



UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF CHEMICAL & BIOENGINEERING
TECHNOLOGY

FINAL EXAMINATION
OCTOBER 2025 SEMESTER

COURSE CODE	: CQB10703
COURSE NAME	: ENGINEERING STATISTICS
PROGRAMME NAME (FOR MPU: PROGRAMME LEVEL)	: BACHELOR OF FOOD SAFETY AND QUALITY TECHNOLOGY
DATE	: 24 JANUARY 2026
TIME	: 2:00 PM – 5:00 PM
DURATION	: 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please read **CAREFULLY** the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of **TWO (2)** sections; Section A and Section B.
4. Answer **ALL** questions in Section A. For Section B, answer **THREE (3)** questions **ONLY**.
5. Please write your answers on the answer booklet provided.
6. Answer all questions in English language **ONLY**.
7. Formulas and distribution tables are appended for your reference.

THERE ARE 12 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL questions.**Please use the answer booklet provided.****Question 1**

Consider the data (Table 1) obtained from a chemical process where the yield of the process is thought to be related to the reaction temperature.

Table 1: Data on a chemical process.

Temperature, x	Yield, y
50	122
53	118
54	128
55	121
56	125
59	136
62	144
65	142
67	149
71	161
72	167

- (a) Assuming that a simple linear regression model is appropriate, fit the regression model relating the yield to the temperature. (13 marks)
- (b) Estimate the value of Yield, given Temperature is 70. (2 marks)
- (c) Calculate the value of correlation coefficient, r and interpret the value. (5 marks)

Question 2

- (a) A consumer products company is formulating a new shampoo and is interested in foam height (in mm). Foam height is approximately normally distributed, with standard deviation of 20 mm. A sample of 10 products were tested with mean 200 mm. The output of related analysis is shown in Figure 1.

```

One-sample z-Test

data: Summarized x
z = A, p-value = 7.723e-05
alternative hypothesis: true mean is not equal to 175
95 percent confidence interval:
 187.6041 212.3959
sample estimates:
mean of x
      200

```

Figure 1: Output from R software.

- i. Determine the value of A. (2 marks)
 - ii. Test the hypothesis at 5% significance level, whether the foam height is not equal to 175 mm. (Hint: use p-value) (4 marks)
 - iii. Show that the 95% confidence interval of the mean is as shown in the output. (3 marks)
- (b) Two types of plastic are suitable for use by a food packaging manufacturer. The breaking strength of this plastic is important. It is known that $\sigma_1 = \sigma_2 = 7 \text{ kN/m}^2$. From a random sample of size $n_1 = 10$ and $n_2 = 12$, the technician obtained $\bar{x}_1 = 162.5$ and $\bar{x}_2 = 155.0$. The company will not adopt Plastic 1 unless its mean breaking strength exceeds that of Plastic 2. Figure 2 shows the output generated for the related analysis.

```

Two-sample z-Test

data: Summarized x and y
z = A, p-value = 0.006169
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 [redacted]
sample estimates:
mean of x mean of y
 162.5      155.0

```

Figure 2: Output from R software.

- i. Determine the value of A.
(3 marks)
- ii. Test the hypothesis at 5% significance level, whether the mean breaking strength of Plastic 1 exceeds Plastic 2. (Hint: use p-value)
(4 marks)
- iii. Calculate the 95% confidence interval on the difference in means breaking strength.
(4 marks)

SECTION B (Total: 60 marks)

INSTRUCTION: Answer THREE (3) questions ONLY.

Please use the answer booklet provided.

Question 3

- (a) In a study, it is suspected that strength is related to the percentage of cotton in the fiber. Five levels of cotton percentage are used, and five replicates are run in random order. The data is presented in Table 2.

Table 2: The measurements of breaking strengths using five levels of cotton percentage.

Cotton Percentage	Observations					Σx_i	Σx_i^2
	1	2	3	4	5		
15	7	7	15	11	9	49	525
20	12	17	12	18	18	77	1225
25	14	18	18	19	19	88	1566
30	19	25	22	19	23	108	2360
35	7	10	11	15	11	54	616

Test at 1% significance level, does the cotton percentage affect the breaking strength.

(10 marks)

- (b) The viscosity of two different brands of car oil was measured. Brand 1 included six samples, and Brand 2 included seven samples. The objective was to assess the difference in viscosity between the two brands. The output of related analysis is shown in Figure 3.

