UNIVERSITI KUALA LUMPUR
MALAYSIA INSTITUTE OF CHEMICAL AND BIOENGINEERING TECHNOLOGY

FINAL EXAMINATION
OCTOBER 2025 SEMESTER

COURSE CODE	:	CCB 10003
COURSE NAME	:	MATHEMATICS FOR ENGINEERS 1
PROGRAMME NAME	:	BACHELOR OF CHEMICAL ENGINEERING WITH HONOURS
DATE	:	24 JANUARY 2026
TIME	:	9.00 AM – 12.00 PM
DURATION	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper has information printed on both sides of the paper.
3. This question paper has FOUR (4) questions. Answer ALL questions.
4. Please WRITE your answers on the answer booklet provided.
5. Please answer all questions in English only.

THERE ARE 13 PAGES OF THIS QUESTION AND APPENDICES EXCLUDING THIS PAGE.

(Total: 100 marks)

INSTRUCTION: Answer ALL questions.

Please use the answer booklet provided.

QUESTION 1

(a) Consider the chemical reaction:

$$NaOH + H_2SO_4 \rightarrow Na_2SO_4 + H_2O$$

Here the elements involved are sodium hydroxide which reacts with sulfuric acid (H_2SO_4) to yield sodium sulfate (Na_2SO_4) and water. Using the chemical reactions provided above, calculate the initial quantities of each element and calculate how much of each element participates in the reaction. Apply the row-echelon form method to solve the system.

(9 marks)

(b) A chemical engineer is designing a metal feed blend for a high-temperature catalytic reactor. The blend is prepared from three commercially available alloy streams: Stainless Steel, Inconel, and Hastelloy. The engineer is interested in controlling the mass fractions of iron (Fe), chromium (Cr), and nickel (Ni) in the final feed, as these metals influence catalyst performance and corrosion resistance.

Laboratory analysis provides the fractional content by weight of Fe, Cr, and Ni in each alloy, as shown in Table 1.

Table 1: Fractional content of Al, Mg and Zn for three minerals

Element	Stainless steel	Inconel	Hastelloy
Fe	0.2	0.2	0.3
Cr	0.4	0.3	0.3
Ni	0.3	0.2	0.2

If the alloys are blended in quantities of $x_1$kg of Stainless Steel, $x_2$kg of Inconel, and $x_3$kg of Hastelloy, the total masses (in kg) of iron, chromium, and nickel required in the final mixture are specified by the process design constraints. This leads to the following system of linear equations:

$$\begin{aligned} 0.2x_1 + 0.2x_2 + 0.3x_3 &= 3.9 \\ 0.4x_1 + 0.3x_2 + 0.3x_3 &= 5.2 \\ 0.3x_1 + 0.2x_2 + 0.2x_3 &= 1.8 \end{aligned}$$

(i) Rewrite the above series of equations in matrix form, $AX = B$.

(1 mark)

(ii) Calculate the weight of each element by L-U decomposition method. Show all steps of the computation.

(11 marks)

(c) The equation $Ax = \lambda x$ below has nonzero solutions for the vector x if and only if the matrix has zero determinant. Calculate the eigenvalues of the matrix A.

$$A = \begin{pmatrix} 7 & 0 & -3 \\ -9 & -2 & 3 \\ 18 & 0 & -8 \end{pmatrix}$$

(4 marks)

QUESTION 2

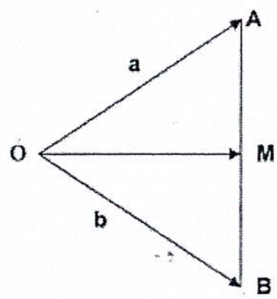

Figure 1: A triangle OAB

(a) The diagram in Figure 1 shows a triangle OAB with M is the midpoint of the line AB.

(i) Calculate $\overrightarrow{OM}$ in terms of a and b.

(2 marks)

(ii) If $\overrightarrow{OM} = i - j + 2k$ and $\overrightarrow{OB} = -i + 2j + 2k$, calculate $\overrightarrow{AM}$ in cartesian form.

(2 marks)

(iii) Hence compute the length of $\overrightarrow{AM}$.

(2 marks)

(iv) Calculate the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ correct to two decimal places.

