CONFIDENTIAL

UNIVERSITI KUALA LUMPUR
Malaysian Institute of Marine Engineering Technology

FINAL EXAMINATION
JULY 2025 SEMESTER SESSION

SUBJECT CODE	:	**LEB22503**
SUBJECT TITLE	:	**SIGNALS AND SYSTEMS**
PROGRAMME NAME (FOR MPU: PROGRAMME LEVEL)	:	**BACHELOR OF ELECTRICAL AND ELECTRONICS ENGINEERING TECHNOLOGY (MARINE) WITH HONOURS**
TIME / DURATION	:	**09.00 AM – 12.00 PM (3 HOURS)**
DATE	:	**18 DECEMBER 2025**

INSTRUCTIONS TO CANDIDATES

1. Please **CAREFULLY** read the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of **TWO (2)** sections; Section A and Section B.
4. Answer **ALL** questions in Section A. For Section B, answer **THREE (3)** questions.
5. Please write your answers on the answer booklet provided.
6. Answer **ALL** questions in English language **ONLY**.
7. Formula sheet has been appended for your reference.

THERE ARE 6 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question 1 (CLO 1, C2, SK3, SP1, SP3, SP4)

(a) Identify whether the system: $y(t) = cos(x(t))$, are:

i. Causal

(3 marks)

ii. Invertible

(3 marks)

iii. Linear

(3 marks)

iv. Memoryless

(3 marks)

v. Time invariant

(3 marks)

Clarify your answer.

(b) Systems are used in signal processing to modify or extract information. They can be classified into several categories. List **FIVE (5)** classification of systems.

(5 marks)

Question 2 (CLO 1, C2, SK3, SP1, SP3, SP4)

(a) Identify **THREE (3)** properties of convolution.

(6 marks)

(b) Describe the complete response of a system.

(4 marks)

(c) Identify **FOUR (4)** basic elements required for the synthesis of an LTCI system (or a given transfer function).

(4 marks)

(d) Identify the **THREE (3)** steps to generate a transpose of any realization.

(6 marks)

SECTION B (Total: 60 marks)

INSTRUCTION: Answer THREE (3) questions ONLY.
Please use the answer booklet provided.

Question 3 (CLO 2, C6, SK6, SP1, SP3, SP4)

(a) Analyse the circuit shown in Figure 1 if given that $i_s(t) = 5\, cos\, 4\, tA$ and using the Fourier transform method, answer the following questions:

i. Determine the source current, $I_s(\omega)$.

(2 marks)

ii. Determine frequency response, $H(\omega)$.

(2 marks)

iii. Determine output current in frequency domain, $I_o(\omega)$.

(2 marks)

iv. Determine output current in time domain, $i_o(t)$.

(4 marks)

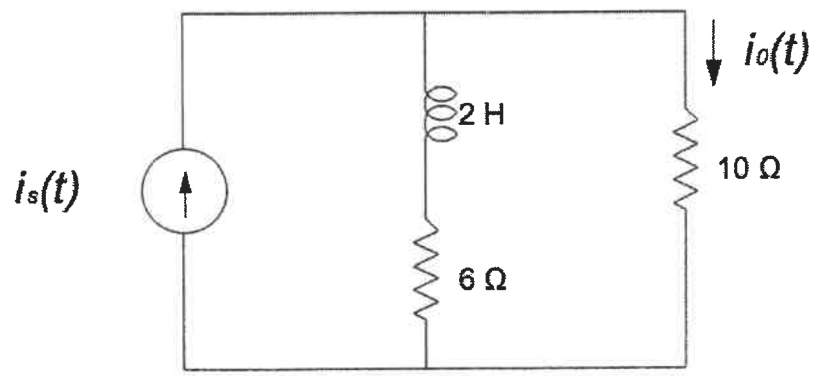

Figure 1

(b) A Chebyshev low-pass filter has the specification as below:

- Ripple in the pass-band is 1 dB.
- Cut-off frequency is 2 kHz.
- Amplitude attenuation at least 10 dB at 3 kHz.

