UNIVERSITI KUALA LUMPUR
Malaysian Institute of Marine Engineering Technology

FINAL EXAMINATION
JANUARY 2016 SESSION

SUBJECT CODE	:	LEB 40403
SUBJECT TITLE	:	DIGITAL SIGNAL PROCESSING
LEVEL	:	DEGREE
TIME / DURATION	:	9.00 AM – 12.00 PM / 3 HOURS
DATE	:	26 MAY 2016 / THURSDAY

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. Begin **EACH** answer on a new page in the Answer Booklet.
3. Where applicable, show clearly steps taken in arriving at the solutions and indicate **ALL** assumptions.
4. This question paper consists of 5 Questions. **Answer Four (4) Questions ONLY.**
5. Tables, Formulae and Charts are appended.
6. Answer all questions in English.

THERE ARE TEN (10) PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

Question 1

a. Consider the signal $x(t)$ given as per equation below. Sketch the respective graph according to the equation given.

$$x(t) = \begin{cases} t+1, -1 \le t \le 0 \\ 1, 0 < t \le 2 \\ -t+3, 2 < t \le 3 \\ 0, otherwiswe \end{cases}$$

(8 marks)

b. Define the difference between energy signal and power signal.

(5 marks)

c. Sketch as accurately as possible the real and imaginary parts of a complex exponential sequence $x(n)$, given by following mathematical expression and provide the basis for the sketches:

$$x(n) = 2e^{(k+j\omega_0)}n\ ; k = -\frac{1}{2}, \omega_0 = \frac{\pi}{6}, n = 0,1,2, \ldots..$$

(7 marks)

d. Based on the part(a) for equation $x(n)$, determine the first four values of the real part for $x(n); n = 0,1,2,3.$

(5 marks)

Question 2

a. Determine and sketch the discrete-time Fourier transform (DTFT) of a sequence,

$$x(n) = \cos(\omega_0 n),\ \omega_0 = 2\pi 2000$$

(10 marks)

b. In the double sideband suppressed carrier (DSBSC) modulation as shown in Figure 2 (b), the sequence in part (a) is multiplied by a high frequency cosine carrier, $x_{\text{carrier}}[n]$, of 100 kHz. Perform a Fourier analysis of the modulation system and determine the resultant spectrum.

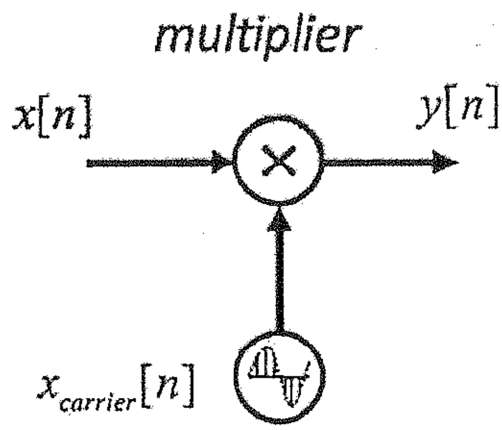


Figure 2 (b)

(10 marks)

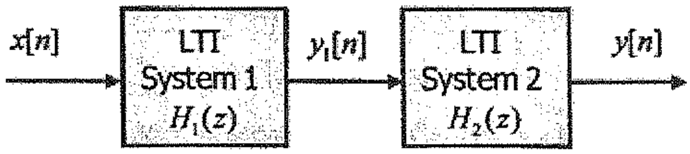
c. Prove that equation given satisfy the Fourier transform characteristics.

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

(5 marks)

Question 3

Figure 3.1 depicts a cascade connection of two discrete-time LTI systems.



$$y_1[n] = 3x[n] - 2x[n-1] + 4x[n-2]$$

$$H_2(z) = 1 + \tfrac{7}{8}z^{-1}$$

Figure 3.1

a. Use z-transforms to find $H_1(z)$

(3 marks)

b. Determine the transfer function, $H(z)$ for the overall system.

(5 marks)

c. Plot the poles and zeros of overall system, $H(z)$ in the complex plane.

(6 marks)

d. Consider the system described by the difference equation

$$y(n) - 2y(n-1) + 2y(n-2) = x(n) + \frac{1}{2}x(n-1)$$

Analyse the equation into a z-transform and form pole-zero plot.

