



UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF CHEMICAL AND BIO-ENGINEERING TECHNOLOGY

FINAL EXAMINATION
OCTOBER 2025 SEMESTER

COURSE CODE : CLB20903
COURSE TITLE : ENGINEERING STATISTIC
PROGRAMME NAME : TECHNICAL FOUNDATION - BACHELOR
DATE : 02 FEBRUARY 2026
TIME : 9:00AM - 12:00PM
DURATION : 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. This question paper consist of TWO sections.
4. Answer ALL questions for Section A.
5. Section B consist of four questions. Answer THREE (3) questions only.
6. Please write your answer on the answer booklet provided.
7. Please answer all questions in English only.
8. Please answer MCQ/EMQ questions using OMR sheet. ☐ *Tick if applicable*
9. Refer to the attached Formula/ Appendies. ☒ *Tick if applicable*

THERE ARE 10 PAGES OF QUESTIONS INCLUDING THIS PAGE

SECTION A (Total:40 marks)

Answer ALL questions.

Please use the answer booklet provided

Question 1

- (a) A company operates three machines in three shifts each day. From production records, data on the number of breakdowns are collected, and the MINITAB output is shown in Figure 1.

Tabulated statistics: Shift, Machines				
Rows: Shift		Columns: Machines		
	A	B	C	All
1	41	20	12	73
	36.92	20.37	15.70	73.00
2	31	11	9	51
	25.80	14.23	A	51.00
3	15	17	16	48
	24.28	13.40		
All	87	48	37	172
	87.00	48.00	37.00	172.00
Cell Contents: Count				
Expected count				
Pearson Chi-Square = 11.103, DF = B, P-Value = 0.025				
Likelihood Ratio Chi-Square = 11.223, DF = 4, P-Value = 0.024				

Figure 1: MINITAB output

- Determine the values of A and B. (5 marks)
- Test the hypothesis that machine breakdowns and shifts are independent at a 1% significance level by using the P-value approach. (5 marks)

- (b) An electrical engineer is studying whether replacing fluorescent bulbs with LED bulbs reduces the energy consumption of lighting systems in office rooms. To test this, the engineer measures the daily energy consumption (kWh) of 8 office rooms before and after switching to LED bulbs. The MINITAB result is shown in Figure 2.

Paired T-Test and CI: LED (After), Fluorescent (Before)				
Paired T for LED (After) - Fluorescent (Before)				
	N	Mean	StDev	SE Mean
LED (After)	8	10.813	0.290	0.103
Fluorescent (Before)	8	13.013	0.348	0.123
Difference	8	-2.2000	0.2070	0.0732
99% upper bound for mean difference: -1.9806				
T-Test of mean difference = 0 (vs < 0): T-Value = -30.06 P-Value = 0.000				

Figure 2: MINITAB output

- i. Based on the P-value, test at a 1% significance level whether the LED bulbs significantly reduce daily energy consumption compared to fluorescent bulbs.
(5 marks)
- ii. Construct the 90% confidence interval for the mean daily energy consumption.
(5 marks)

Question 2

- (a) The manager of a lemonade bottling plant is interested in comparing the performance of two production lines. Ten observations are taken from each line, and the MINITAB output is shown in Figure 3.

Two-sample T for line 1 vs line 2				
	N	Mean	StDev	SE Mean
line 1	10	82.40	3.13	0.99
line 2	10	78.00	4.76	1.5
Difference = mu (line 1) - mu (line 2)				
Estimate for difference: 4.40				
95% CI for difference: (0.61, 8.19)				
T-Test of difference = 0 (vs not =): T-Value = ■ P-Value = 0.025 DF = B				
Both use Pooled StDev = A				

Figure 3: MINITAB output

- i. Determine the values of A and B.

(6 marks)

- ii. Based on the P-value, test at 5% significant level whether the means of the two production lines are different.

(5 marks)

- (b) The extraction rate of a certain polymer is known to depend on the reaction temperature and the amount of catalyst used. An experiment was conducted at four levels of temperature and five levels of the catalyst, and the extraction rate is shown in Figure 4.

ANOVA: Extraction Rate versus Catalyst, Temperature					
Source	DF	SS	MS	F	P
Catalyst	3	2235.79	A	■	0.000
Temperature	2	458.58	229.292	B	0.000
Interaction	6	69.08	11.514	■	0.305
Error	12	101.50	8.458		
Total	23	2864.96			
S = 2.908 R-Sq = 96.46% R-Sq(adj) = 93.21%					

Figure 4: MINITAB output

- i. Calculate the values of A and B.