Given Chebyshev polynomials as shown in Table 1 and Chebyshev low-pass filter approximation is given as:

$$|H_{Cn}(j\omega)| = \frac{1}{\sqrt{[1+\varepsilon^2 C_n^2(\omega)]}}$$

where

$C_n(\omega)$ = Chebyshev polynomial.

ε = ripple factor

n = filter order

Design a Chebyshev low-pass filter by determine:

i. Ripple factor in decibel, ε.

(2 marks)

ii. Normalized angular frequency, ω.

(2 marks)

iii. Minimum order of a Chebyshev low-pass filter as specification given.

(6 marks)

Table 1

n	$C_n(\omega)$
0	1
1	ω
2	$2\omega^2 - 1$
3	$4\omega^3 - 3\ \omega$
4	$8\omega^4 - 8\omega^2 + 1$
5	$16\omega^5 - 20\omega^3 + 5\omega$
6	$32\omega^7 - 48\omega^4 + 18\omega^2 - 1$
7	$74\omega^7 - 112\omega^5 + 57\omega^3 - 7\omega$
8	$128\omega^8 - 256\omega^7 + 170\omega^4 - 32\omega^2_{+1}$

Question 4 (CLO 2, C6, SK6, SP1, SP3, SP4)

(a) Find the Fourier Transform, $X_2(\omega)$, for the signal shown in Figure 2 using definition of Fourier Transform.

(5 marks)

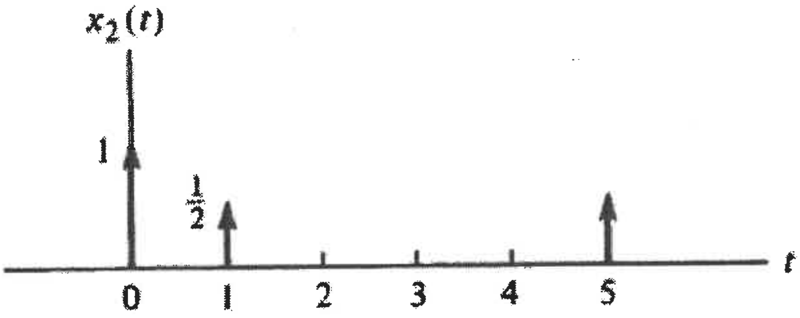

Figure 2

(b) Determine the signal corresponding to the Fourier transforms shown in Figure 3 using definition of inverse Fourier transform.

(5 marks)

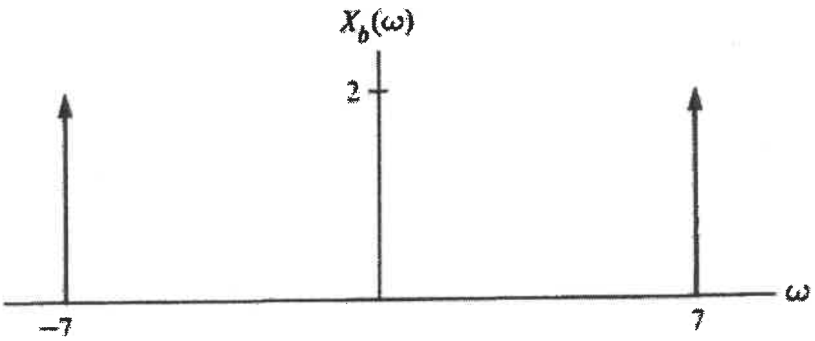

Figure 3

(c) Consider a continuous-time Linear Time-Invariant (LTI) system with system function:

$$H(s) = \frac{3s + 7}{(s + 1)(s + 2)(s + 5)}$$

Create a state representation of the system in matrix form.

