



UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF CHEMICAL & BIOENGINEERING
TECHNOLOGY

FINAL EXAMINATION
OCTOBER 2025 SEMESTER

COURSE CODE	: CCB21203
COURSE NAME	: STATISTICS FOR ENGINEERS
PROGRAMME NAME (FOR MPU: PROGRAMME LEVEL)	: BACHELOR OF CHEMICAL ENGINEERING WITH HONOURS
DATE	: 24 JANUARY 2026
TIME	: 2:00 PM – 5:00 PM
DURATION	: 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please read **CAREFULLY** the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of **TWO (2)** sections; Section A and Section B.
4. Answer **ALL** questions in Section A. For Section B, answer **THREE (3)** questions **ONLY**.
5. Please write your answers on the answer booklet provided.
6. Answer all questions in English language **ONLY**.
7. Formulas and distribution tables are appended for your reference.

THERE ARE 14 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

SECTION A (Total: 40 marks)**INSTRUCTION: Answer ALL questions.****Please use the answer booklet provided.****Question 1**

Consider the data (Table 1) obtained from a chemical process where the yield of the process is thought to be related to the reaction temperature.

Table 1: Data on a chemical process.

Temperature, x	Yield, y
50	122
53	118
54	128
55	121
56	125
59	136
62	144
65	142
67	149
71	161
72	167

- (a) Assuming that a simple linear regression model is appropriate, fit the regression model relating the yield to the temperature. (13 marks)
- (b) Estimate the value of Yield, given Temperature is 70. (2 marks)
- (c) Calculate the value of correlation coefficient, r and interpret the value. (5 marks)

Question 2

- (a) An experiment was performed to determine whether the annealing temperature of ductile iron affects its tensile strength. Five specimens were annealed at each of four temperatures. The tensile strength (in ksi) was measured for each temperature. A related analysis to study the effect of different temperatures on the tensile strength is presented in Figure 1.

One-way ANOVA: Tensile Strength versus Temperature					
Source	DF	SS	MS	F	P
Temperature	A	58.65	19.55	C	0.001
Error	B	36.84	B		
Total	19	95.49			

S = 1.517 R-Sq = 61.42% R-Sq(adj) = 54.19%

Figure 1: MINITAB output to study the effect of different temperatures on the tensile strength.

- Determine the values of A, B and C.
(6 marks)
 - Test at 1% significance level, the effect of Temperature on Tensile Strength.
(Hint: use p-value)
(4 marks)
- (b) An experiment in which the amount of radon released in showers was investigated. Radon-enriched water was used in the experiment, and four different orifice diameters were tested in shower heads. A related analysis to study whether the size of the orifice does affect the mean percentage of radon released is presented in Figure 2.

One-way ANOVA: Radon Released versus Diameter					
Source	DF	SS	MS	F	P
Diameter	B	256.25	C	14.34	0.000
Error	A	71.50	5.96		
Total	15	B			

S = 2.441 R-Sq = 78.18% R-Sq(adj) = 72.73%

Figure 2: MINITAB output to study the effect of the size of the orifice does affect the mean percentage of radon was released.

- Determine the values of A, B and C.
(6 marks)
- Test at 5% significance level, whether the size of the orifice does affect the mean percentage of radon released. (Hint: use p-value)
(4 marks)

SECTION B (Total: 60 marks)**INSTRUCTION: Answer THREE (3) questions ONLY.****Please use the answer booklet provided.****Question 3**

- (a) A machine produces metal rods used in an automobile suspension system. A random sample of 12 rods is selected, and the diameter is measured. The resulting data in millimeters are shown in Table 2.

Table 2: The diameter of 12 rods.

8.23	8.31	8.42
8.29	8.19	8.24
8.19	8.29	8.30
8.14	8.32	8.40

Test the hypothesis at 5% significance level, to indicate that mean rod diameter is not 8.20mm.

(10 marks)

- (b) Ninety-one coils of galvanized steel are inspected across 7 categories of flaws. The MINITAB output of the analysis is shown in Figure 3.

Chi-Square Goodness-of-Fit Test for Observed Counts in Variable: Observed				
Category	Observed	Test Proportion	Expected	Contribution to Chi-Sq
1	10	0.142857	13	0.692308
2	15	0.142857	13	A
3	12	0.142857	13	0.076923
4	16	0.142857	13	B
5	15	0.142857	13	0.307692
6	12	0.142857	13	0.076923
7	11	0.142857	13	0.307692
N	DF	Chi-Sq	P-Value	
91	C	2.46154	0.873	

Figure 3: MINITAB output of Chi-Square Goodness of Fit Test.

- i. Determine the values of A, B and C.
- ii. Test at 5% significance level, whether the distribution across the seven categories of flaws is equally likely. (Hint: use p-value)

(6 marks)

(4 marks)

Question 4

- (a) A geologist collects hand-specimen sized pieces of limestone from a particular area. A qualitative assessment of both texture and colour is made and the results are shown in Table 3.

Table 3: The observed number of defects.

Texture	Colour		
	Light	Medium	Dark
Fine	4	20	8
Medium	5	23	12
Coarse	21	23	4

Test the hypothesis at 5% significance level, is there an association between colour and texture for these lime stones.

(10 marks)

- (b) Ten individuals have participated in a diet-modification program to stimulate weight loss. The relevant analysis is conducted and shown in Figure 4.