(9 marks)

e. Provide your justification on the system

(2 marks)

Question 4

a. Consider a Linear Time-Invariant (LTI) system described by

$$y(n) - \frac{1}{5}y(n-1) = x(n) + \frac{1}{5}x(n-1)$$

Determine the frequency response of the system.

(8 marks)

b. A sinusoidal signal of frequency 26 Hz is sampled at 20 Hz. Using frequency domain analysis, predict the resulting aliasing effects in the time-domain.

(4 marks)

c. The response of a discrete-time system is characterized in the form of

$$y(nT + 2T) = e^{nT} + 5x(nT + 2T)$$

Analyse the

i. Linearity of the system (5 marks)

ii. Time-invariance of the system (5 marks)

d. An audio signal has a peak-to-mean power ratio of 12 dB. For a linear quantiser, determine a number of bits needed for the A-to D conversion, in order to achieve a signal to quantization noise in excess of 90 dB – a value typically required for audio CD formats.

(3 marks)

Question 5

a. Explain the difference between Chebychev and Butterworth filter.

[4 marks]

b. You are required to design a FIR lowpass filter with the following specifications.

Sampling frequency = 8000 Hz
Passband frequency = 1500 Hz
Stopband frequency = 2000 Hz
3-dB Cutoff frequency = 1800 Hz
Stopband ripple = 0.01

A Kaiser window is to be used in the design of the filter.

i. Design a minimum order filter that will meet the specifications above, and determine the expression for its impulse response.

[10 marks]

ii. You decide to choose a Kaiser window parameter, β, that is larger than the one you used in part (a)(ii). Analyse how this change will affect the response of the filter that you have designed.

[5 marks]

c. Digital filters can be either Infinite Impulse Response (IIR) or Finite Impulse Response (FIR) types. Explain both characteristics.

(6 marks)

APPENDIX I

TABLE AI.1 Common DTFT Pairs

Sequence	DTFT
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-jn_0\omega}$
1	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$
$a^n u[n]$, $\lvert a\rvert<1$	$\frac{1}{1-ae^{-j\omega}}$
$-a^n u[-n-1]$, $\lvert a\rvert>1$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}}+\sum_{k=-\infty}^{\infty}\pi\delta(\omega+2\pi k)$
$(n+1)a^n u[n]$, $\lvert a\rvert<1$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p}u[n]$, $\lvert r\rvert<1$	$\frac{1}{1-2r\cos\omega_p\, e^{-j\omega}+r^2e^{-j2\omega}}$
$\frac{\sin\omega_c}{\pi n}$	$X(e^{j\omega})=\begin{cases}1, & \lvert\omega\rvert<\omega_c\\ 0, & \omega_c<\lvert\omega\rvert\le\pi\end{cases}$
$x[n]=\begin{cases}1, & 0\le n\le M\\ 0, & otherwise\end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)}e^{-j\omega M/2}$
$e^{jn\omega_0}$	$\sum_{k=-\infty}^{\infty}2\pi\delta(\omega-\omega_0+2\pi k)$
$\cos(\omega_0 n+\varphi)$	$\sum_{k=-\infty}^{\infty}[\pi e^{j\varphi}\delta(\omega-\omega_0+2\pi k)+\pi e^{-j\varphi}\delta(\omega+\omega_0+2\pi k)$

TABLE AI.2 Properties of DTFT

Property	Sequence	DTFT
Linearity	$ax[n]+by[n]$	$aX(e^{j\omega})+bY(e^{j\omega})$
Time-shift	$x[n-n_0]$	$e^{-jn_0\omega}X(e^{j\omega})$
Time-reversal	$x[-n]$	$X(e^{-j\omega})$
Modulation	$e^{jn\omega_0}x[n]$	$X(e^{j(\omega-\omega_0)})$
Convolution	$x[n]*y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Derivative	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\theta})Y(e^{j(\omega-\omega_0)})d\theta$