(10 marks)

Question 5 (CLO 2, C6, SK6, SP1, SP3, SP4)

(a) A system given represented by the differential equation:

$$\frac{d}{dt}y(t) + 5y(t) = 5x(t)$$

in response to the input $x(t) = \left(\frac{3}{5}\right)e^{-2t}u(t)$. Using the Laplace transform method, determine:

i. output response, Y(s).

(3 marks)

ii. input response, X(s).

(2 marks)

iii. output response, Y(s), when the initial condition on the system is y(0) = -2.

(3 marks)

iv. output response, y(t), when the initial condition on the system is y(0) = -2.

(2 marks)

(b) The circuit parameters for series RLC bandstop filter are $R = 2k\Omega, L = 0.1H,$ and $C = 40pF$. Determine:

i. the center frequency.

(2 marks)

ii. the half-power frequencies

(4 marks)

iii. the quality factor.

(2 marks)

iv. the bandwidth

(2 marks)

Question 6 (CLO 2, C6, SK6, SP1, SP3, SP4)

(a) The switch in the circuit of Figure 4 is closed for a long time and then opened instantaneously at $t = 0$. Using Laplace transform method, answer the following questions:

i. Sketch the given circuit in s-domain.

(4 marks)

ii. Determine the total impedance, Z(s).

(2 marks)

iii. Determine the loop current, Y(s).

(2 marks)

iv. Determine the current, y(t).

(2 marks)

Assume that all initial conditions are zero.

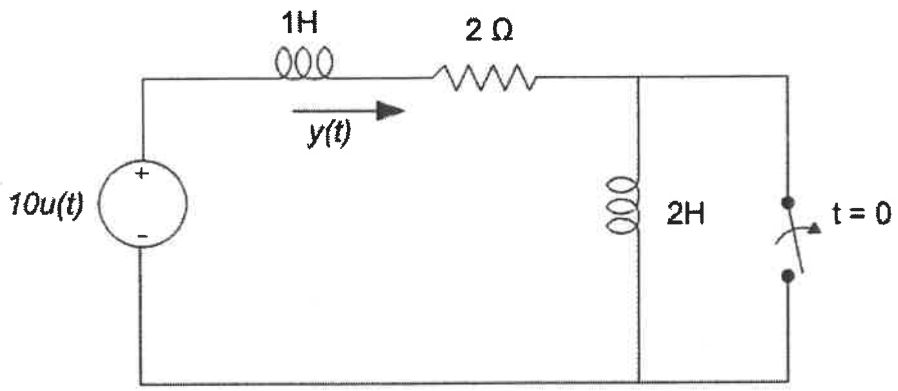

Figure 4

(b) Consider a continuous-time LTI system with the system transfer function as following:

$$H(s) = \frac{3s + 7}{(s + 1)(s + 2)}$$

Identify a state representation of the system using parallel form realization in matrix form.

(10 marks)

END OF EXAMINATION PAPER

Table of formulae for LEB22503 Signals and Systems

(For use during examination only)