(4 marks)

- ii. Test the hypothesis to investigate the effects of the different catalysts on the reaction time, using 5% significance level.

(5 marks)

SECTION B (Total: 60 marks)

INSTRUCTION: Answer THREE(3) questions ONLY.**Please use the answer booklet provided.****Question 1**

A biological research company is investigating the relationship between a specific yeast strain's temperature (x) and the growth rate (y). Table 1 shows the data collected for different temperatures.

Table 1: Temperature and yeast growth rate

Temperature (°C)	20	25	30	35	40
Growth Rate (cells/hour)	60	90	120	140	160

Given: $\sum xy = 18350$

- (a) Calculate the least squares regression line to predict yeast growth rate based on temperature.
(12 marks)
- (b) Estimate the yeast growth rate when the temperature is 32°C.
(2 marks)
- (c) Determine the correlation coefficient (r) between temperature and yeast growth rate.
(4 marks)
- (d) Identify the relationship between temperature and yeast growth rate.
(2 marks)

Question 2

A chemical company is investigating the effect of three different chemical concentrations (R, S, and T) on the reaction time (in minutes) required for a particular chemical reaction to complete. The company records the reaction times in Table 2 for each concentration in several trials.

Given: $\sum \sum x_{ij}^2 = 4353.70$

Refer Below - Table2 : The reaction time (in minutes) .

Table 2: The reaction time (in minutes)

R	S	T
18.2	22.1	19.8
18.5	21.8	19.5
18.0	22.3	20.0
18.3		19.7

- (a) Test at 5% significance level, whether there is enough evidence to conclude that the mean reaction times are different across the three concentrations.

(14 marks)

- (b) Construct a 90% confidence interval for the difference in mean reaction time for chemical concentrations R and chemical concentrations S.

(6 marks)

Question 3

- (a) The following measurements were recorded for the drying time, in hours, of a certain brand of latex paint. Assuming the distribution of the measurement is approximately normal.

3.4 2.5 4.8 2.9 3.6 2.8 3.3 5.6 3.7 2.8 4.4 4.8

Test the hypothesis that the mean drying time is greater than 3.5 hours at the 5% significance level.

(10 marks)

- (b) A researcher is studying the distribution of different species of insects in a certain habitat. The researcher expects the four species (Q, X, Y, and Z) to be found in equal proportions. Table 3 shows the frequency of each species the researcher recorded over a month of observation.

Table 3: The frequency of insects

Species	Q	X	Y	Z
Frequency	42	50	34	27

Conduct a Chi-Square goodness of fit test by using a 5% significance level to determine if the observed distribution of insect species is equally likely.

(10 marks)

Question 4

- (a) A chemical manufacturing plant is evaluating the performance of a newly implemented quality control system for a specific chemical product. The target specification for the concentration of a critical component in the product is 45 grams per liter (g/L). A random sample of 36 product batches produced under the new system is tested, yielding a sample mean concentration of 44.6 g/L and a sample standard deviation of 1.2 g/L. Determine whether there is sufficient evidence to conclude that the new quality control system is meeting the target specification for the concentration of the critical component at 5% significance level.

(7 marks)

- (b) Trace metals in drinking water affect the flavour, and an unusually high concentration can pose a health hazard. Eight pairs of data were collected, measuring zinc concentrations in the bottom and surface water. Data are shown in Table 4.

Table 4: Zinc concentration

Location	Zinc concentration in bottom water	Zinc concentration in surface water
1	0.43	0.41
2	0.26	0.23
3	0.56	0.39
4	0.53	0.41
5	0.70	0.60
6	0.71	0.60
7	0.65	0.63
8	0.58	0.52

Test at 1% level of significance whether the zinc concentration in the bottom water exceeds the zinc concentration in the surface water. (Hint: $D = \text{bottom-surface}$)

(13 marks)