Two Sample t-test	
data: Brand1 and Brand2	
t = 0.40561, df = A, p-value = 0.6928	
alternative hypothesis: true difference in means is not equal to 0	
95 percent confidence interval:	
-0.08325831 0.12087736	
sample estimates:	
mean of x	mean of y
10.57167	10.55286

Figure 3: Output from R software.

- i. Determine the value of A.

(2 marks)

- ii. Show the calculation of the 95% confidence interval is as shown in the output.
(Given the Pooled Variance, $S^2_p = 0.006946$)
(4 marks)
- iii. Test at 5% significance level, is there a difference in viscosity between the two brands. (Hint: use p-value)
(4 marks)

Question 4

- (a) A machine produces metal rods used in an automobile suspension system. A random sample of 12 rods is selected, and the diameter is measured. The resulting data in millimeters are shown in Table 3.

Table 3: The diameter of 12 rods.

8.23	8.31	8.42
8.29	8.19	8.24
8.19	8.29	8.30
8.14	8.32	8.40

Test the hypothesis at 5% significance level to indicate that mean rod diameter is not 8.2mm.

(10 marks)

- (b) An experiment in which the amount of radon released in showers was investigated. Radon-enriched water was used in the experiment, and three different orifice diameters were tested in shower heads. For each diameter, six samples of reading were taken. The researcher was interested to see whether there is a difference between orifice diameters. The related analysis is shown in Figure 4.

	Df	Sum Sq	Mean Sq	F value	Pry(>F)						
Diameter	A	160.8	80.39	C	5.21e-05 ***						
Residuals	15	59.0	B								

Signif. codes:	0	'***'	0.001	'**'	0.01	'*'	0.05	'.'	0.1	' '	1

Figure 4: Output from R software.

- i. Determine the values of A, B and C.
(6 marks)
- ii. Test at 5% significance level, whether there is a difference between orifice diameters. (Hint: use p-value)
(4 marks)

Question 5

- (a) Jar of honey are filled by a machine. It has been found that the quantity of honey in the jar has mean 460.3 g and standard deviation 3.2 g. A random sample of 60 jars is taken and the mean quantity of honey per jar is found to be 461.2 g.

- i. Test at 5% significance level, the hypothesis that the mean quantity of honey may have changed from 460.3 g.

(6 marks)

- ii. Construct a 90% confidence interval on the mean quantity of honey.

(4 marks)

- (b) A study was made on the amount of converted sugar (Y) in a certain process at various temperatures (X). The output of related analysis is shown in Figure 5.

```
Call:
lm(formula = Sugar ~ Temperature)

Residuals:
    Min       1Q   Median       3Q      Max
-0.7082 -0.4868 -0.1227  0.5109  1.0346

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   6.4136     0.9246   6.936 6.79e-05 ***
Temperature   1.8091     0.6032   2.999  0.015 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6326 on 9 degrees of freedom
Multiple R-squared:  0.4999,    Adjusted R-squared:  0.4443
F-statistic: 8.996 on 1 and 9 DF,  p-value: 0.01497
```

Figure 5: Output from R software.

- i. Write the full regression equation based on the output.
- (1 mark)
- ii. Estimate the mean amount of converted sugar produced when the coded temperature is 1.75.
- (2 marks)
- iii. Determine the correlation coefficient, r of the given output and comment on the value.
- (3 marks)
- iv. State the value of standard error estimate, S_e and comment on the value.
- (2 marks)
- v. Test for significance of the slope, β_1 at 5% significance level. (Hint: use p-value)
- (2 marks)

Question 6

- (a) An article describes the effect of delamination on the natural frequency of beams made from composite laminates. Five such delaminated beams were subjected to loads, and the resulting frequencies were as follows (in hertz):

230.66 233.05 232.58 229.48 232.58

Test at 1% significance level that the mean natural frequency is less than 240.

(10 marks)

- (b) A chemical engineer is studying the effects of various reagents and catalysts on the yield of a certain process. Yield is expressed as a percentage of a theoretical maximum. Four runs of the process were made for each combination of 3 reagents and 4 catalysts. The output of related analysis is shown in Figure 6.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
catalyst	A	877.6	292.52	9.358	0.000104	***
reagent	2	327.1		B	5.233	0.010118 *
catalyst:reagent	6	157.0	26.16		C	0.549603
Residuals	36	1125.3	31.26			

Signif. codes:	0	'***'	0.001	'**'	0.01	'*' 0.05 '.' 0.1 ' ' 1

Figure 6: Output from R software.