Convolution Table

No.	$x_1(t)$	$x_2(t)$	$x_1(t) * x_2(t) = x_2(t) * x_1(t)$
1	$x(t)$	$\delta(t-T)$	$x(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\dfrac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\dfrac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\dfrac{1}{2}t^2e^{\lambda t}u(t)$
7	$t^N u(t)$	$e^{\lambda t}u(t)$	$\dfrac{N!\,e^{\lambda t}}{\lambda^{N+1}}u(t)-\sum_{k=0}^{N}\dfrac{N!\,t^{N-k}}{\lambda^{k+1}(N-k)!}u(t)$
8	$t^M u(t)$	$t^N u(t)$	$\dfrac{M!N!}{(M+N+1)!}t^{M+N+1}u(t)$
9	$te^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\dfrac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$
10	$t^M e^{\lambda t}u(t)$	$t^N e^{\lambda t}u(t)$	$\dfrac{M!\,N!}{(N+M+1)!}t^{M+N+1}e^{\lambda t}u(t)$
11	$t^M e^{\lambda_1 t}u(t)$ $\lambda_1 \neq \lambda_2$	$t^N e^{\lambda_2 t}u(t)$	$\sum_{k=0}^{M}\dfrac{(-1)^k M!(N+k)!\,t^{M-k}e^{\lambda_1 t}}{k!(M-k)!(\lambda_1-\lambda_2)^{N+k+1}}u(t) + \sum_{k=0}^{N}\dfrac{(-1)^k N!(M+k)!\,t^{N-k}e^{\lambda_2 t}}{k!(N-k)!(\lambda_2-\lambda_1)^{M+k+1}}u(t)$
12	$e^{-\alpha t}\cos(\beta t+\theta)u(t)$	$e^{\lambda t}u(t)$	$\dfrac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$ $\phi=\tan^{-1}[-\beta/(\alpha+\lambda)]$
13	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(-t)$	$\dfrac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1} \quad \mathrm{Re}\,\lambda_2 > \mathrm{Re}\,\lambda_1$
14	$e^{\lambda_1 t}u(-t)$	$e^{\lambda_2 t}u(-t)$	$\dfrac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$

Table of formulae for LEB22503 Signals and Systems

(For use during examination only)

Laplace Transform Table

No.	$x(t)$	$X(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{\lambda t} u(t)$	$\frac{1}{s-\lambda}$
6	$te^{\lambda t} u(t)$	$\frac{1}{(s-\lambda)^2}$
7	$t^n e^{\lambda t} u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$
8a	$\cos bt\, u(t)$	$\frac{s}{s^2+b^2}$
8b	$\sin bt\, u(t)$	$\frac{b}{s^2+b^2}$
9a	$e^{-at}\cos bt\, u(t)$	$\frac{s+a}{(s+a)^2+b^2}$
9b	$e^{-at}\sin bt\, u(t)$	$\frac{b}{(s+a)^2+b^2}$
10a	$re^{-at}\cos(bt+\theta)\, u(t)$	$\frac{(r\cos\theta)s+(ar\cos\theta-br\sin\theta)}{s^2+2as+(a^2+b^2)}$
10b	$re^{-at}\cos(bt+\theta)\, u(t)$	$\frac{0.5re^{j\theta}}{s+a-jb}+\frac{0.5re^{-j\theta}}{s+a+jb}$
10c	$re^{-at}\cos(bt+\theta)\, u(t)$	$\frac{As+B}{s^2+2as+c}$
	$r=\sqrt{\frac{A^2c+B^2-2ABa}{c-a^2}}$ $\theta=\tan^{-1}\left(\frac{Aa-B}{A\sqrt{c-a^2}}\right)$ $b=\sqrt{c-a^2}$	
10d	$e^{-at}\left[A\cos bt+\frac{B-Aa}{b}\sin bt\right]u(t)$	$\frac{As+B}{s^2+2as+c}$
	$b=\sqrt{c-a^2}$	

Table of formulae for LEB22503 Signals and Systems

(For use during examination only)

Summary of Laplace Transform Operation

Operation	$x(t)$	$X(s)$
Addition	$x_1(t) + x_2(t)$	$X_1(s) + X_2(s)$
Scalar multiplication	$kx(t)$	$kX(s)$
Time differentiation	$\frac{dx}{dt}$	$sX(s) - x(0^-)$
	$\frac{d^2x}{dt^2}$	$s^2X(s) - sx(0^-) - \dot{x}(0^-)$
	$\frac{d^3x}{dt^3}$	$s^3X(s) - s^2x(0^-) - s\dot{x}(0^-) - \ddot{x}(0^-)$
	$\frac{d^nx}{dt^n}$	$s^nX(s) - \sum_{k=1}^{n} s^{n-k}x^{(k-1)}(0^-)$
Time integration	$\int_{0^-}^{t} x(\tau)\,d\tau$	$\frac{1}{s}X(s)$
	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{s}X(s) + \frac{1}{s}\int_{-\infty}^{0^-} x(t)\,dt$