END OF EXAMINATION PAPER

APPENDIX

Standard deviation	$s = \sqrt{\frac{(\sum x_i^2) - \frac{(\sum x_i)^2}{n}}{n-1}}$
Confidence interval	$\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
	$\bar{x} \pm t_{\frac{\alpha}{2}, \nu} \frac{s}{\sqrt{n}} \quad ; \nu = n-1$
	$(\bar{x}_1 - \bar{x}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} @ (\bar{x}_1 - \bar{x}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
	$(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}, \nu} \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} ; s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} ; \nu = n_1 + n_2 - 2$
Test statistic	$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$
	$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} ; \nu = n-1$
	$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} @ z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$
	$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} ; s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} ; \nu = n_1 + n_2 - 2$
	$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} ; \nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2}{\left(\frac{s_1^2}{n_1} \right)^2 + \left(\frac{s_2^2}{n_2} \right)^2}$
	$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} ; \nu = n-1 ; d = x_1 - x_2$

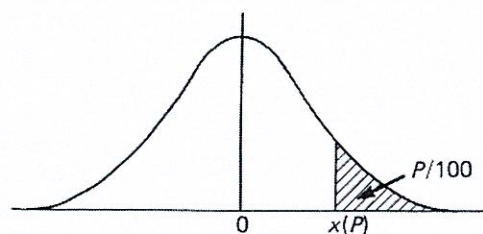
Analysis of Variance (ANOVA)	$SSA = \sum \left[\frac{(\sum x_i)^2}{n_i} \right] - \frac{(\sum \sum x_{ij})^2}{n}$ $SST = \sum \sum x_{ij}^2 - \frac{(\sum \sum x_{ij})^2}{n}; SSE = SST - SSA$ $\bar{x} \pm t_{\frac{\alpha}{2}, n-a} \sqrt{\frac{MSE}{n_i}}$ $(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}, n-a} \sqrt{\frac{2MSE}{n_i}} @ (\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}, n-a} \sqrt{MSE \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$
Chi square test	$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} ; v = (a-1)(b-1) @ v = k-1$
Regression model	$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} ; \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ $S_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n} ; S_{yy} = \sum y_i^2 - \frac{(\sum y_i)^2}{n} ; S_{xy} = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}$
β_1 test statistic	$t = \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\frac{s_e^2}{S_{xx}}}} ; s_e^2 = \frac{S_{yy} - \hat{\beta}_1 S_{xy}}{n-2} ; v = n-2$
ρ test statistic	$t = r \sqrt{\frac{n-2}{1-r^2}} ; v = n-2$
Correlation coefficient	$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}} \int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2} dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geq x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geq x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0.0000	5.0	1.6449	3.0	1.8808	2.0	2.0537	1.0	2.3263	0.10	3.0902		
45	0.1257	4.8	1.6646	2.9	1.8957	1.9	2.0749	0.9	2.3656	0.09	3.1214		
40	0.2533	4.6	1.6849	2.8	1.9110	1.8	2.0969	0.8	2.4089	0.08	3.1559		
35	0.3853	4.4	1.7060	2.7	1.9268	1.7	2.1201	0.7	2.4573	0.07	3.1947		
30	0.5244	4.2	1.7279	2.6	1.9431	1.6	2.1444	0.6	2.5121	0.06	3.2389		
25	0.6745	4.0	1.7507	2.5	1.9600	1.5	2.1701	0.5	2.5758	0.05	3.2905		
20	0.8416	3.8	1.7744	2.4	1.9774	1.4	2.1973	0.4	2.6521	0.04	3.7190		
15	1.0364	3.6	1.7991	2.3	1.9954	1.3	2.2262	0.3	2.7478	0.005	3.8906		
10	1.2816	3.4	1.8250	2.2	2.0141	1.2	2.2571	0.2	2.8782	0.001	4.2649		
5	1.6449	3.2	1.8522	2.1	2.0335	1.1	2.2904	0.1	3.0902	0.0005	4.4172		

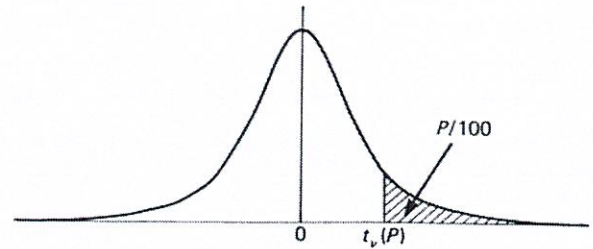
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PERCENTAGE POINTS OF THE *t*-DISTRIBUTION

This table gives percentage points $t_p(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}(\nu+1))}{\Gamma(\frac{1}{2}\nu)} \int_{t_p(P)}^{\infty} \frac{1}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}} dt$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2 -distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t -distribution with ν degrees of freedom, and the probability that $t \geq t_p(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_p(P)$, and the probability that $|t| \geq t_p(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