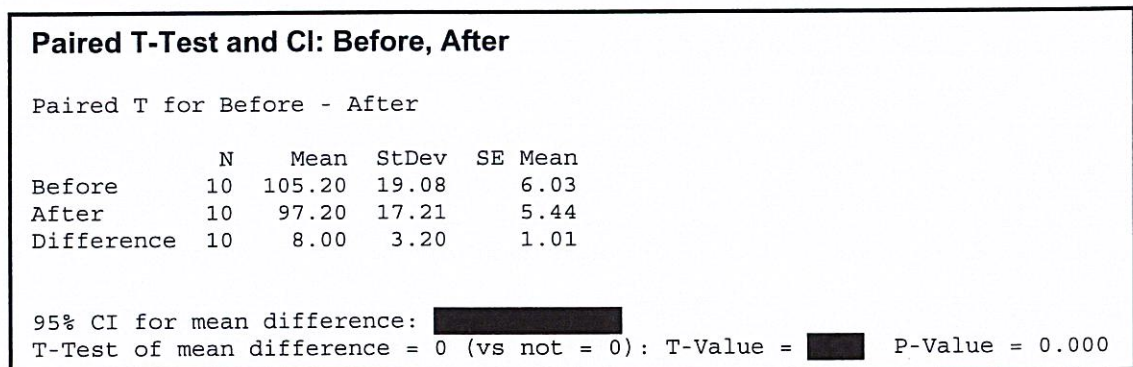


Figure 4: MINITAB output of a study on the effectiveness of a diet-modification program.

- Calculate the test value (T-value).
(2 marks)
- Test at 5% significance level, whether the diet-modification program is effective in producing mean weight reduction. (Hint: use p-value)
(4 marks)
- Calculate the 95% confidence interval of the mean difference.
(4 marks)

Question 5

- (a) In a study, it is suspected that strength is related to the percentage of cotton in the fiber. Five levels of cotton percentage are used, and five replicates are run in random order. The data is presented in Table 4.

Table 4: The measurements of breaking strengths using five levels of cotton percentage.

Cotton Percentage	Observations					$\sum x_i$	$\sum x_i^2$
	1	2	3	4	5		
15	7	7	15	11	9	49	525
20	12	17	12	18	18	77	1225
25	14	18	18	19	19	88	1566
30	19	25	22	19	23	108	2360
35	7	10	11	15	11	54	616

Test at 1% significance level, does the cotton percentage affect the breaking strength.

(10 marks)

- (b) An article describes an experiment to investigate flatness distortion from heat treating for two types of gears and two heat treating times. The MINITAB output of the relevant analysis is shown in Figure 5.

Factorial Fit: Flatness Distortion versus Gear Type, Time						
Estimated Effects and Coefficients for Flatness Distortion (coded units)						
Term	Effect	Coef	SE Coef	T	P	
Constant		0.054438	0.002673	20.37	0.000	
Gear Type	0.018125	0.009062	0.002673	3.39	0.028	
Time	0.031625	0.015812	0.002673	5.92	0.004	
Gear Type*Time	0.001375	0.000688	0.002673	0.26	0.810	
S = 0.00756018 PRESS = 0.0009145						
R-Sq = 92.09% R-Sq(pred) = 68.35% R-Sq(adj) = 86.15%						
Analysis of Variance for Flatness Distortion (coded units)						
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	0.00265731	0.00265731	0.00132866	23.25	0.006
2-Way Interactions	1	0.00000378	0.00000378	0.00000378	0.07	0.810
Residual Error	4	0.00022863	0.00022863	0.00005716		
Pure Error	4	0.00022862	0.00022862	0.00005716		
Total	7	0.00288972				

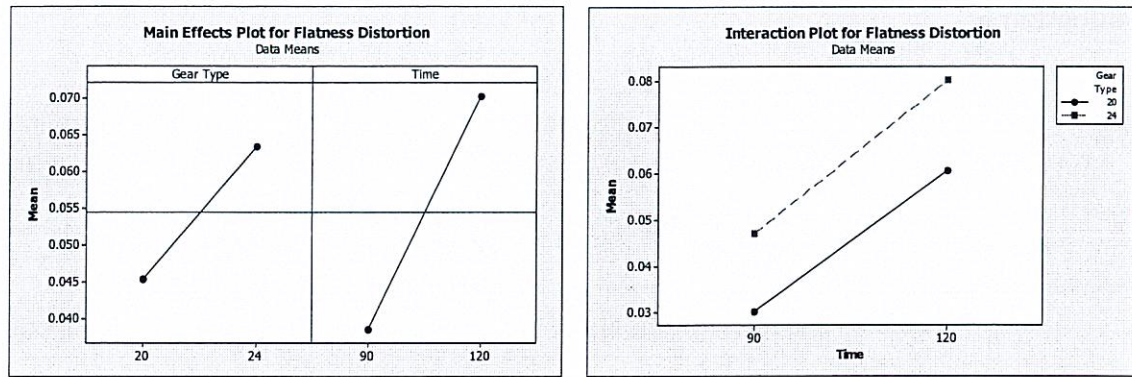


Figure 5: MINITAB output of the analysis on the Flatness Distortion.

