UNIVERSITI KUALA LUMPUR

MALAYSIA INSTITUTE OF CHEMICAL AND BIOENGINEERING TECHNOLOGY

FINAL EXAMINATION

OCTOBER 2025 SEMESTER

COURSE CODE	:	CPB20203
COURSE NAME	:	NUMERICAL METHODS IN CHEMICAL ENGINEERING
PROGRAMME NAME	:	BACHELOR OF CHEMICAL ENGINEERING TECHNOLOGY
DATE	:	27 JANUARY 2026
TIME	:	9.00 AM – 12.00 PM
DURATION	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please CAREFULLY read the instructions given in the question paper.
2. This question paper consists of FIVE (5) questions. Answer ONLY FOUR (4) questions.
3. Write your answers in the answer booklet provided.
4. Answer all questions in English language ONLY.
5. Formula table has been appended for your reference.

(Total: 100 marks)

INSTRUCTIONS: Answer Four Questions Only.

Use at least 4 decimal places for all calculations.

Please use the answer booklet provided.

Question 1 (done)

Evaluate the root of $f(x) = x^3 - e^{-x}$ by using:

(a) the bisection method. Start with $a = 0$ and $b = 1$, and calculate until x_5.

(13 marks)

(b) the secant method. Start with these two points, $x_1 = 0$, and $x_2 = 1$, and calculate until x_5.

(6 marks)

(c) Newton Raphson's method. Start at $x_1 = 0$, and calculate until x_5.
$(Hint: f'(x) = 3x^2 + e^{-x})$

(6 marks)

Question 2

Given below is a system of linear equations.

$$4x + y = 4 - z$$

$$x + z = 6 - 5y$$

$$y + 6z = 7 - x$$

(a) Determine whether the system is diagonally dominant. Proof it.

(6 marks)

(b) Rewrite the equations following the Gauss-Seidel iterative method.

(6 marks)

(c) Hence, solve the equations (until $n = 3$) and complete the table below. No need to show detailed calculations.

n	x	y	z	$Maximum$ $Error$
0	0	0	0	–
1				
2				
3				

(12 marks)

(d) Write down the solutions.

(1 mark)

Question 3

(a) A manufacturing company wants to predict the daily production output (y) based on two factors:

- x_1: Number of skilled workers
- x_2: Number of machine operating hours

The following data were collected over five days:

Day	x_1	x_2	y
1	2	4	50
2	3	6	65
3	5	5	78
4	6	8	95
5	8	7	110

i. Determine the multiple linear regression model that best fits the data.

(12 marks)

ii. From the model, calculate the production output when $x_1 = 7$ and $x_2 = 6$.

(2 marks)

(b) The temperature (°C) along a thin metal rod varies with a distance x (m) according to the model:

$$T(x) = 2.5x^3 - 1.2x^2 + 0.8e^{0.5x} + 10$$

The rate of change of temperature with respect to distance is required at a specific location to assess thermal stress. Using $h = 0.4$, calculate the first derivative $T'(x)$ at $x = 1.0$, using:

i. forward difference formula $O(h)$.

(4 marks)

ii. centered difference formula $O(h^2)$.

(7 marks)

Question 4

(a) A factory produces goods over a 24-hour production cycle. Its emissions rate of CO_2 (in kg/hour) varies with time due to changes in machinery load and energy use, modeled by:

$$E(t) = 50 + 20\sin\left(\frac{\pi t}{12}\right) + 10\cos\left(\frac{\pi t}{6}\right)$$

Where t is the time in hours. Estimate the total CO_2 emissions over the 24-hour cycle:

$$Q = \int_0^{24} E(t)\ dt$$

Using the following methods:

i. Multiple Simpson's 1/3 with 12 subintervals.

(5 marks)

ii. Multiple Simpson's 3/8 method with 12 subintervals.

(5 marks)

(b) Consider the first-order ODE:

$$\frac{dy}{dx} = yx - x^3,\ \ 0 \le x \le 1.8, y(0) = 1$$

i. solve the ODE using Euler's method with $h = 0.6$.

(9 marks)

ii. analytical solution of the ODE is

$$y = x^2 - e^{\frac{x^2}{2}} + 2$$

Calculate the percent relative error between the analytical and numerical solutions. Complete the table below.

x	y_exact	$y\_Euler's$	% Relative Error
0.6			
1.2			
1.8			

(6 marks)

Question 5

A package of seafood is removed from a refrigerated truck and placed in a warehouse. We need to track its temperature rise to ensure it does not exceed safety limits.