<i>P</i>	40	30	25	20	15	10	5	2.5	1	0.5	0.1	0.05
$\nu = 1$	0.3249	0.7265	1.0000	1.3764	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.2887	0.6172	0.8165	1.0607	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.2767	0.5844	0.7649	0.9785	1.250	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.2707	0.5686	0.7407	0.9410	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.2672	0.5594	0.7267	0.9195	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.2648	0.5534	0.7176	0.9057	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.2632	0.5491	0.7111	0.8960	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.2619	0.5459	0.7064	0.8889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.2610	0.5435	0.7027	0.8834	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.2602	0.5415	0.6998	0.8791	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.2596	0.5399	0.6974	0.8755	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.2590	0.5386	0.6955	0.8726	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.2586	0.5375	0.6938	0.8702	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.2582	0.5366	0.6924	0.8681	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.2579	0.5357	0.6912	0.8662	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.2576	0.5350	0.6901	0.8647	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.2573	0.5344	0.6892	0.8633	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.2571	0.5338	0.6884	0.8620	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.2569	0.5333	0.6876	0.8610	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.2567	0.5329	0.6870	0.8600	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.2566	0.5325	0.6864	0.8591	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.2564	0.5321	0.6858	0.8583	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.2563	0.5317	0.6853	0.8575	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.2562	0.5314	0.6848	0.8569	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.2561	0.5312	0.6844	0.8562	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.2560	0.5309	0.6840	0.8557	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.2559	0.5306	0.6837	0.8551	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.2558	0.5304	0.6834	0.8546	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.2557	0.5302	0.6830	0.8542	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.2556	0.5300	0.6828	0.8538	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	0.2555	0.5297	0.6822	0.8530	1.054	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	0.2553	0.5294	0.6818	0.8523	1.052	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	0.2552	0.5291	0.6814	0.8517	1.052	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	0.2551	0.5288	0.6810	0.8512	1.051	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	0.2550	0.5286	0.6807	0.8507	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.2547	0.5278	0.6794	0.8489	1.047	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.2545	0.5272	0.6786	0.8477	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.2539	0.5258	0.6765	0.8446	1.041	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.2533	0.5244	0.6745	0.8416	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291

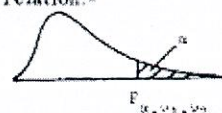
PERCENTAGE POINTS OF THE F DISTRIBUTION

The table gives the values of $F_{\alpha; \nu_1, \nu_2}$ the 100 α percentage point of the F distribution having ν_1 degrees of freedom in the numerator and ν_2 degrees of freedom in the denominator.

For each pair of values of ν_1 and ν_2 , $F_{\alpha; \nu_1, \nu_2}$ is tabulated for $\alpha = 0.05, 0.025, 0.01, 0.001$, the 0.025 values being bracketed.

The lower percentage points of the distribution may be obtained from the relation:-