- Write the full regression equation based on the output.
(2 marks)
- Test for significance of the individual coefficients using $\alpha = 0.01$.
(4 marks)
- Test for significance of the interaction using $\alpha = 0.05$.
(2 marks)
- Determine the Gear Type and Time Alignment Method that can minimize the Flatness Distortion. (Hint: refer to the Main Effects Plot)
(2 marks)

Question 6

- In semiconductor manufacturing, wet chemical etching is often used to remove silicon from the backs of wafers prior to metallization. The etch rate is important characteristic in this process. Two different etching solutions have been compared, using two random samples of 10 wafers for etch solution. The observed etch rates (in mils/min) are shown in Table 5.

Table 5: Comparison of two etching solutions.

Solution 1		Solution 2	
9.9	10.6	10.2	10.0
9.4	10.3	10.6	10.2
9.3	10.0	10.7	10.7
9.6	10.3	10.4	10.4
10.2	10.1	10.5	10.3
$\sum X_1 = 99.7$		$\sum X_2 = 104$	
$\sum X_1^2 = 995.61$		$\sum X_2^2 = 1082.08$	

Test at 5% significance level whether the mean etch rate is the same for both solutions with the assumption that both population variances are equal.

(10 marks)

- (b) An article describes an experiment for investigating a method for aligning optical chips onto circuit boards. The method involves placing solder bumps onto the bottom of the chip. The experiment used three solder bump sizes and three alignment methods. The response variable is alignment accuracy (in micrometers). The MINITAB output of the relevant analysis is shown in Figure 6.

Response Surface Regression: Accuracy versus Bump Size, Alignment Method						
The analysis was done using coded units.						
Estimated Regression Coefficients for Accuracy						
Term		Coef	SE Coef	T		P
Constant		1.1973	0.3204	3.737		0.003
Bump Size		0.4558	0.1515	3.008		0.011
Alignment Method		-1.2040	0.1535	-7.845		0.000
Bump Size*Bump Size		1.1540	0.3225	3.578		0.004
Alignment Method*Alignment Method		0.7017	0.2624	2.674		0.020
Bump Size*Alignment Method		0.1922	0.1807	1.064		0.308
S = 0.524873 PRESS = 7.05716						
R-Sq = 89.51% R-Sq(pred) = 77.61% R-Sq(adj) = 85.14%						
Analysis of Variance for Accuracy						
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	5	28.2115	28.2115	5.64230	20.48	0.000
Linear	2	22.4037	19.4493	9.72467	35.30	0.000
Square	2	5.4962	5.4962	2.74808	9.98	0.003
Interaction	1	0.3116	0.3116	0.31163	1.13	0.308
Residual Error	12	3.3059	3.3059	0.27549		
Lack-of-Fit	3	3.1885	3.1885	1.06282	81.44	0.000
Pure Error	9	0.1175	0.1175	0.01305		
Total	17	31.5174				

Figure 6: MINITAB output of the analysis on the alignment accuracy.

- Write the full regression equation based on the output.
(2 marks)
- Test for significance of the individual coefficients using $\alpha = 0.01$.
(4 marks)
- Test for significance of the model and lack-of-fit using $\alpha = 0.05$.
(4 marks)

END OF EXAMINATION PAPER

APPENDIX

Sample Mean	$\bar{x} = \frac{\sum x_i}{n}$	Sample Standard deviation	$s = \sqrt{\frac{(\sum x_i^2) - \frac{(\sum x_i)^2}{n}}{n-1}}$
Confidence interval (One Sample)	CASE 1 : σ known $\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ CASE 2 : σ unknown ($n \geq 30$) $\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$ CASE 3 : σ unknown ($n < 30$) $\bar{X} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}$		
Confidence interval (Two Samples)	CASE 1 : σ known $(\bar{X}_1 - \bar{X}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ CASE 2 : σ unknown ($n \geq 30$) $(\bar{X}_1 - \bar{X}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ Case 3(a) σ unknown ($n < 30$), assume unequal variances $(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}, v} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}$ Case 3(b) σ unknown ($n < 30$), assume equal variances $(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}, v} \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}} \quad s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} \quad v = n_1 + n_2 - 2$		
Test statistic (One Sample)	CASE 1 : σ known $Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ CASE 2 : σ unknown ($n \geq 30$) $Z = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$ CASE 3 : σ unknown ($n < 30$) $T = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$		

Test statistic (Two Samples)	CASE 1 : σ known $Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma^2_1}{n_1} + \frac{\sigma^2_2}{n_2}}}$ CASE 2 : σ unknown ($n \geq 30$) $Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}}}$ Case 3(a) σ unknown ($n < 30$), assume unequal variances $T = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}}}$ $v = \frac{\left(\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}\right)^2}{\frac{\left(\frac{s^2_1}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s^2_2}{n_2}\right)^2}{n_2 - 1}}$ Case 3(b) σ unknown ($n < 30$), assume equal variances $T = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s^2_p}{n_1} + \frac{s^2_p}{n_2}}} \quad s^2_p = \frac{(n_1 - 1)s^2_1 + (n_2 - 1)s^2_2}{n_1 + n_2 - 2} \quad v = n_1 + n_2 - 2$
Dependent samples	Test statistic $T = \frac{\bar{D} - \mu_d}{\frac{s_d}{\sqrt{n}}} \quad ; \quad v = n - 1; \quad \bar{D} = \frac{\sum D}{n}; \quad s_D = \sqrt{\frac{\sum D^2 - \frac{(\sum D)^2}{n}}{n - 1}}$ Confidence Interval $\bar{D} \pm t_{\frac{\alpha}{2}, v} \frac{s_d}{\sqrt{n}}$
Analysis of Variance (ANOVA)	$SSA = \sum \left[\frac{(\sum x_i)^2}{n_i} \right] - \frac{(\sum \sum x_{ij})^2}{n}$ $SST = \sum \sum x_{ij}^2 - \frac{(\sum \sum x_{ij})^2}{n}$ $SSE = SST - SSA$
Chi Square Test	$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$
Regression model	$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ $S_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n} \quad S_{xy} = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}$