Operation	$x(t)$	$X(s)$
Time shifting	$x(t - t_0)u(t - t_0)$	$X(s)e^{-st_0} \quad t_0 \geq 0$
Frequency shifting	$x(t)e^{s_0t}$	$X(s - s_0)$
Frequency differentiation	$-tx(t)$	$\frac{dX(s)}{ds}$
Frequency integration	$\frac{x(t)}{t}$	$\int_s^\infty X(z)\,dz$
Scaling	$x(at),\ a \geq 0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi j}X_1(s) * X_2(s)$
Initial value	$x(0^+)$	$\lim_{s\to\infty} sX(s) \quad (n > m)$
Final value	$x(\infty)$	$\lim_{s\to 0} sX(s) \quad$ [poles of $sX(s)$ in LHP]

Table of formulae for LEB22503 Signals and Systems

(For use during examination only)

Fourier Transform Table

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a > 0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a > 0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a > 0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a > 0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a > 0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a > 0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a > 0$

16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\text{rect}\left(\dfrac{t}{\tau}\right)$	$\tau\,\text{sinc}\left(\dfrac{\omega\tau}{2}\right)$	
18	$\dfrac{W}{\pi}\,\text{sinc}\,(Wt)$	$\text{rect}\left(\dfrac{\omega}{2W}\right)$	
19	$\Delta\left(\dfrac{t}{\tau}\right)$	$\dfrac{\tau}{2}\,\text{sinc}^2\left(\dfrac{\omega\tau}{4}\right)$	
20	$\dfrac{W}{2\pi}\,\text{sinc}^2\left(\dfrac{Wt}{2}\right)$	$\Delta\left(\dfrac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\dfrac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table of formulae for LEB22503 Signals and Systems

(For use during examination only)

Summary of Fourier Transform Operation

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\lvert a \rvert} X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$

Operation	$x(t)$	$X(\omega)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\,du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

Table of formulae for LEB22503 Signals and Systems

(For use during examination only)

Z- Transform Table

No.	x[n]	X[z]
1	$\delta[n-k]$	z^{-k}
2	$u[n]$	$\frac{z}{z-1}$
3	$nu[n]$	$\frac{z}{(z-1)^2}$
4	$n^2u[n]$	$\frac{z(z+1)}{(z-1)^3}$
5	$n^3u[n]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^n u[n]$	$\frac{z}{z-\gamma}$
7	$\gamma^{n-1}u[n-1]$	$\frac{1}{z-\gamma}$
8	$n\gamma^n u[n]$	$\frac{\gamma z}{(z-\gamma)^2}$
9	$n^2\gamma^n u[n]$	$\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\frac{n(n-1)(n-2)\ldots(n-m+1)}{\gamma^m m!}\gamma^n u[n]$	$\frac{z}{(z-\gamma)^{m+1}}$
11a	$\lvert\gamma^n\rvert\, cos\beta n\, u[n]$	$\frac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
11b	$\lvert\gamma\rvert^n \sin\beta n\ u[n]$	$\frac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12a	$r\lvert\gamma\rvert^n\cos(\beta n+\theta)\,u[n]$	$\frac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12b	$r\lvert\gamma\rvert^n\cos(\gamma n+\theta)\,u[n] \qquad \gamma=\lvert\gamma\rvert e^{j\beta}$	$\frac{(0.5re^{j\theta})z}{z-\gamma}+\frac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r\lvert\gamma\rvert^n\cos(\gamma n+\theta)u[n]$ $r=\sqrt{\frac{A^2\lvert\gamma\rvert^2+B^2-2AaB}{\lvert\gamma\rvert^2-a^2}}$ $\beta=cos^{-1}\frac{-a}{\lvert\gamma\rvert},$ $\theta=tan^{-1}\frac{Aa-B}{A\sqrt{\lvert\gamma\rvert^2-a^2}}$	$\frac{z-(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$