- Determine the values of A, B and C.
(6 marks)
- Test at 5% significance level, is there an effect due to reagent. (Hint: use p-value)
(4 marks)

END OF EXAMINATION PAPER

APPENDIX

Sample Mean	$\bar{x} = \frac{\sum x_i}{n}$	Sample Standard deviation	$s = \sqrt{\frac{(\sum x_i^2) - \frac{(\sum x_i)^2}{n}}{n-1}}$
Confidence interval (One Sample)	CASE 1 : σ known $\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ CASE 2 : σ unknown ($n \geq 30$) $\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$ CASE 3 : σ unknown ($n < 30$) $\bar{X} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}$		
Confidence interval (Two Samples)	CASE 1 : σ known $(\bar{X}_1 - \bar{X}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ CASE 2 : σ unknown ($n \geq 30$) $(\bar{X}_1 - \bar{X}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$		
Test statistic (One Sample)	CASE 1 : σ known $Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ CASE 2 : σ unknown ($n \geq 30$) $Z = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$ CASE 3 : σ unknown ($n < 30$) $T = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$		
Test statistic (Two Samples)	CASE 1 : σ known $Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ CASE 2 : σ unknown ($n \geq 30$) $Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$		

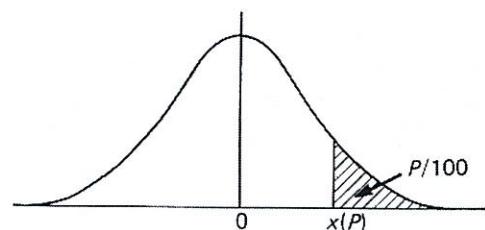
Analysis of Variance (ANOVA)	$SSA = \sum \left[\frac{(\sum x_i)^2}{n_i} \right] - \frac{(\sum \sum x_{ij})^2}{n}$ $SST = \sum \sum x_{ij}^2 - \frac{(\sum \sum x_{ij})^2}{n}$ $SSE = SST - SSA$
Regression model	$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ $S_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$ $S_{yy} = \sum y_i^2 - \frac{(\sum y_i)^2}{n}$ $S_{xy} = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}$ $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ <i>Correlation coefficient</i> $r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$ <i>Standard error estimate</i> $s_e = \sqrt{\frac{S_{yy} - \hat{\beta}_1 S_{xy}}{n-2}}$

PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}} \int_{x(P)}^{\infty} e^{-t^2/2} dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geq x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geq x(P)$ is $2P/100$.



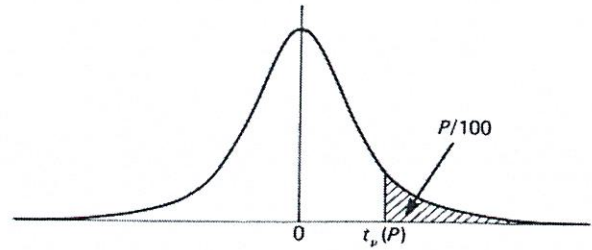
P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0.0000	5.0	1.6449	3.0	1.8808	2.0	2.0537	1.0	2.3263	0.10	3.0902
45	0.1257	4.8	1.6646	2.9	1.8957	1.9	2.0749	0.9	2.3656	0.09	3.1214
40	0.2533	4.6	1.6849	2.8	1.9110	1.8	2.0969	0.8	2.4089	0.08	3.1559
35	0.3853	4.4	1.7060	2.7	1.9268	1.7	2.1201	0.7	2.4573	0.07	3.1947
30	0.5244	4.2	1.7279	2.6	1.9431	1.6	2.1444	0.6	2.5121	0.06	3.2389
25	0.6745	4.0	1.7507	2.5	1.9600	1.5	2.1701	0.5	2.5758	0.05	3.2905
20	0.8416	3.8	1.7744	2.4	1.9774	1.4	2.1973	0.4	2.6521	0.04	3.7190
15	1.0364	3.6	1.7991	2.3	1.9954	1.3	2.2262	0.3	2.7478	0.005	3.8906
10	1.2816	3.4	1.8250	2.2	2.0141	1.2	2.2571	0.2	2.8782	0.001	4.2649
5	1.6449	3.2	1.8522	2.1	2.0335	1.1	2.2904	0.1	3.0902	0.0005	4.4172

