\theta) = -\sin\theta$ |
| $\sec\theta = \dfrac{1}{\cos\theta}$ | $\cos(-\theta) = \cos\theta$ |
| $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$ | $\tan(-\theta) = -\tan\theta$ |
| $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ | $\csc(-\theta) = -\csc\theta$ |
| $\sin^2\theta + \cos^2\theta = 1$ | $\sec(-\theta) = \sec\theta$ |
| $1 + \tan^2\theta = \sec^2\theta$ | $\cot(-\theta) = -\cot\theta$ |
| $1 + \cot^2\theta = \csc^2\theta$ | |

| ADDITION FORMULAS | SUBTRACTION FORMULAS |
|---|---|
| $\sin(A+B) = \sin A\cos B + \cos A\sin B$ | $\sin(A-B) = \sin A\cos B - \cos A\sin B$ |
| $\cos(A+B) = \cos A\cos B - \sin A\sin B$ | $\cos(A-B) = \cos A\cos B + \sin A\sin B$ |
| $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$ | $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$ |

| HALF-ANGLE FORMULAS | DOUBLE-ANGLE FORMULAS |
|---|---|
| $\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$ | $\sin 2\theta = 2\sin\theta\cos\theta$ |
| $\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$ | $\cos 2\theta = \cos^2\theta - \sin^2\theta$<br>.......... $= 1 - 2\sin^2\theta$<br>.......... $= 2\cos^2\theta - 1$ |
| $\tan\dfrac{\theta}{2} = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$ | $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ |

| PRODUCT-TO-SUM FORMULAS | SUM-TO-PRODUCT FORMULAS |
|---|---|
| $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$ | $\sin\alpha+\sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$ |
| $\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$ | $\sin\alpha-\sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$ |
| $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$ | $\cos\alpha+\cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$ |
| $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$ | $\cos\alpha-\cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$ |

## DIFFERENTIATION

| STANDARD FORM | GENERAL FORM |
|---|---|
| $\frac{d}{dx}(\sin x) = \cos x$ | $\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$ |
| $\frac{d}{dx}(\cos x) = -\sin x$ | $\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$ |
| $\frac{d}{dx}(\tan x) = \sec^2 x$ | $\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$ |
| $\frac{d}{dx}(\csc x) = -\csc x\cot x$ | $\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$ |
| $\frac{d}{dx}(\sec x) = \sec x\tan x$ | $\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$ |
| $\frac{d}{dx}(\cot x) = -\csc^2 x$ | $\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$ |

## EXPONENTIAL FUNCTION

| STANDARD FORM | GENERAL FORM |
|---|---|
| $\frac{d}{dx}e^x = e^x$ | $\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$ |

## LOGARITHMIC FUNCTION

| STANDARD FORM | GENERAL FORM |
|---|---|
| $\frac{d}{dx}\ln x = \frac{1}{x}$ | $\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$ |

## INTEGRATION

| STANDARD FORM | GENERAL FORM<br>Where : $f(x) = ax + b$ |
|---|---|
| $\int \cos x\, dx = \sin x + c$ | $\int \cos f(x) dx = \dfrac{\sin f(x)}{f'(x)} + c$ |
| $\int \sin x\, dx = -\cos x + c$ | $\int \sin f(x) dx = \dfrac{-\cos f(x)}{f'(x)} + c$ |
| $\int \sec^2 x\, dx = \tan x + c$ | $\int \sec^2 f(x) dx = \dfrac{\tan f(x)}{f'(x)} + c$ |
| $\int \sec x \tan x\, dx = \sec x + c$ | $\int \sec f(x) \tan f(x) dx = \dfrac{\sec f(x)}{f'(x)} + c$ |
| $\int \csc x \cot x\, dx = -\csc x + c$ | $\int \csc f(x) \cot f(x) dx = \dfrac{-\csc f(x)}{f'(x)} + c$ |
| $\int \csc^2 x\, dx = -\cot x + c$ | $\int \csc^2 f(x) dx = \dfrac{-\cot f(x)}{f'(x)} + c$ |

## EXPONENTIAL FUNCTION

| STANDARD FORM | GENERAL FORM<br>Where : $f(x) = ax + b$ |
|---|---|
| $\int e^x dx = e^x + c$ | $\int e^{f(x)} dx = \dfrac{e^{f(x)}}{f'(x)} + c$ |

## LOGARITHMIC FUNCTION

| STANDARD FORM | GENERAL FORM<br>Where : $f(x) = ax + b$ |
|---|---|
| $\int \frac{1}{x} dx = \ln\lvert x\rvert + c$ | $\int \dfrac{1}{f(x)} dx = \dfrac{\ln\lvert f(x)\rvert}{f'(x)} + c$ |