Table of formulae for LEB22503 Signals and Systems

(For use during examination only)

Summary of Z-Transform Operation

Operation	x[n]	X[z]
Addition	$x_1[n] + x_2[n]$	$X_1[z] + X_2[z]$
Scalar multiplication	$ax[n]$	$aX[z]$
Right-shifting	$x[n-m]u[n-m]$	$\frac{1}{z^m}X[z]$
	$x[n-m]u[n]$	$\frac{1}{z^m}X[z] + \frac{1}{z^m}\sum_{n=1}^{m} x[-n]z^n$
	$x[n-1]u[n]$	$\frac{1}{z}X[z] + x[-1]$
	$x[n-2]u[n]$	$\frac{1}{z^2}X[z] + \frac{1}{z}x[-1] + x[-2]$
	$x[n-3]u[n]$	$\frac{1}{z^3}X[z] + \frac{1}{z^2}x[-1] + \frac{1}{z}x[-2] + x[-3]$
Left-shiting	$x[n+m]u[n]$	$z^m X[z] - z^m \sum_{n=0}^{m-1} x[n]z^{-n}$
	$x[n+1]u[n]$	$zX[z] - zx[0]$
	$x[n+2]u[n]$	$z^2X[z] - z^2x[0] - zx[1]$
	$x[n+3]u[n]$	$z^3X[z] - z^3x[0] - z^2x[1] - zx[2]$
Multiplication by γ^n	$\gamma^n x[n]u[n]$	$X\left[\frac{z}{\gamma}\right]$
Multiplication by n1	$nx[n]u[n]$	$-z\frac{d}{dz}X[z]$
Time convolution	$x_1[n] * x_2[n]$	$X_1[z]X_2[z]$
Frequency convolution	$x_1[n]x_2[n]$	$\frac{1}{2\pi j}\oint X_1[u]X_2\left[\frac{z}{u}\right]u^{-1}du$
Time reversal	$x[-n]$	$X[1/z]$
Initial value	$x[0]$	$\lim_{z\to}(z-1)X[z]$ poles of $z(z-1)X[z]$ inside the unit circle

Table of formulae for LEB22503 Signals and Systems

(For use during examination only)

Formulae

Series Resonance

Resonance frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}}\ rad/s$$

Half-power frequency,

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Quality factor,

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

Bandwidth,

$$B = \frac{R}{L} = \frac{\omega_0}{Q}$$

For $Q \geq 10$,

$$\omega_1 \cong \omega_0 - \frac{B}{2}, \qquad \omega_2 \cong \omega_0 + \frac{B}{2},$$

Parallel Resonance

$$\omega_0 = \frac{1}{\sqrt{LC}}\ rad/s$$

Half-power frequency,

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Bandwidth,

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

Quality factor,

$$Q = \frac{\omega_o}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

For $Q \geq 10$,

$$\omega_1 \cong \omega_0 - \frac{B}{2}, \qquad \omega_2 \cong \omega_0 + \frac{B}{2},$$

Table of formulae for LEB22503 Signals and Systems

(For use during examination only)

PASSIVE FILTER

Highpass filter

$$H(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

$$\omega_c = \frac{1}{RC}$$

Lowpass filter

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

$$\omega_c = \frac{1}{RC}$$

Bandpass filter

$$H(\omega) = \frac{R}{R + j(\omega L - \frac{1}{\omega C})}$$

$$\omega_c = \frac{1}{\sqrt{RC}}$$

Bandstop filter

$$H(\omega) = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$$

$$\omega_c = \frac{1}{\sqrt{LC}}$$

ACTIVE FILTERS

First-order Lowpass filter

$$H(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_f R_f}$$

$$\omega_c = \frac{1}{R_f C_f}$$

First-order Highpass Filter

$$H(\omega) = -\frac{j\omega C_i R_f}{1 + j\omega C_i R_i}$$

$$\omega_c = \frac{1}{R_i C_i}$$

Bandpass filter

$$H(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_1 R}\frac{j\omega C_2 R}{1 + j\omega C_2 R}$$

$$\omega_2 = \frac{1}{RC_1}$$

$$\omega_1 = \frac{1}{RC_2}$$

$$K = \frac{R_f}{R_i}\frac{\omega_2}{\omega_1 + \omega_2}$$

Table of formulae for LEB22503 Signals and Systems

(For use during examination only)