$$F_{1-\alpha; \nu_1, \nu_2} = 1/F_{\alpha; \nu_2, \nu_1}$$



$$\text{e.g. } F_{.95; 12, 8} = 1/F_{.05; 8, 12} = 1/2.85 = 0.351$$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	10	12	24	∞
1	161.4 (548) 4052 4053*	199.5 (800) 5000 5000*	215.7 (864) 5403 5404*	224.6 (900) 5625 5625*	230.2 (922) 5764 5764*	234.0 (937) 5859 5859*	236.8 (948) 5928 5929*	238.9 (957) 5981 5981*	241.9 (969) 6056 6056*	243.9 (977) 6106 6107*	249.0 (997) 6235 6235*	254.3 (1018) 6366 6366*
2	18.5 (38.5) 98.5 998.5	19.0 (39.0) 99.2 999.0	19.2 (39.2) 99.2 999.2	19.2 (39.2) 99.3 999.2	19.3 (39.3) 99.3 999.3	19.3 (39.3) 99.3 999.3	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.5 (39.5) 99.5 999.5	19.5 (39.5) 99.5 999.5
3	10.13 (17.4) 34.1 167.0	9.55 (16.0) 30.8 148.5	9.28 (15.4) 29.5 141.1	9.12 (15.1) 28.7 137.1	9.01 (14.9) 28.2 134.6	8.94 (14.7) 27.9 132.8	8.89 (14.6) 27.7 131.5	8.85 (14.5) 27.5 130.6	8.79 (14.4) 27.2 129.2	8.74 (14.3) 27.1 128.3	8.64 (14.1) 26.6 125.9	8.53 (13.9) 26.1 123.5
4	7.71 (12.22) 21.2 74.14	6.94 (10.65) 18.0 61.25	6.59 (9.98) 16.7 56.18	6.39 (9.60) 16.0 53.44	6.26 (9.36) 15.5 51.71	6.16 (9.20) 15.2 50.53	6.09 (9.07) 15.0 49.66	6.04 (8.98) 14.8 49.00	5.96 (8.84) 14.5 48.05	5.91 (8.75) 14.4 47.41	5.77 (8.51) 13.9 45.77	5.63 (8.26) 13.5 44.05
5	6.61 (10.01) 16.26 47.16	5.79 (8.43) 13.27 37.12	5.41 (7.76) 12.06 33.20	5.19 (7.39) 11.39 31.09	5.05 (7.15) 10.97 29.75	4.95 (6.98) 10.67 28.83	4.88 (6.85) 10.46 28.16	4.82 (6.76) 10.29 27.65	4.74 (6.62) 10.05 26.92	4.68 (6.52) 9.89 26.42	4.53 (6.28) 9.47 25.14	4.36 (6.02) 9.02 23.79
6	5.99 (8.81) 13.74 35.51	5.14 (7.26) 10.92 27.00	4.76 (6.60) 9.78 23.70	4.53 (6.23) 9.15 21.92	4.39 (5.99) 8.75 20.80	4.28 (5.82) 8.47 20.03	4.21 (5.70) 8.26 19.46	4.15 (5.60) 8.10 19.03	4.08 (5.46) 7.87 18.41	4.00 (5.37) 7.72 17.99	3.84 (5.12) 7.31 16.90	3.67 (4.85) 6.88 15.75
7	5.59 (8.07) 12.25 29.25	4.74 (6.54) 9.55 21.69	4.35 (5.89) 8.45 18.77	4.12 (5.52) 7.85 17.20	3.97 (5.29) 7.46 16.21	3.87 (5.12) 7.19 15.52	3.79 (4.99) 6.99 15.02	3.73 (4.90) 6.84 14.63	3.64 (4.76) 6.62 14.08	3.57 (4.67) 6.47 13.71	3.41 (4.42) 6.07 12.73	3.23 (4.14) 5.65 11.70
8	5.32 (7.57) 11.26 25.42	4.46 (6.06) 8.65 18.49	4.07 (5.42) 7.59 15.83	3.84 (5.05) 7.01 14.39	3.69 (4.82) 6.63 13.48	3.58 (4.65) 6.37 12.86	3.50 (4.53) 6.18 12.40	3.44 (4.43) 6.03 12.05	3.35 (4.30) 5.81 11.54	3.28 (4.20) 5.67 11.19	3.12 (3.95) 5.28 10.30	2.93 (3.67) 4.86 9.34
9	5.12 (7.21) 10.56 22.86	4.26 (5.71) 8.02 16.39	3.86 (5.08) 6.99 13.90	3.63 (4.72) 6.42 12.56	3.48 (4.48) 6.06 11.71	3.37 (4.32) 5.80 11.13	3.29 (4.20) 5.61 10.69	3.23 (4.10) 5.47 10.37	3.14 (3.96) 5.26 9.87	3.07 (3.87) 5.11 9.57	2.90 (3.61) 4.73 8.72	2.71 (3.33) 4.31 7.81
10	4.96 (6.94) 10.04 21.04	4.10 (5.46) 7.56 14.91	3.71 (4.83) 6.55 12.55	3.48 (4.47) 5.99 11.28	3.33 (4.24) 5.64 10.48	3.22 (4.07) 5.39 9.93	3.14 (3.95) 5.20 9.52	3.07 (3.85) 5.06 9.20	2.98 (3.72) 4.85 8.74	2.91 (3.62) 4.71 8.44	2.74 (3.37) 4.33 7.64	2.54 (3.08) 3.91 6.76
11	4.84 (6.72) 9.65 19.69	3.98 (5.26) 7.21 13.81	3.59 (4.63) 6.22 11.56	3.36 (4.28) 5.67 10.35	3.20 (4.04) 5.32 9.58	3.09 (3.88) 5.07 9.05	3.01 (3.78) 4.89 8.66	2.95 (3.66) 4.74 8.35	2.85 (3.53) 4.54 7.92	2.79 (3.43) 4.40 7.63	2.61 (3.17) 4.02 6.85	2.40 (2.88) 3.60 6.00
12	4.75 (6.55) 9.33 18.64	3.89 (5.10) 6.93 12.97	3.49 (4.47) 5.95 10.80	3.26 (4.12) 5.41 9.63	3.11 (3.89) 5.06 8.89	3.00 (3.73) 4.82 8.38	2.91 (3.61) 4.64 8.00	2.85 (3.51) 4.50 7.71	2.75 (3.37) 4.30 7.29	2.69 (3.28) 4.16 7.00	2.51 (3.02) 3.78 6.25	2.30 (2.72) 3.36 5.42
13	4.67 (6.41) 9.07 17.82	3.81 (4.97) 6.70 12.31	3.41 (4.35) 5.74 10.21	3.18 (4.00) 5.21 9.07	3.03 (3.77) 4.86 8.35	2.92 (3.60) 4.62 7.86	2.83 (3.48) 4.44 7.49	2.77 (3.39) 4.30 7.21	2.67 (3.25) 4.10 6.80	2.60 (3.15) 3.96 6.52	2.42 (2.89) 3.59 5.78	2.21 (2.60) 3.17 4.97