PERCENTAGE POINTS OF THE t -DISTRIBUTION

This table gives percentage points $t_p(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_p(P)}^{\infty} \frac{dt}{(1+t^2/\nu)^{(\nu+1)/2}}$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2 -distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t -distribution with ν degrees of freedom, and the probability that $t \geq t_p(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_p(P)$, and the probability that $|t| \geq t_p(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P	40	30	25	20	15	10	5	2.5	1	0.5	0.1	0.05
$\nu = 1$	0.3249	0.7265	1.0000	1.3764	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.2887	0.6172	0.8165	1.0607	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.2767	0.5844	0.7649	0.9785	1.250	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.2707	0.5686	0.7407	0.9410	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.2672	0.5594	0.7267	0.9195	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.2648	0.5534	0.7176	0.9057	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.2632	0.5491	0.7111	0.8960	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.2619	0.5459	0.7064	0.8889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.2610	0.5435	0.7027	0.8834	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.2602	0.5415	0.6998	0.8791	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.2596	0.5399	0.6974	0.8755	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.2590	0.5386	0.6955	0.8726	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.2586	0.5375	0.6938	0.8702	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.2582	0.5366	0.6924	0.8681	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.2579	0.5357	0.6912	0.8662	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.2576	0.5350	0.6901	0.8647	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.2573	0.5344	0.6892	0.8633	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.2571	0.5338	0.6884	0.8620	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.2569	0.5333	0.6876	0.8610	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.2567	0.5329	0.6870	0.8600	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.2566	0.5325	0.6864	0.8591	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.2564	0.5321	0.6858	0.8583	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.2563	0.5317	0.6853	0.8575	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.2562	0.5314	0.6848	0.8569	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.2561	0.5312	0.6844	0.8562	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.2560	0.5309	0.6840	0.8557	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.2559	0.5306	0.6837	0.8551	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.2558	0.5304	0.6834	0.8546	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.2557	0.5302	0.6830	0.8542	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.2556	0.5300	0.6828	0.8538	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	0.2555	0.5297	0.6822	0.8530	1.054	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	0.2553	0.5294	0.6818	0.8523	1.052	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	0.2552	0.5291	0.6814	0.8517	1.052	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	0.2551	0.5288	0.6810	0.8512	1.051	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	0.2550	0.5286	0.6807	0.8507	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.2547	0.5278	0.6794	0.8489	1.047	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.2545	0.5272	0.6786	0.8477	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.2539	0.5258	0.6765	0.8446	1.041	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.2533	0.5244	0.6745	0.8416	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291

PERCENTAGE POINTS OF THE F DISTRIBUTION

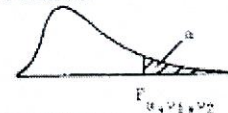
The table gives the values of $F_{\alpha; \nu_1, \nu_2}$ the 100 α percentage point of the F distribution having ν_1 degrees of freedom in the numerator and ν_2 degrees of freedom in the denominator.

For each pair of values of ν_1 and ν_2 , $F_{\alpha; \nu_1, \nu_2}$ is tabulated for $\alpha = 0.05, 0.025, 0.01, 0.001$, the 0.025 values being bracketed.

The lower percentage points of the distribution may be obtained from the relation:-