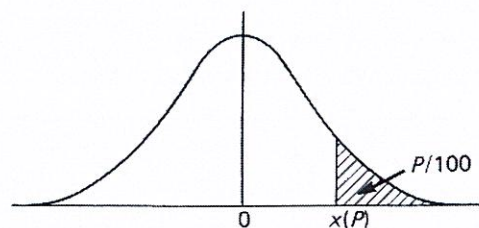
$S_{yy} = \sum y_i^2 - \frac{(\sum y_i)^2}{n}$ $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ Correlation coefficient $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$ Standard error estimate $s_e = \sqrt{\frac{S_{yy} - \hat{\beta}_1 S_{xy}}{n-2}}$	
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PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}} \int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2} dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geq x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geq x(P)$ is $2P/100$.



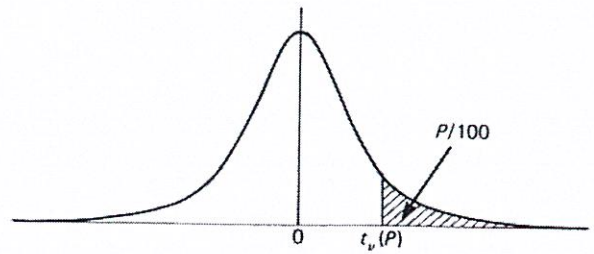
P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0.0000	5.0	1.6449	3.0	1.8808	2.0	2.0537	1.0	2.3263	0.10	3.0902
45	0.1257	4.8	1.6646	2.9	1.8957	1.9	2.0749	0.9	2.3656	0.09	3.1214
40	0.2533	4.6	1.6849	2.8	1.9110	1.8	2.0969	0.8	2.4089	0.08	3.1559
35	0.3853	4.4	1.7060	2.7	1.9268	1.7	2.1201	0.7	2.4573	0.07	3.1947
30	0.5244	4.2	1.7279	2.6	1.9431	1.6	2.1444	0.6	2.5121	0.06	3.2389
25	0.6745	4.0	1.7507	2.5	1.9600	1.5	2.1701	0.5	2.5758	0.05	3.2905
20	0.8416	3.8	1.7744	2.4	1.9774	1.4	2.1973	0.4	2.6521	0.01	3.7190
15	1.0364	3.6	1.7991	2.3	1.9954	1.3	2.2262	0.3	2.7478	0.005	3.8906
10	1.2816	3.4	1.8250	2.2	2.0141	1.2	2.2571	0.2	2.8782	0.001	4.2649
5	1.6449	3.2	1.8522	2.1	2.0335	1.1	2.2904	0.1	3.0902	0.0005	4.4172

PERCENTAGE POINTS OF THE t -DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1+t^2/\nu)^{(\frac{1}{2}\nu+1)}}$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2 -distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t -distribution with ν degrees of freedom, and the probability that $t \geq t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geq t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P	40	30	25	20	15	10	5	2.5	1	0.5	0.1	0.05
$\nu = 1$	0.3249	0.7265	1.0000	1.3764	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.2887	0.6172	0.8165	1.0607	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.2767	0.5844	0.7649	0.9785	1.250	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.2707	0.5686	0.7407	0.9410	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.2672	0.5594	0.7267	0.9195	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.2648	0.5534	0.7176	0.9057	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.2632	0.5491	0.7111	0.8960	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.2619	0.5459	0.7064	0.8889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.2610	0.5435	0.7027	0.8834	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.2602	0.5415	0.6998	0.8791	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.2596	0.5399	0.6974	0.8755	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.2590	0.5386	0.6955	0.8726	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.2586	0.5375	0.6938	0.8702	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.2582	0.5366	0.6924	0.8681	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.2579	0.5357	0.6912	0.8662	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.2576	0.5350	0.6901	0.8647	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.2573	0.5344	0.6892	0.8633	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.2571	0.5338	0.6884	0.8620	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.2569	0.5333	0.6876	0.8610	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.2567	0.5329	0.6870	0.8600	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.2566	0.5325	0.6864	0.8591	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.2564	0.5321	0.6858	0.8583	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.2563	0.5317	0.6853	0.8575	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.2562	0.5314	0.6848	0.8569	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.2561	0.5312	0.6844	0.8562	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.2560	0.5309	0.6840	0.8557	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.2559	0.5306	0.6837	0.8551	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.2558	0.5304	0.6834	0.8546	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.2557	0.5302	0.6830	0.8542	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.2556	0.5300	0.6828	0.8538	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	0.2555	0.5297	0.6822	0.8530	1.054	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	0.2553	0.5294	0.6818	0.8523	1.052	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	0.2552	0.5291	0.6814	0.8517	1.052	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	0.2551	0.5288	0.6810	0.8512	1.051	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	0.2550	0.5286	0.6807	0.8507	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.2547	0.5278	0.6794	0.8489	1.047	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.2545	0.5272	0.6786	0.8477	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.2539	0.5258	0.6765	0.8446	1.041	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.2533	0.5244	0.6745	0.8416	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291

PERCENTAGE POINTS OF THE F DISTRIBUTION

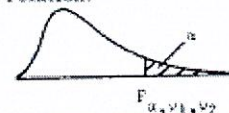
The table gives the values of $F_{\alpha; \nu_1, \nu_2}$ the 100 α percentage point of the F distribution having ν_1 degrees of freedom in the numerator and ν_2 degrees of freedom in the denominator.