APPENDIX II

TABLE AII.1 Common z-Transform Pairs

Sequence	z-Transform	ROC
$\delta[n]$	1	All z
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
$(\cos\omega_0 n)u[n]$	$\frac{1-(\cos\omega_0)z^{-1}}{1-(2\cos\omega_0)z^{-1}+z^{-2}}$	$\|z\| > 1$
$(\sin\omega_0 n)u[n]$	$\frac{(\sin\omega_0)z^{-1}}{1-(2\cos\omega_0)z^{-1}+z^{-2}}$	$\|z\| > 1$
$(r^n\cos\omega_0 n)u[n]$	$\frac{1-(r\cos\omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\|z\| > r$
$(r^n\sin\omega_0 n)u[n]$	$\frac{(r\sin\omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\|z\| > r$
$x(n) = \begin{cases} a^n, 0 \le n \le N-1 \\ 0, otherwise \end{cases}$	$\frac{1-a^N z^{-N}}{1-z^{-1}}$	$\|z\| > 0$

TABLE AII.2 Properties of z-Transform

Property	Sequence	z-transform	ROC
Linearity	$ax[n]+by[n]$	$aX(z)+bY(z)$	Contains $R_x \cap R_y$
Time-shift	$x[n-n_0]$	$z^{-n_0}X(z)$	R_x
Time-reversal	$x[-n]$	$X(z^{-1})$	$1/R_x$
Exponentiation	$\alpha^n x[n]$	$X(\alpha^{-1}z)$	$\|\alpha\|R_x$
Convolution	$x[n] * y[n]$	$X(z)Y(z)$	Contains $R_x \cap R_y$
Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
Derivative	$nx[n]$	$-z\frac{dX(z)}{dz}$	R_x

APPENDIX III

TABLE AIII.1 Some Common Windows

Rectangular	$w(n)=\begin{cases}1 & 0\le n\le N\\ 0 & else\end{cases}$
Hanning[1]	$w(n)=\begin{cases}0.5-0.5\cos\left(\frac{2\pi n}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Hamming	$w(n)=\begin{cases}0.54-0.46\cos\left(\frac{2\pi n}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Blackman	$w(n)=\begin{cases}0.42-0.5\cos\left(\frac{2\pi n}{N}\right)+0.08\cos\left(\frac{4\pi n}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Kaiser	$w(n)=\begin{cases}\frac{I_0[\beta(1-[(n-\alpha)/\alpha]^2)^{1/2}]}{I_0(\beta)} & 0\le n\le N\\ 0 & else\end{cases}$

[1] In the literature, this window is also called a Hann window or a von Hann window.

[2] The unit sample response of an ideal low pass filter

$$h_d(n)=\frac{\sin(n-\alpha)\omega_c}{(n-\alpha)\pi}, 0\le n\le N-1, \alpha=\frac{N}{2}$$

TABLE AIII.2 The Peak Side-Lobe Amplitude of Some Common Windows and the Approximate Transition Width and Stopband Attenuation of an *N* th-Order Low-Pass Filter Designed Using the Given Window.

Window	Side-Lobe Amplitude (dB)	Transition Width (Δf)	Stopband Attenuation (dB)
Rectangular	-13	0.9/*N*	-21
Hanning	-31	3.1/*N*	-44
Hamming	-41	3.3/*N*	-53
Blackman	-57	5.5/*N*	-74

APPENDIX III (CONT.)

TABLE AIII.3 Characteristics of the Kaiser Window as a Function of β.

Parameter, β	Sidelobe(dB)	Transition Width (NΔf)	Stopband Attenuation(dB)
2.0	-19	1.5	-29
3.0	-24	2.0	-37
4.0	-30	2.6	-45
5.0	-37	3.2	-54
6.0	-44	3.8	-63
7.0	-51	4.5	-72
8.0	-59	5.1	-81
9.0	-67	5.7	-90
10.0	-74	6.4	-99

Stopband attenuation α_s is related to parameter β as follows:

$$\beta = \begin{cases} 0.1102(\alpha_s - 8.7) & \alpha_s > 50 \\ 0.5842(\alpha_s - 21)^{0.4} + 0.07886(\alpha_s - 21) & 21 \le \alpha_s \le 50 \\ 0.0 & \alpha_s < 21 \end{cases}$$

N is related to the transition width Δf and the stopband attenuation as α_s follows:

$$N = \frac{\alpha_s - 7.95}{14.36\Delta f} \qquad \alpha_s \ge 21$$

END OF QUESTION PAPER