Notch filter

$$H(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_1 R} - \frac{j\omega C_2 R}{1 + j\omega C_2 R}$$

$$\omega_2 = \frac{1}{RC_1}$$

$$\omega_1 = \frac{1}{RC_2}$$

$$K = \frac{R_f}{R_i}\frac{2\omega_1}{\omega_1 + \omega_2}$$

OTHER TYPES OF FILTER

Butterworth Filters (BF):

Roots of the Butterworth polynomial,

$$s_m = -\sin[(2m-1)(\pi/2n)] + j\cos[(2m-1)(\pi/2n)] = \sigma_m + j\omega_m; \qquad m = 1,2,\ldots,2n$$

The magnitude function of the nth order Butterworth filter:

$$|H_{Bn}(j\omega)| = \frac{1}{\sqrt{[1+(\omega/\omega_c)^{2n}}}; \qquad n = positive\ integer;\ \omega_c = \text{cut-off frequency}$$

Normalizing magnitude function of the nth order Butterworth filter is

$$|H_{Bn}(j\omega)| = \frac{1}{\sqrt{1 + (\omega)^{2n}}}$$

List of polynomials Butterworth Filters (BF) up to n=7:

n	*Polynomial*
1	$(s + 1)$
2	$(s^2 + 1.414s + 1)$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)$
5	$(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)$
6	$(s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.939s + 1)$

Table of formulae for LEB22503 Signals and Systems

(For use during examination only)

Chebyshev Filters (CF):

Minimum value of ripple (dB),

$$dB(\gamma) = 10\log_{10}(1+\varepsilon^2)$$

Roots of the Chebyshev polynomial,

$$s_m = -\left[\sin\left((2m-1)\left(\frac{\pi}{2n}\right)\right)\right]\sinh\left[\left(\frac{1}{n}\right)\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right] + j\left[\cos\left((2m-1)\left(\frac{\pi}{2n}\right)\right)\right]\cosh\left[\left(\frac{1}{n}\right)\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right]$$

Normalizing magnitude function of the nth order Chebyshev filter is

$$|H_{Cn}(j\omega)| = \frac{1}{\sqrt{[1+\xi^2 C_n^2(\omega)]}}$$

Transfer function of CF for normalized frequency,

$$H_{cn}(s) = \frac{K}{(-1)^n \prod_{m=1}^{n}\left(\frac{s}{s_m}-1\right)}$$, K = specified gain.

Transfer function of CF for denormalized frequency,

$$H_{cn}(s) = \frac{K}{(-1)^n \prod_{m=1}^{n}\left(\frac{s}{s_m\omega_c}-1\right)}$$

List of polynomials Chebyshev Filters (CF) up to n=8:

n	$C_n(\omega)$
0	1
1	ω
2	$2\omega^2 - 1$
3	$4\omega^3 - 3\ \omega$
4	$8\omega^4 - 8\omega^2 + 1$
5	$16\omega^5 - 20\omega^3 + 5\omega$
6	$32\omega^7 - 48\omega^4 + 18\omega^2 - 1$
7	$74\omega^7 - 112\omega^5 + 57\omega^3 - 7\omega$
8	$128\omega^8 - 256\omega^7 + 170\omega^4 - 32\omega^2_{+1}$

Table of formulae for LEB22503 Signals and Systems

(For use during examination only)