$$F_{1-\alpha; \nu_1, \nu_2} = 1/F_{\alpha; \nu_2, \nu_1}$$

$$\text{e.g. } F_{0.95; 12, 8} = 1/F_{0.05; 8, 12} = 1/2.85 = 0.351$$



$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	10	12	24	∞
1	161.4 (548) 4052 4053*	199.5 (800) 5000 5000*	215.7 (864) 5403 5404*	224.6 (900) 5625 5625*	230.2 (922) 5764 5764*	234.0 (937) 5859 5859*	236.8 (948) 5928 5929*	238.9 (957) 5981 5981*	241.9 (969) 6056 6056*	243.9 (977) 6106 6107*	249.0 (997) 6235 6235*	254.3 (1018) 6366 6366*
2	18.5 (38.5) 98.5 998.5	19.0 (39.0) 99.0 999.0	19.2 (39.2) 99.2 999.2	19.2 (39.2) 99.2 999.2	19.3 (39.3) 99.3 999.3	19.3 (39.3) 99.3 999.3	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.5 (39.5) 99.5 999.5	19.5 (39.5) 99.5 999.5
3	10.13 (17.4) 34.1 167.0	9.55 (16.0) 30.8 148.5	9.28 (15.4) 29.5 141.1	9.12 (15.1) 28.7 137.1	9.01 (14.9) 28.2 134.6	8.94 (14.7) 27.9 132.8	8.89 (14.6) 27.7 131.5	8.85 (14.5) 27.5 130.6	8.79 (14.4) 27.2 129.2	8.74 (14.3) 27.1 128.3	8.64 (14.1) 26.6 125.9	8.53 (13.9) 26.1 123.5
4	7.71 (12.22) 21.2 74.14	6.94 (10.65) 18.0 61.25	6.59 (9.98) 16.7 56.18	6.39 (9.60) 16.0 53.44	6.26 (9.36) 15.5 51.71	6.16 (9.20) 15.2 50.53	6.09 (9.07) 15.0 49.66	6.04 (8.98) 14.8 49.00	5.96 (8.84) 14.5 48.05	5.91 (8.75) 14.4 47.41	5.77 (8.51) 13.9 45.77	5.63 (8.26) 13.5 44.05
5	6.61 (10.01) 16.26 47.18	5.79 (8.43) 13.27 37.12	5.41 (7.78) 12.06 33.20	5.19 (7.39) 11.39 31.09	5.05 (7.15) 10.97 29.75	4.95 (6.98) 10.67 28.83	4.88 (6.85) 10.46 28.16	4.82 (6.76) 10.29 27.65	4.74 (6.62) 10.05 26.92	4.68 (6.52) 9.89 26.42	4.53 (6.28) 9.47 25.14	4.36 (6.02) 9.02 23.79
6	5.99 (8.81) 13.74 35.51	5.14 (7.26) 10.92 27.00	4.76 (6.60) 9.78 23.70	4.53 (6.23) 9.15 21.92	4.39 (5.99) 8.75 20.80	4.28 (5.82) 8.47 20.03	4.21 (5.70) 8.26 19.46	4.15 (5.60) 8.10 19.03	4.08 (5.46) 7.87 18.41	4.00 (5.37) 7.72 17.99	3.84 (5.12) 7.31 16.90	3.67 (4.85) 6.88 15.75
7	5.59 (8.07) 12.25 29.25	4.74 (6.54) 9.55 21.69	4.35 (5.89) 8.45 18.77	4.12 (5.52) 7.85 17.20	3.97 (5.29) 7.46 16.21	3.87 (5.12) 7.19 15.52	3.79 (4.99) 6.99 15.02	3.73 (4.90) 6.84 14.63	3.64 (4.76) 6.62 14.08	3.57 (4.67) 6.47 13.71	3.41 (4.42) 6.07 12.73	3.23 (4.14) 5.65 11.70
8	5.32 (7.57) 11.26 25.42	4.46 (6.06) 8.65 18.49	4.07 (5.42) 7.59 15.83	3.84 (5.05) 7.01 14.39	3.69 (4.82) 6.63 13.48	3.58 (4.65) 6.37 12.86	3.50 (4.53) 6.18 12.40	3.44 (4.43) 6.03 12.05	3.35 (4.30) 5.81 11.54	3.28 (4.20) 5.67 11.19	3.12 (3.95) 5.28 10.30	2.93 (3.67) 4.86 9.34
9	5.12 (7.21) 10.56 22.86	4.26 (5.71) 8.02 16.39	3.86 (5.08) 6.99 13.90	3.63 (4.72) 6.42 12.56	3.48 (4.48) 6.06 11.71	3.37 (4.32) 5.80 11.13	3.29 (4.20) 5.61 10.69	3.23 (4.10) 5.47 10.37	3.14 (3.96) 5.26 9.87	3.07 (3.87) 5.11 9.57	2.90 (3.61) 4.73 8.72	2.71 (3.33) 4.31 7.81
10	4.96 (6.94) 10.04 21.04	4.10 (5.46) 7.56 14.91	3.71 (4.83) 6.55 12.55	3.48 (4.47) 5.99 11.28	3.33 (4.24) 5.64 10.48	3.22 (4.07) 5.39 9.93	3.14 (3.95) 5.20 9.52	3.07 (3.85) 5.06 9.20	2.98 (3.72) 4.85 8.74	2.91 (3.62) 4.71 8.44	2.74 (3.37) 4.33 7.64	2.54 (3.08) 3.91 6.76
11	4.84 (6.72) 9.65 19.69	3.96 (5.26) 7.21 13.81	3.59 (4.63) 6.22 11.56	3.36 (4.28) 5.67 10.35	3.20 (4.04) 5.32 9.58	3.09 (3.88) 5.07 9.05	3.01 (3.76) 4.89 8.66	2.95 (3.66) 4.74 8.35	2.85 (3.53) 4.54 7.92	2.79 (3.43) 4.40 7.63	2.61 (3.17) 4.02 6.85	2.40 (2.88) 3.60 6.00
12	4.75 (6.55) 9.33 18.64	3.89 (5.10) 6.93 12.97	3.49 (4.47) 5.95 10.80	3.26 (4.12) 5.41 9.63	3.11 (3.89) 5.06 8.89	3.00 (3.73) 4.82 8.38	2.91 (3.61) 4.64 8.00	2.85 (3.51) 4.50 7.71	2.75 (3.37) 4.30 7.29	2.69 (3.28) 4.16 7.00	2.51 (3.02) 3.78 6.25	2.30 (2.72) 3.36 5.42
13	4.67 (6.41) 9.07 17.82	3.81 (4.97) 6.70 12.31	3.41 (4.35) 5.74 10.21	3.18 (4.00) 5.21 9.07	3.03 (3.77) 4.86 8.35	2.92 (3.60) 4.62 7.86	2.83 (3.48) 4.44 7.49	2.77 (3.39) 4.30 7.21	2.67 (3.25) 4.10 6.80	2.60 (3.15) 3.96 6.52	2.42 (2.89) 3.59 5.78	2.21 (2.60) 3.17 4.97