- Initial temperature (T_0): 4°C (temperature inside the truck).
- Ambient temperature (T_a): 25°C (temperature of the warehouse).
- Exposure duration: 10 minutes.
- Freshness threshold: The seafood must stay below 10°C to remain fresh.

The temperature change is modeled by Newton's Law of Cooling.

$$\frac{dT}{dt} = -k(T - T_a)$$

Where:

- T is temperature of the seafood at time t,
- T_a is the ambient temperature,
- $k = 0.05$ is a positive constant.

(a) Calculate the temperature of the seafood from $t = 0$ to $t = 10$ minutes. Use Euler's method with a step size $h = 1$ minute.

(22 marks)

(b) Based on your calculations, how fast must the seafood be returned to refrigeration to ensure it never exceeds the 10°C freshness limit?

(3 marks)

END OF EXAMINATION PAPER

FORMULAE TABLE

Percent Relative Error:

$$\varepsilon_t = \left|\frac{current\ approximation - previous\ approximation}{current\ approximation}\right| \times 100\%$$

$$\varepsilon_t = \left|\frac{Exact\ Value - Approximate\ Value}{Exact\ Value}\right| \times 100\%$$

Bisection Method:

$$x_m = \frac{x_a + x_b}{2}$$

Newton-Raphson Method:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Secant Method:

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Finite Difference Formulas:

(a) First Derivatives:

Forward $O(h)$:	$f'(x) \approx \frac{f(x+h) - f(x)}{h}$
Backward $O(h)$:	$f'(x) \approx \frac{f(x) - f(x-h)}{h}$
Central $O(h^2)$:	$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$
Forward $O(h^2)$:	$f'(x) \approx \frac{1}{2h}[-f(x+2h) + 4f(x+h) - 3f(x)]$
Backward $O(h^2)$:	$f'(x) \approx \frac{1}{2h}[3f(x) - 4f(x-h) + f(x-2h)]$
Central $O(h^4)$:	$f'(x) \approx \frac{1}{12h}[-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)]$

b) Second Derivatives:

Forward $O(h)$:	$f''(x) \approx \frac{1}{h^2}[f(x) - 2f(x+h) + f(x+2h)]$
Backward $O(h)$:	$f''(x) \approx \frac{1}{h^2}[f(x-2h) - 2f(x-h) + f(x)]$
Central $O(h^2)$:	$f''(x) \approx \frac{1}{h^2}[f(x+h) - 2f(x) + f(x-h)]$

Euler's Method:

$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$$

Heun's Method:

$$y_{n+1}^0 = y_n + f(x_n, y_n)h$$

$$y_{n+1} = y_n + \frac{f(x_n, y_n) + f(x_{n+1}, y_{n+1}^0)}{2}h$$

Ralston's Method:

$$y_{n+1} = y_n + \left(\frac{1}{3}k_1 + \frac{2}{3}k_2\right)h$$

$$k_1 = f(x_n, y_n), \qquad k_2 = f\left(x_n + \frac{3}{4}h, y_n + \frac{3}{4}k_1 h\right)$$

4th Order RK Method:

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$k_1 = f(x_n, y_n), \qquad k_2 = f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1 h\right)$$

$$k_3 = f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2 h\right), \qquad k_4 = f(x_n + h, y_n + k_3 h)$$

Multiple Simpson's 1/3 Rule:

$$\int_{x_a}^{x_b} f(x)\,dx \approx \frac{h}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

Multiple Simpson's 3/8 Rule:

$$\int_{x_a}^{x_b} f(x)\,dx \approx \frac{3h}{8}\left[f(x_0) + 3\big(f(x_1) + f(x_2) + \cdots + f(x_{n-2})\big) + 2\big(f(x_3) + \cdots + f(x_{n-3})\big) + f(x_n)\right]$$

Linear Regression:

$$y = a + bx$$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$a = \frac{\sum y}{n} - \frac{b\sum x}{n}$$

Polynomial Regression:

$$y = a + bx + cx^2$$

$$\sum y = an + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

Multiple Linear Regression:

$$y = a + bx_1 + cx_2$$

$$\sum y = an + b\sum x_1 + c\sum x_2$$

$$\sum x_1 y = a\sum x_1 + b\sum x_1^2 + c\sum x_1 x_2$$

$$\sum x_2 y = a\sum x_2 + b\sum x_1 x_2 + c\sum x_2^2$$