* Entries marked thus must be multiplied by 100

2	1	2	3	4	5	6	7	8	10	12	24	∞
14	4.60 (6.30) 8.86 17.14	3.74 (4.86) 6.51 11.78	3.34 (4.24) 5.56 9.73	3.11 (3.89) 5.04 8.62	2.96 (3.66) 4.70 7.92	2.85 (3.50) 4.46 7.44	2.76 (3.38) 4.28 7.08	2.70 (3.29) 4.14 6.80	2.60 (3.15) 3.94 6.40	2.53 (3.05) 3.80 6.13	2.35 (2.79) 3.43 5.41	2.13 (2.49) 3.00 4.60
16	4.49 (5.12) 8.53 16.12	3.63 (4.69) 6.01 10.97	3.24 (4.08) 5.09 9.01	3.01 (3.73) 4.77 7.94	2.85 (3.50) 4.44 7.27	2.74 (3.34) 4.20 6.80	2.66 (3.22) 4.03 6.46	2.59 (3.12) 3.89 6.19	2.49 (2.99) 3.69 5.81	2.42 (2.89) 3.55 5.55	2.24 (2.63) 3.18 4.85	2.01 (2.32) 2.75 4.06
18	4.41 (5.98) 8.29 15.38	3.55 (4.56) 6.01 10.39	3.16 (3.95) 5.09 8.49	2.93 (3.61) 4.58 7.46	2.77 (3.38) 4.25 6.81	2.66 (3.22) 4.01 6.35	2.58 (3.10) 3.84 6.02	2.51 (3.01) 3.71 5.76	2.41 (2.87) 3.51 5.39	2.34 (2.77) 3.37 5.13	2.15 (2.50) 3.00 4.45	1.92 (2.19) 2.57 3.67
20	4.35 (5.87) 8.10 14.82	3.49 (4.46) 5.85 9.95	3.10 (3.86) 4.94 8.10	2.87 (3.51) 4.43 7.10	2.71 (3.29) 4.10 6.46	2.60 (3.13) 3.87 6.02	2.51 (3.01) 3.70 5.69	2.45 (2.91) 3.56 5.44	2.35 (2.77) 3.37 5.08	2.28 (2.68) 3.23 4.82	2.08 (2.41) 2.86 4.15	1.84 (2.09) 2.42 3.38
22	4.30 (5.79) 7.95 14.38	3.44 (4.38) 5.72 9.61	3.05 (3.78) 4.82 7.80	2.82 (3.44) 4.31 6.81	2.66 (3.22) 3.99 6.19	2.55 (3.05) 3.76 5.76	2.46 (2.93) 3.59 5.44	2.40 (2.84) 3.45 5.19	2.30 (2.70) 3.26 4.83	2.23 (2.60) 3.12 4.58	2.03 (2.33) 2.75 3.92	1.78 (2.00) 2.31 3.15
24	4.26 (5.72) 7.82 14.03	3.40 (4.32) 5.61 9.34	3.01 (3.72) 4.72 7.55	2.78 (3.38) 4.22 6.59	2.62 (3.15) 3.90 5.98	2.51 (2.99) 3.67 5.55	2.42 (2.87) 3.50 5.23	2.36 (2.78) 3.36 4.99	2.25 (2.64) 3.17 4.64	2.18 (2.54) 3.03 4.39	1.98 (2.27) 2.66 3.74	1.73 (1.94) 2.21 2.97
26	4.23 (5.66) 7.72 13.74	3.37 (4.27) 5.53 9.12	2.98 (3.67) 4.64 7.36	2.74 (3.33) 4.14 6.41	2.59 (3.10) 3.82 5.80	2.47 (2.94) 3.59 5.38	2.39 (2.82) 3.42 5.07	2.32 (2.73) 3.29 4.83	2.22 (2.59) 3.09 4.48	2.15 (2.49) 2.96 4.24	1.95 (2.22) 2.58 3.59	1.69 (1.88) 2.13 2.82