* Entries marked thus must be multiplied by 100

$\frac{1}{2}$	1	2	3	4	5	6	7	8	10	12	24	∞
14	4.60 (6.30) 8.66 17.14	3.74 (4.86) 6.51 11.78	3.34 (4.24) 5.56 9.73	3.11 (3.89) 5.04 8.62	2.96 (3.66) 4.70 7.92	2.85 (3.50) 4.46 7.44	2.76 (3.38) 4.28 7.08	2.70 (3.29) 4.14 6.80	2.60 (3.15) 3.94 6.40	2.53 (3.05) 3.80 6.13	2.35 (2.79) 3.43 5.41	2.13 (2.49) 3.00 4.60
16	4.49 (6.12) 8.53 16.12	3.63 (4.69) 6.23 10.97	3.24 (4.08) 5.29 9.01	3.01 (3.73) 4.77 7.94	2.85 (3.50) 4.44 7.27	2.74 (3.34) 4.20 6.80	2.66 (3.22) 4.03 6.46	2.59 (3.12) 3.89 6.19	2.49 (2.99) 3.69 5.81	2.42 (2.89) 3.55 5.55	2.24 (2.63) 3.18 4.85	2.01 (2.32) 2.75 4.06
18	4.41 (5.98) 8.29 15.38	3.55 (4.56) 6.01 10.39	3.16 (3.95) 5.09 8.49	2.93 (3.61) 4.58 7.46	2.77 (3.38) 4.25 6.81	2.66 (3.22) 4.01 6.35	2.58 (3.10) 3.84 6.02	2.51 (3.01) 3.71 5.76	2.41 (2.87) 3.51 5.39	2.34 (2.77) 3.37 5.13	2.15 (2.50) 3.00 4.45	1.92 (2.19) 2.57 3.67
20	4.35 (5.87) 8.10 14.82	3.49 (4.46) 5.85 9.95	3.10 (3.86) 4.94 8.10	2.87 (3.51) 4.43 7.10	2.71 (3.29) 4.10 6.46	2.60 (3.13) 3.87 6.02	2.51 (3.01) 3.70 5.69	2.45 (2.91) 3.56 5.44	2.35 (2.77) 3.37 5.08	2.28 (2.68) 3.23 4.82	2.08 (2.41) 2.86 4.15	1.84 (2.09) 2.42 3.38
22	4.30 (5.79) 7.95 14.38	3.44 (4.38) 5.72 9.61	3.05 (3.78) 4.82 7.80	2.82 (3.44) 4.31 6.81	2.66 (3.22) 3.99 6.19	2.55 (3.05) 3.76 5.76	2.46 (2.93) 3.59 5.44	2.40 (2.84) 3.45 5.19	2.30 (2.70) 3.26 4.83	2.23 (2.60) 3.12 4.58	2.03 (2.33) 2.75 3.92	1.78 (2.00) 2.31 3.15
24	4.26 (5.72) 7.82 14.03	3.40 (4.32) 5.61 9.34	3.01 (3.72) 4.72 7.55	2.78 (3.38) 4.22 6.59	2.62 (3.15) 3.90 5.98	2.51 (2.99) 3.67 5.55	2.42 (2.87) 3.50 5.23	2.36 (2.78) 3.36 4.99	2.25 (2.64) 3.17 4.64	2.18 (2.54) 3.03 4.39	1.98 (2.27) 2.66 3.74	1.73 (1.94) 2.21 2.97