(3 marks)

(b) Figure 2 shows a right prism with triangular ends ABC and DEF, and parallel edges AD, BE, CF. Given that A = (2, 7, -1), B = (5, 8, 2) C = (6, 7, 4) and D = (12, 1, -9).

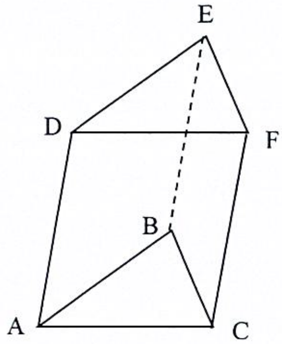

Figure 2: A right prism with triangular ends

(i) Calculate $\vec{AB} \times \vec{AC}$.

(3 marks)

(ii) Compute $\left(\vec{AB} \times \vec{AC}\right) \bullet \vec{AD}$.

(3 marks)

(iii) Hence, calculate the volume of the prism.

(2 marks)

(c) Given that $A = (1, 2, -3)$ and $B = (2, -1, 2)$ and $C = (3, 1, -1)$. Compute *A.(B X C)* and show that *A*, *B* and *C* are coplanar.

(3 marks)

(d) Evaluate the magnitude and direction of the resultant vector of the force system shown in Figure 3.

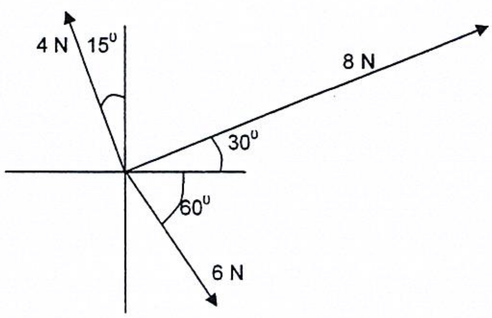

Figure 3: Magnitude and direction of the resultant vector

(5 marks)

QUESTION 3

(a) An open-top box is to be made by cutting small congruent squares from the corners of a 12-in.-by-12-in. sheet of tin and bending up the sides. Compute on how large should the squares cut from the corners be to make the box hold as much as possible.

(5 marks)

(b) A closed rectangular box (Figure 4) is to be constructed from material that cost RM 3 per square foot for the top and bottom and RM 1 per square foot its sides. If the volume of the box is 1000 cubic feet, determine the dimensions of the box *x, y, z* that will minimize the cost.

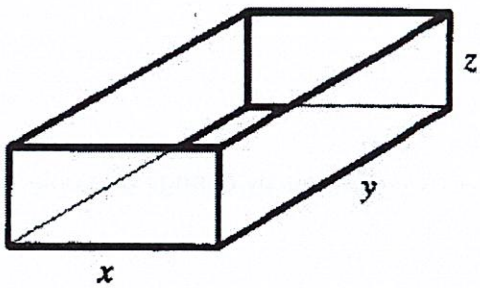

Figure 4: A closed rectangular box with dimensions x, y and z

(12 marks)

(c) An object is dropped and its position vector, r, is given by;

$$r = (t^4 - 3t^3)i + (6t^2 - 2t)j$$

Calculate;

(i) the velocity, v.

(2 marks)

(ii) the acceleration, *a*.

(2 marks)

(d) The voltage *V* in a circuit satisfies the law *V* = *IR* is slowly dropping as the battery wears out (see Figure 5). At the same time, the resistance *R* is increasing as the resistor heats up. By using the equation below, compute how much current is changing at the instant when *R* = 600 ohms, *I* = 0.04 amp, *dR/dt* = 0.5 ohm/sec, and *dV/dt* = -0.01 volt/sec.

$$\frac{\partial V}{\partial t} = \frac{\partial V}{\partial I}\frac{\partial I}{\partial t} + \frac{\partial V}{\partial R}\frac{\partial R}{\partial t}$$

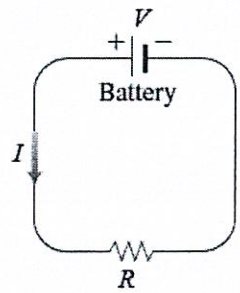

Figure 5: Electrical circuit

(4 marks)

QUESTION 4

(a) Solve the following double integrals by changing to polar coordinates.