For each pair of values of ν_1 and ν_2 , $F_{\alpha; \nu_1, \nu_2}$ is tabulated for $\alpha = 0.05, 0.025, 0.01, 0.001$, the 0.025 values being bracketed.

The lower percentage points of the distribution may be obtained from the relation:-

$$F_{1-\alpha; \nu_1, \nu_2} = 1/F_{\alpha; \nu_2, \nu_1}$$

$$\text{e.g. } F_{0.95; 12, 8} = 1/F_{0.05; 8, 12} = 1/2.85 = 0.351$$

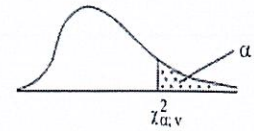


$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	10	12	24	∞
1	161.4 (648) 4052 4053*	199.5 (800) 5000 5000*	215.7 (864) 5403 5404*	224.6 (900) 5625 5625*	230.2 (922) 5764 5764*	234.0 (937) 5859 5859*	236.8 (948) 5928 5928*	238.9 (957) 5981 5981*	241.9 (969) 6056 6056*	243.9 (977) 6106 6107*	249.0 (997) 6235 6235*	254.3 (1018) 6366 6366*
2	18.5 (38.5) 98.5 998.5	19.0 (39.0) 99.0 999.0	19.2 (39.2) 99.2 999.2	19.2 (39.2) 99.2 999.2	19.3 (39.3) 99.3 999.3	19.3 (39.3) 99.3 999.3	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.5 (39.5) 99.5 999.5	19.5 (39.5) 99.5 999.5
3	10.13 (17.4) 34.1 167.0	9.55 (16.0) 30.8 148.5	9.28 (15.4) 29.5 141.1	9.12 (15.1) 28.7 137.1	9.01 (14.9) 28.2 134.6	8.94 (14.7) 27.9 132.8	8.89 (14.6) 27.7 131.5	8.85 (14.5) 27.5 130.6	8.79 (14.4) 27.2 129.2	8.74 (14.3) 27.1 128.3	8.64 (14.1) 26.8 125.9	8.53 (13.9) 26.1 123.5
4	7.71 (12.22) 21.2 74.14	6.94 (10.65) 18.0 61.25	6.59 (9.98) 16.7 56.18	6.39 (9.60) 16.0 53.44	6.26 (9.36) 15.5 51.71	6.16 (9.20) 15.2 50.53	6.09 (9.07) 15.0 49.66	6.04 (8.98) 14.8 49.00	5.96 (8.84) 14.5 48.05	5.91 (8.75) 14.4 47.41	5.77 (8.51) 13.9 45.77	5.63 (8.26) 13.5 44.05
5	6.61 (10.01) 16.26 47.18	5.79 (8.43) 13.27 37.12	5.41 (7.76) 12.06 33.20	5.19 (7.39) 11.39 31.09	5.05 (7.15) 10.97 29.75	4.95 (6.98) 10.67 28.83	4.88 (6.85) 10.46 28.16	4.82 (6.76) 10.29 27.65	4.74 (6.62) 10.05 26.92	4.68 (6.52) 9.89 26.42	4.53 (6.28) 9.47 25.14	4.36 (6.02) 9.02 23.79
6	5.99 (8.81) 13.74 35.51	5.14 (7.26) 10.92 27.00	4.76 (6.60) 9.78 23.70	4.53 (6.23) 9.15 21.92	4.39 (5.99) 8.75 20.80	4.28 (5.82) 8.47 20.03	4.21 (5.70) 8.26 19.46	4.15 (5.60) 8.10 19.03	4.08 (5.46) 7.87 18.41	4.00 (5.37) 7.72 17.99	3.84 (5.12) 7.31 16.90	3.67 (4.85) 6.88 15.75