FIR FILTER:

$$H_{lc}(z) = H_l(z)\ z^{-(N-1)/2}$$

$$H_l(z) = h(0) + \sum_{\ell=1}^{\frac{N-1}{2}} h(\ell T)(z^{\ell} + z^{-\ell})$$

IIR FILTER :

Bilinear z-transform characteristic:

$$s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$$

$$\omega_{ac} = \frac{2}{T} tan\left(\frac{\omega_{dc}T}{2}\right)$$

$n = \frac{log(A^2-1)}{2log\left(\frac{\omega_a}{\omega_c}\right)}$ or since usually A>>1, $n \approx \frac{log\ A}{log\left(\frac{\omega_a}{\omega_c}\right)}$

STATE-SPACE ANALYSIS – Continuous-Time Systems

$$Q(s) = \phi(s)q(0) + \phi(s)BX(s)$$

$$\phi(s) = (sI - A)^{-1}$$

$$Y(s) = CQ(s) + DX(s)$$

$$H(s) = C\phi(s)B + D$$

$$e^{At} = \beta_0 I + \beta_1 A + \beta_2 A^2 + \cdots + \beta_{N-1}A^{N-1}$$

where $\begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{N-1} \end{bmatrix} = \begin{bmatrix} 1 & \lambda_1 & \lambda_1^2 \cdots & \lambda_1^{N-1} \\ 1 & \lambda_2 & \lambda_2^2 \cdots & \lambda_2^{N-1} \\ \vdots & \vdots & \vdots \cdots & \vdots \\ 1 & \lambda_N & \lambda_N^2 \cdots & \lambda_N^{N-1} \end{bmatrix}^{-1} \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \\ \vdots \\ e^{\lambda_N t} \end{bmatrix}$

$$q(t) = e^{At}q(0) + e^{At} * Bx(t)$$

$$y(t) = Ce^{At}q(0) + [Ce^{At}B + D\delta(t)] * x(t) = e^{At}q(0) + h(t) * x(t)$$

$$h(t) = C\phi B + D\delta(t)$$

$$\dot{w} = \hat{A}w + \hat{B}x, \qquad y = \hat{C}w + Dx$$

$$\hat{A} = PAP^{-1},\ \hat{B} = PB,\ \hat{C} = CP^{-1},\ w = Pq$$

$\dot{z} = \Lambda z + \hat{B}x, \quad Y = \hat{C}z + Dx$

$\Lambda = PAP^{-1}$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0\cdots & 0 \\ 0 & \lambda_2 & 0\cdots & 0 \\ \vdots & \vdots & \vdots\cdots & \vdots \\ 0 & 0 & \ldots & \lambda_N \end{bmatrix}$$

STATE-SPACE ANALYSIS – Discrete-Time Systems

$q[n] = A^n q[0] + A^{n-1}u[n-1] * Bx[n]$

$y[n] = Cq + Dx = CA^n q[0] + CA^{n-1}u[n-1] * Bx[n] + Dx$

$A^n = \beta_0 I + \beta_1 A + \beta_2 A^2 + \cdots + \beta_{N-1}A^{N-1}$

where $\begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{N-1} \end{bmatrix} = \begin{bmatrix} 1 & \lambda_1 & \lambda_1^2\cdots & \lambda_1^{N-1} \\ 1 & \lambda_2 & \lambda_2^2\cdots & \lambda_2^{N-1} \\ \vdots & \vdots & \vdots\cdots & \vdots \\ 1 & \lambda_N & \lambda_N^2\cdots & \lambda_N^{N-1} \end{bmatrix}^{-1} \begin{bmatrix} \lambda_1^n \\ \lambda_2^n \\ \vdots \\ \lambda_N^n \end{bmatrix}$

$Q[z] = (I - z^{-1}A)^{-1}q[0] + (zI - A)^{-1}BX[z]$

$Y[z] = C(I - z^{-1}A)^{-1}q[0] + H[z]X[z]$

$H[z] = C(zI - A)^{-1}B + D$