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \cos(x^2 + y^2) dydx$$

(10 marks)

(b) Sketch the region of integration which is bounded by lines |x| + |y| = 1, and compute the following double integral.

$$\iint y - 2x^2 dydx$$

(10 marks)

(c) Sketch the region of integration that is bounded by lines y = 2, y = 0, x =3, and x = 0, and compute the following double integral.

$$\iint 4 - y^2 \, dydx$$

(5 marks)

END OF EXAMINATION QUESTIONS

APPENDICES

Differentiation	Integration
$\frac{d}{dx}a = 0, a\ is\ a\ constant$	$\int a\,dx = ax + C$
$\frac{d}{dx}\,x^n = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C\ ,\qquad n \neq -1$
$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$	$\int \frac{1}{x}\,dx = ln\lvert x\rvert + C$
$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}\,e^x = e^x$	$\int sinx\,dx = -cosx + C$
$\frac{d}{dx}lnx = \frac{1}{x}$	$\int cosx\,dx = sinx + C$
$\frac{d}{dx}(cosx) = -sinx$	$\int \sec^2 x\,dx = tanx + C$
$\frac{d}{dx}(sinx) = cosx$	$\int cosec^2x\,dx = \ -cotx + C$
$\frac{d}{dx}(tanx) = \sec^2 x$	$\int secx\,tanx\,dx = secx + C$
Product Rule: $[uv]' = uv' + vu'$ Quotient Rule: $\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$	Integration by Parts: $\int u\,dv = uv - \int v\,du$

Fundamental Identities	**Formulas for Negatives**
$\csc\theta = \dfrac{1}{sin\theta}$	$\sin(-\theta) = -sin\theta$
$\sec\theta = \dfrac{1}{cos\theta}$	$\cos(-\theta) = cos\theta$
$cot\theta = \dfrac{1}{tan\theta} = \dfrac{cos\theta}{sin\theta}$	$\tan(-\theta) = -tan\theta$
$tan\theta = \dfrac{sin\theta}{cos\theta}$	$\csc(-\theta) = -csc\theta$
$\sin^2\theta + \cos^2\theta = 1$	$\sec(-\theta) = sec\theta$
$1 + \tan^2\theta = \sec^2\theta$	$\cot(-\theta) = -cot\theta$
$1 + \cot^2\theta$	
Addition Formulas	**Substraction Formulas**
$\sin(A+B) = sinAcosB + cosAsinB$	$\sin(A-b) = sinAcosB - cosAsinB$
$\cos(A+B) = cosAcosB - sinAsinB$	$\cos(A-B) = cosAcosB + sinAsinB$
$\tan(A+B) = \dfrac{tanA + tanB}{1 - tanA\,tanB}$	$\tan(A-B) = \dfrac{tanA - tanB}{1 + tanA\,tanB}$
Half-Angle Formulas	Double-Angel Formulas
$\sin\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 - cos\theta}{2}}$	$\sin 2\theta = 2sin\theta cos\theta$
$\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 + cos\theta}{2}}$	$\cos 2\theta = 2sin\theta cos\theta$ $\cdots = 1 - 2\sin^2\theta$ $\cdots = 2\cos^2\theta - 1$
$\tan\left(\dfrac{\theta}{2}\right) = \dfrac{1 - cos\theta}{sin\theta} = \dfrac{sin\theta}{1 + cos\theta}$	$tan2\theta = \dfrac{2tan\theta}{1 - \tan^2\theta}$

Product-to-sum formulas	Sum-to-product formulas
$sin\alpha\, cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$	$sin\alpha + sin\beta = 2sin\frac{\alpha+\beta}{2}\, cos\frac{\alpha-\beta}{2}$
$cos\alpha\, sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$	$sin\alpha - sin\beta = 2cos\frac{\alpha+\beta}{2}\, sin\frac{\alpha-\beta}{2}$
$cos\alpha\, cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$	$cos\alpha + cos\beta = 2cos\frac{\alpha+\beta}{2}\, cos\frac{\alpha-\beta}{2}$
$sin\alpha\, sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$	$cos\alpha - cos\beta = -2sin\frac{\alpha+\beta}{2}\, sin\frac{\alpha-\beta}{2}$