7	5.59 (8.07) 12.25 29.25	4.74 (6.54) 9.55 21.69	4.35 (5.89) 8.45 18.77	4.12 (5.52) 7.85 17.20	3.97 (5.29) 7.46 16.21	3.87 (5.12) 7.19 15.52	3.79 (4.99) 6.99 15.02	3.73 (4.90) 6.84 14.63	3.64 (4.76) 6.62 14.08	3.57 (4.67) 6.47 13.71	3.41 (4.42) 6.07 12.73	3.23 (4.14) 5.65 11.70
8	5.32 (7.57) 11.26 25.42	4.46 (6.06) 8.65 18.49	4.07 (5.42) 7.59 15.83	3.84 (5.05) 7.01 14.39	3.69 (4.82) 6.63 13.48	3.58 (4.65) 6.37 12.86	3.50 (4.53) 6.18 12.40	3.44 (4.43) 6.03 12.05	3.35 (4.30) 5.81 11.54	3.28 (4.20) 5.67 11.19	3.12 (3.95) 5.28 10.30	2.93 (3.67) 4.86 9.34
9	5.12 (7.21) 10.56 22.86	4.26 (5.71) 8.02 16.39	3.86 (5.08) 6.99 13.90	3.63 (4.72) 6.42 12.56	3.48 (4.48) 6.06 11.71	3.37 (4.32) 5.80 11.13	3.29 (4.20) 5.61 10.69	3.23 (4.10) 5.47 10.37	3.14 (3.96) 5.26 9.87	3.07 (3.87) 5.11 9.57	2.90 (3.61) 4.73 8.72	2.71 (3.33) 4.31 7.81
10	4.96 (6.94) 10.04 21.04	4.10 (5.46) 7.56 14.91	3.71 (4.83) 6.55 12.55	3.48 (4.47) 5.99 11.28	3.33 (4.24) 5.64 10.48	3.22 (4.07) 5.39 9.93	3.14 (3.95) 5.20 9.52	3.07 (3.85) 5.06 9.20	2.98 (3.72) 4.85 8.74	2.91 (3.62) 4.71 8.44	2.74 (3.37) 4.33 7.64	2.54 (3.08) 3.91 6.76
11	4.84 (6.72) 9.65 19.69	3.96 (5.26) 7.21 13.81	3.59 (4.63) 6.22 11.56	3.36 (4.28) 5.67 10.35	3.20 (4.04) 5.32 9.58	3.09 (3.88) 5.07 9.05	3.01 (3.76) 4.89 8.66	2.95 (3.66) 4.74 8.35	2.85 (3.53) 4.54 7.92	2.79 (3.43) 4.40 7.63	2.61 (3.17) 4.02 6.85	2.40 (2.88) 3.60 6.00
12	4.75 (6.55) 9.33 18.64	3.89 (5.10) 6.93 12.97	3.49 (4.47) 5.95 10.80	3.26 (4.12) 5.41 9.63	3.11 (3.89) 5.06 8.89	3.00 (3.73) 4.82 8.38	2.91 (3.61) 4.64 8.00	2.85 (3.51) 4.50 7.71	2.75 (3.37) 4.30 7.29	2.69 (3.28) 4.16 7.00	2.51 (3.02) 3.78 6.25	2.30 (2.72) 3.36 5.42
13	4.67 (6.41) 9.07 17.82	3.81 (4.97) 6.70 12.31	3.41 (4.35) 5.74 10.21	3.18 (4.00) 5.21 9.07	3.03 (3.77) 4.86 8.35	2.92 (3.60) 4.62 7.86	2.83 (3.48) 4.44 7.49	2.77 (3.39) 4.30 7.21	2.67 (3.25) 4.10 6.80	2.60 (3.15) 3.96 6.52	2.42 (2.89) 3.59 5.76	2.21 (2.60) 3.17 4.97