26	4.23 (5.66) 7.72 13.74	3.37 (4.27) 5.53 9.12	2.98 (3.67) 4.64 7.36	2.74 (3.33) 4.14 6.41	2.59 (3.10) 3.82 5.80	2.47 (2.94) 3.59 5.38	2.39 (2.82) 3.42 5.07	2.32 (2.73) 3.29 4.83	2.22 (2.59) 3.09 4.48	2.15 (2.49) 2.96 4.24	1.95 (2.22) 2.58 3.59	1.69 (1.88) 2.13 2.82
28	4.20 (5.61) 7.64 13.50	3.34 (4.22) 5.45 8.93	2.95 (3.63) 4.57 7.19	2.71 (3.29) 4.07 6.25	2.56 (3.06) 3.75 5.66	2.45 (2.90) 3.53 5.24	2.36 (2.78) 3.36 4.93	2.29 (2.69) 3.23 4.69	2.19 (2.55) 3.03 4.35	2.12 (2.45) 2.90 4.11	1.91 (2.17) 2.52 3.46	1.65 (1.83) 2.06 2.69
30	4.17 (5.57) 7.56 13.29	3.32 (4.18) 5.39 8.77	2.92 (3.59) 4.51 7.05	2.69 (3.25) 4.02 6.12	2.53 (3.03) 3.70 5.53	2.42 (2.87) 3.47 5.12	2.33 (2.75) 3.30 4.82	2.27 (2.65) 3.17 4.58	2.16 (2.51) 2.98 4.24	2.09 (2.41) 2.84 4.00	1.89 (2.14) 2.47 3.36	1.62 (1.79) 2.01 2.59
40	4.08 (5.42) 7.31 12.61	3.23 (4.05) 5.18 8.25	2.84 (3.46) 4.31 6.59	2.61 (3.13) 3.83 5.70	2.45 (2.90) 3.51 5.13	2.34 (2.74) 3.29 4.73	2.25 (2.62) 3.12 4.44	2.18 (2.53) 2.99 4.21	2.08 (2.39) 2.80 3.87	2.00 (2.29) 2.66 3.64	1.79 (2.01) 2.29 3.01	1.51 (1.64) 1.80 2.23
60	4.00 (5.29) 7.08 11.97	3.15 (3.93) 4.98 7.77	2.76 (3.34) 4.13 6.17	2.53 (3.01) 3.65 5.31	2.37 (2.79) 3.34 4.76	2.25 (2.63) 3.12 4.37	2.17 (2.51) 2.95 4.09	2.10 (2.41) 2.82 3.86	1.99 (2.27) 2.63 3.54	1.92 (2.17) 2.50 3.32	1.70 (1.88) 2.12 2.69	1.39 (1.48) 1.60 1.89
120	3.92 (5.15) 6.85 11.38	3.07 (3.80) 4.79 7.32	2.68 (3.23) 3.95 5.78	2.45 (2.89) 3.48 4.95	2.29 (2.67) 3.17 4.42	2.18 (2.52) 2.96 4.04	2.09 (2.39) 2.79 3.77	2.02 (2.30) 2.66 3.55	1.91 (2.16) 2.47 3.24	1.83 (2.05) 2.34 3.02	1.61 (1.76) 1.95 2.40	1.25 (1.31) 1.38 1.54
∞	3.84 (5.02) 6.63 10.83	3.00 (3.69) 4.61 6.91	2.60 (3.12) 3.78 5.42	2.37 (2.79) 3.32 4.62	2.21 (2.57) 3.02 4.10	2.10 (2.41) 2.80 3.74	2.01 (2.29) 2.64 3.47	1.94 (2.19) 2.51 3.27	1.83 (2.05) 2.32 2.96	1.75 (1.94) 2.18 2.74	1.52 (1.64) 1.79 2.13	1.00 (1.00) 1.00 1.00