De Moivre's Theorem

$$Z^n = r^n(\cos n\theta + j\sin n\theta) \qquad Z^n = r^n e^{j\,n\theta} \qquad Z^n = r^n\angle n\theta$$

$$\sqrt[n]{z} = \sqrt[n]{r}\left[cos\left(\frac{\theta+2\pi k}{n}\right) + i\sin\left(\frac{\theta+2\pi k}{n}\right)\right];\ k = 0, 1, 2, \dots, n-1$$

Volume

$$Tetrahedron = \frac{1}{6}\left|\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\cdot\overrightarrow{AD}\right|$$

$$Prism = \frac{1}{2}\left|\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\cdot\overrightarrow{AD}\right|$$

Cramer's Rule (2 x 2 matrix)

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

Cramer's Rule (3 x 3 matrix)

$$X = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{|A|} \qquad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{|A|} \qquad z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{|A|}$$

Eigenvalues & Eigenvectors

$|\boldsymbol{A} - \lambda\, \boldsymbol{I}| = \boldsymbol{0}$

Scalar Product

$\vec{A} \cdot \vec{B} = ABcos\theta$

$\vec{A} \cdot \vec{B} = a_1a_2 + b_1b_2 + c_1c_2$

$$cos\theta = \frac{A.B}{|A||B|}$$

Vector / Cross Product

$\vec{A}\, x\, \vec{B} = ABsin\theta$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$\cos\theta = ll' + mm' + nn'$

Scalar Triple Product

$$A\,.(B\;x\;C) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

Coplanar Vectors

$A\,.(B\;x\;C) = 0$

Vector Triple Product

$$(\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = \mathbf{i}\begin{vmatrix} b_y & b_z \\ c_y & c_z \end{vmatrix} - \mathbf{j}\begin{vmatrix} b_x & b_z \\ c_x & c_z \end{vmatrix} + \mathbf{k}\begin{vmatrix} b_x & b_y \\ c_x & c_y \end{vmatrix}$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ \begin{vmatrix} b_y & b_z \\ c_y & c_z \end{vmatrix} & -\begin{vmatrix} b_x & b_z \\ c_x & c_z \end{vmatrix} & \begin{vmatrix} b_x & b_y \\ c_x & c_y \end{vmatrix} \end{vmatrix}$$

DIFFERENTIATION

$$\frac{d}{dx}a = 0, a \text{ is a constant}$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}lnx = \frac{1}{x}$$

$$\frac{d}{dx}(cosx) = -sinx$$

$$\frac{d}{dx}(sinx) = cosx$$

$$\frac{d}{dx}(tanx) = \sec^2 x$$

Product Rule:

$$[uv]' = uv' + vu'$$

Quotient Rule:

$$\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(a^{f(x)}) = a^{f(x)} (\ln a)\, f'(x)$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(e^{f(x)}) = e^{f(x)} f'(x)$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)} f'(x)$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cos ecx) = -\cos ecx \cot x$$

$$\frac{d}{dx}(\cot x) = -\cos ec^2 x$$

$$\frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos ec u) = -\cos ec u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}(\cot u) = -\cos ec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\cot^{-1} x\right) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\csc^{-1} x\right) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)=\frac{\partial^2 f}{\partial x^2}=f_{xx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)=\frac{\partial^2 f}{\partial y^2}=f_{yy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)=\frac{\partial^2 f}{\partial y\partial x}=f_{xy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)=\frac{\partial^2 f}{\partial x\partial y}=f_{yx}$$

$$\delta z=f_X\,\delta x+f_y\,\delta y$$

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial x},$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial y},$$

$$\frac{dy}{dx}=-\frac{F_x(x,y)}{F_y(x,y)}$$

$$\frac{dz}{dx}=-\frac{F_x(x,y,z)}{F_z(x,y,z)},\ \frac{dz}{dy}=-\frac{F_y(x,y,z)}{F_z(x,y,z)}$$

$$\left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right)-\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2;$$

$$\frac{\partial^2 f}{\partial x^2}\qquad\frac{\partial^2 f}{\partial y^2}\qquad\frac{\partial^2 f}{\partial x\partial y}$$