* Entries marked thus must be multiplied by 100

2	1	2	3	4	5	6	7	8	10	12	24	∞
14	4.60 (6.30) 8.86 17.14	3.74 (4.86) 6.51 11.78	3.34 (4.24) 5.56 9.73	3.11 (3.89) 5.04 8.62	2.96 (3.66) 4.70 7.92	2.85 (3.50) 4.46 7.44	2.76 (3.38) 4.28 7.08	2.70 (3.29) 4.14 6.80	2.60 (3.15) 3.94 6.40	2.53 (3.05) 3.80 6.13	2.35 (2.79) 3.43 5.41	2.13 (2.49) 3.00 4.60
16	4.49 (6.12) 8.53 16.12	3.63 (4.69) 6.23 10.97	3.24 (4.08) 5.29 9.01	3.01 (3.73) 4.77 7.94	2.85 (3.50) 4.44 7.27	2.74 (3.34) 4.20 6.80	2.66 (3.22) 4.03 6.46	2.59 (3.12) 3.89 6.19	2.49 (2.99) 3.69 5.81	2.42 (2.89) 3.55 5.55	2.24 (2.63) 3.18 4.85	2.01 (2.32) 2.75 4.06
18	4.41 (5.98) 8.29 15.38	3.55 (4.56) 6.01 10.39	3.16 (3.95) 5.09 8.49	2.93 (3.61) 4.58 7.46	2.77 (3.38) 4.25 6.81	2.66 (3.22) 4.01 6.35	2.58 (3.10) 3.84 6.02	2.51 (3.01) 3.71 5.76	2.41 (2.87) 3.51 5.39	2.34 (2.77) 3.37 5.13	2.15 (2.50) 3.00 4.45	1.92 (2.19) 2.57 3.67
20	4.35 (5.87) 8.10 14.82	3.49 (4.46) 5.85 9.95	3.10 (3.86) 4.94 8.10	2.87 (3.51) 4.43 7.10	2.71 (3.29) 4.10 6.46	2.60 (3.13) 3.87 6.02	2.51 (3.01) 3.70 5.69	2.45 (2.91) 3.56 5.44	2.35 (2.77) 3.37 5.08	2.28 (2.68) 3.23 4.82	2.08 (2.41) 2.86 4.15	1.84 (2.09) 2.42 3.38
22	4.30 (5.79) 7.95 14.38	3.44 (4.38) 5.72 9.61	3.05 (3.78) 4.82 7.80	2.82 (3.44) 4.31 6.81	2.66 (3.22) 3.99 6.19	2.55 (3.05) 3.76 5.76	2.46 (2.93) 3.59 5.44	2.40 (2.84) 3.45 5.19	2.30 (2.70) 3.26 4.83	2.23 (2.60) 3.12 4.58	2.03 (2.33) 2.75 3.92	1.78 (2.00) 2.31 3.15
24	4.26 (5.72) 7.82 14.03	3.40 (4.32) 5.61 9.34	3.01 (3.72) 4.72 7.55	2.78 (3.38) 4.22 6.59	2.62 (3.15) 3.90 5.98	2.51 (2.99) 3.67 5.55	2.42 (2.87) 3.50 5.23	2.36 (2.78) 3.36 4.99	2.25 (2.64) 3.17 4.64	2.18 (2.54) 3.03 4.39	1.98 (2.27) 2.66 3.74	1.73 (1.94) 2.21 2.97
26	4.23 (5.66) 7.72 13.74	3.37 (4.27) 5.53 9.12	2.98 (3.67) 4.64 7.36	2.74 (3.33) 4.14 6.41	2.59 (3.10) 3.82 5.80	2.47 (2.94) 3.59 5.38	2.39 (2.82) 3.42 5.07	2.32 (2.73) 3.29 4.83	2.22 (2.59) 3.09 4.48	2.15 (2.49) 2.96 4.24	1.95 (2.22) 2.58 3.59	1.69 (1.88) 2.13 2.82
28	4.20 (5.61) 7.64 13.50	3.34 (4.22) 5.45 8.93	2.95 (3.63) 4.57 7.19	2.71 (3.29) 4.07 6.25	2.56 (3.06) 3.75 5.66	2.45 (2.90) 3.53 5.24	2.36 (2.78) 3.36 4.93	2.29 (2.69) 3.23 4.69	2.19 (2.55) 3.03 4.35	2.12 (2.45) 2.90 4.11	1.91 (2.17) 2.52 3.46	1.65 (1.83) 2.06 2.69
30	4.17 (5.57) 7.56 13.29	3.32 (4.18) 5.39 8.77	2.92 (3.59) 4.51 7.05	2.69 (3.25) 4.02 6.12	2.53 (3.03) 3.70 5.53	2.42 (2.87) 3.47 5.12	2.33 (2.75) 3.17 4.82	2.27 (2.65) 3.17 4.58	2.16 (2.51) 2.98 4.24	2.09 (2.41) 2.84 4.00	1.89 (2.14) 2.47 3.36	1.62 (1.79) 2.01 2.59
40	4.08 (5.42) 7.31 12.61	3.23 (4.05) 5.18 8.25	2.84 (3.46) 4.31 6.59	2.61 (3.13) 3.83 5.70	2.45 (2.90) 3.51 5.13	2.34 (2.74) 3.29 4.73	2.25 (2.62) 3.12 4.44	2.18 (2.53) 2.99 4.21	2.08 (2.39) 2.80 3.87	2.00 (2.29) 2.66 3.64	1.79 (2.01) 2.29 3.01	1.51 (1.64) 1.80 2.23
60	4.00 (5.29) 7.08 11.97	3.15 (3.93) 4.98 7.77	2.76 (3.34) 4.13 6.17	2.53 (3.01) 3.65 5.31	2.37 (2.79) 3.34 4.76	2.25 (2.63) 3.12 4.37	2.17 (2.51) 2.95 4.09	2.10 (2.41) 2.82 3.86	1.99 (2.27) 2.63 3.54	1.92 (2.17) 2.50 3.32	1.70 (1.88) 2.12 2.69	1.39 (1.48) 1.60 1.89
120	3.92 (5.15) 6.85 11.38	3.07 (3.80) 4.79 7.32	2.68 (3.23) 3.95 5.78	2.45 (2.89) 3.48 4.95	2.29 (2.67) 3.17 4.42	2.18 (2.52) 2.96 4.04	2.09 (2.39) 2.79 3.77	2.02 (2.30) 2.66 3.55	1.91 (2.16) 2.47 3.24	1.83 (2.05) 2.34 3.02	1.61 (1.76) 1.95 2.40	1.25 (1.31) 1.38 1.54
∞	3.84 (5.02) 6.63 10.83	3.00 (3.69) 4.61 6.91	2.60 (3.12) 3.78 5.42	2.37 (2.79) 3.32 4.62	2.21 (2.57) 3.02 4.10	2.10 (2.41) 2.80 3.74	2.01 (2.29) 2.64 3.47	1.94 (2.19) 2.51 3.27	1.83 (2.05) 2.33 2.96	1.75 (1.94) 2.18 2.74	1.52 (1.64) 1.79 2.13	1.00 (1.00) 1.00 1.00