28	4.20 (5.61) 7.64 13.50	3.34 (4.22) 5.45 8.93	2.95 (3.63) 4.57 7.19	2.71 (3.29) 4.07 6.25	2.56 (3.06) 3.75 5.66	2.45 (2.90) 3.53 5.24	2.36 (2.78) 3.36 4.93	2.29 (2.69) 3.23 4.69	2.19 (2.55) 3.03 4.35	2.12 (2.45) 2.90 4.11	1.91 (2.17) 2.52 3.46	1.65 (1.83) 2.06 2.69
30	4.17 (5.57) 7.56 13.29	3.32 (4.18) 5.39 8.77	2.92 (3.59) 4.51 7.05	2.69 (3.25) 4.02 6.12	2.53 (3.03) 3.70 5.53	2.42 (2.87) 3.47 5.12	2.33 (2.75) 3.30 4.82	2.27 (2.65) 2.98 4.58	2.16 (2.51) 2.98 4.24	2.09 (2.41) 2.84 4.00	1.89 (2.14) 2.47 3.36	1.62 (1.79) 2.01 2.59
40	4.08 (5.42) 7.31 12.61	3.23 (4.05) 5.18 8.25	2.84 (3.46) 4.31 6.59	2.61 (3.13) 3.83 5.70	2.45 (2.90) 3.51 5.13	2.34 (2.74) 3.29 4.73	2.25 (2.62) 3.12 4.44	2.18 (2.53) 2.99 4.21	2.08 (2.39) 2.80 3.87	2.00 (2.29) 2.66 3.64	1.79 (2.01) 2.29 3.01	1.51 (1.64) 1.80 2.23
60	4.00 (5.29) 7.08 11.97	3.15 (3.93) 4.98 7.77	2.76 (3.34) 4.13 6.17	2.53 (3.01) 3.65 5.31	2.37 (2.79) 3.34 4.76	2.25 (2.63) 3.12 4.37	2.17 (2.51) 2.95 4.09	2.10 (2.41) 2.82 3.86	1.99 (2.27) 2.63 3.54	1.92 (2.17) 2.50 3.32	1.70 (1.88) 2.12 2.69	1.39 (1.48) 1.60 1.89
120	3.92 (5.15) 6.85 11.38	3.07 (3.80) 4.79 7.32	2.68 (3.23) 3.95 5.78	2.45 (2.89) 3.48 4.95	2.29 (2.67) 3.17 4.42	2.18 (2.52) 2.96 4.04	2.09 (2.39) 2.79 3.77	2.02 (2.30) 2.68 3.55	1.91 (2.16) 2.47 3.24	1.83 (2.05) 2.34 3.02	1.61 (1.76) 1.95 2.40	1.25 (1.31) 1.38 1.54
∞	3.84 (5.02) 6.63 10.83	3.00 (3.69) 4.61 6.91	2.60 (3.12) 3.78 5.42	2.37 (2.79) 3.32 4.62	2.21 (2.57) 3.02 4.10	2.10 (2.41) 2.80 3.74	2.01 (2.29) 2.64 3.47	1.94 (2.19) 2.51 3.27	1.83 (2.05) 2.32 2.96	1.75 (1.94) 2.18 2.74	1.52 (1.64) 1.79 2.13	1.00 (1.00) 1.00 1.00

Chi-Square (χ^2) Distribution

Area to the Right of Critical Value

Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169