CHI SQUARE DISTRIBUTION



$\alpha =$	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001	$=\alpha$
V = 1	0.0000393	0.000157	0.000628	0.000982	0.00393	0.0158	0.0642	0.1642	0.2706	0.3841	0.5024	0.5412	0.635	0.7879	1.0827	1.3815	1.6268	18.465	20.517	22.457	V = 1
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.605	0.779	0.988	1.143	1.1668	13.277	14.860	18.465	20.517	22.457	24.322	26.125	27.877	2
3	0.0717	0.115	0.185	0.216	0.352	0.584	0.831	1.145	1.610	2.343	3.219	4.605	5.991	7.378	8.784	9.210	10.597	12.838	14.860	16.268	3
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	2.362	3.357	4.605	5.991	7.378	8.784	9.210	10.597	12.838	14.860	16.268	18.465	20.517	4
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	3.219	4.605	5.991	7.378	8.784	9.210	10.597	12.838	14.860	16.268	18.465	20.517	22.457	5
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	4.180	5.591	7.378	9.488	11.070	12.832	13.388	15.086	16.750	18.548	20.278	21.955	23.687	6
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	5.024	6.349	8.032	9.889	11.828	13.388	14.668	16.013	17.459	18.475	20.278	21.955	23.687	7
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.989	7.779	9.488	11.070	12.832	13.388	14.668	16.013	17.459	18.475	20.278	21.955	23.687	8
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	6.898	8.784	10.655	12.592	14.449	15.033	16.622	18.168	19.679	20.909	22.027	23.589	25.188	9
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	7.779	9.488	11.070	12.832	14.449	15.033	16.622	18.168	19.679	20.909	22.027	23.589	25.188	10
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	8.631	10.371	12.196	14.013	15.985	16.618	18.275	19.675	21.026	22.161	23.185	24.225	25.188	11
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	9.581	11.338	13.270	15.013	16.919	17.535	19.168	20.509	21.666	22.618	23.589	24.601	25.188	12
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	10.371	12.196	14.013	15.985	17.828	18.475	20.086	21.337	22.459	23.361	24.322	25.188	26.217	13
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	11.211	13.036	14.841	16.736	18.561	19.219	20.757	21.999	23.027	23.833	24.753	25.188	26.125	14
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	12.151	14.007	15.812	17.696	19.541	20.219	21.678	22.901	23.901	24.753	25.666	26.125	27.001	15
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	13.005	14.841	16.736	18.561	20.413	21.101	22.561	23.785	24.753	25.666	26.579	27.001	27.999	16
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	13.841	15.676	17.541	19.376	21.211	21.901	23.361	24.585	25.553	26.466	27.379	27.801	28.799	17
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	14.691	16.506	18.311	20.146	21.981	22.671	24.131	25.355	26.323	27.236	28.149	28.571	29.569	18
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	15.561	17.376	19.181	20.996	22.831	23.521	24.981	26.205	27.173	28.086	28.999	29.421	30.419	19
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	16.438	18.298	20.103	21.918	23.753	24.443	25.903	27.127	28.095	28.999	29.912	30.334	31.332	20
21	8.034	8.897	9.915	10.283	11.591	13.240	15.145	17.011	18.871	20.676	22.491	24.326	25.016	26.476	27.700	28.668	29.571	30.484	30.896	31.894	21
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	18.171	20.031	21.846	23.661	25.496	26.206	27.666	28.890	29.858	30.761	31.674	32.086	33.084	22
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	19.242	21.067	22.882	24.697	26.532	27.242	28.702	29.926	30.894	31.807	32.720	33.132	34.130	23
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	20.157	22.002	23.817	25.632	27.467	28.177	29.637	30.861	31.829	32.742	33.655	34.067	35.065	24
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	20.965	22.810	24.625	26.440	28.275	28.985	30.445	31.669	32.637	33.550	34.463	35.376	36.374	25
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	21.795	23.640	25.455	27.270	29.105	29.815	31.275	32.499	33.467	34.380	35.293	36.206	37.119	26
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	22.592	24.442	26.257	28.072	29.907	30.617	32.077	33.301	34.269	35.182	36.095	37.008	37.921	27
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	23.427	25.239	27.054	28.869	30.704	31.414	32.874	34.098	35.066	35.979	36.892	37.805	38.718	28
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	24.319	26.130	27.945	29.760	31.595	32.305	33.765	34.989	35.957	36.870	37.783	38.696	39.609	29
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	25.250	27.061	28.876	30.691	32.526	33.236	34.696	35.920	36.888	37.801	38.714	39.627	40.540	30
40	20.706	22.164	23.838	24.433	26.509	29.051	32.345	35.639	38.933	42.227	45.521	48.815	50.423	52.967	55.411	57.955	60.499	63.043	65.587	68.131	40
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	45.164	48.879	52.584	56.289	59.994	62.538	65.082	67.626	70.170	72.714	75.258	77.802	80.346	50
60	35.535	37.485	39.699	40.482	43.188	46.459	50.641	54.326	58.011	61.696	65.381	69.066	71.610	74.154	76.698	79.242	81.786	84.330	86.874	89.418	60
70	43.275	45.442	47.893	48.758	51.739	55.329	59.898	63.971	67.944	71.917	75.890	79.863	82.407	84.951	87.495	89.939	92.383	94.827	97.271	99.715	70
80	51.171	53.539	56.213	57.153	60.391	64.278	69.207	73.630	77.503	81.376	85.249	89.122	91.666	94.210	96.754	99.298	101.842	104.386	106.930	109.474	80
90	59.196	61.754	64.634	65.646	69.126	73.291	78.558	82.981	86.854	90.727	94.599	98.472	100.916	103.460	105.904	108.348	110.792	113.236	115.680	118.124	90
100	67.327	70.065	73.142	74.222	77.929	82.358	87.945	92.768	96.641	100.514	104.387	108.260	110.704	113.248	115.692	118.136	120.580	123.024	125.468	127.912	100