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SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION

SEPTEMBER 2014 SESSION

SUBJECT CODE	:	**NCB10303**
SUBJECT TITLE	:	**MATHEMATICS FOR ENGINEERS 2**
LEVEL	:	**BACHELOR**
TIME / DURATION	:	**9.00 AM – 12.00 PM (3 HOURS)**
DATE	:	**08 JANUARY 2015**

INSTRUCTIONS TO CANDIDATES

1. **Please read the instructions given in the question paper CAREFULLY.**
2. **This question paper is printed on both sides of the paper.**
3. **Please write your answers on the answer booklet provided.**
4. **Answer should be written in blue or black ink except for sketching, graphic and illustration.**
5. **This question paper consists of SIX (6) questions. Answer FIVE (5) questions only.**
6. **Answer all questions in English.**
7. **Appendix is attached.**

THERE ARE 4 PAGES OF QUESTIONS AND 7 PAGES OF APPENDIX, EXCLUDING THIS PAGE.

INSTRUCTION: Answer FIVE (5) questions only.

Please use the answer booklet provided.

Question 1

a) Determine whether $y = 2e^{-x} + 3e^{2x}$ is a solution to the differential equation

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + 2y$$

(6 marks)

b) Solve the given differential equation $x^2 \frac{dy}{dx} = 1 - xy$

(7 marks)

c) The temperature gradient, $\frac{d\theta}{dx}$, of a slab of thickness *t* and with thermal conductivity *k* is given by $\frac{d\theta}{dx} = C$ where C is a constant and θ is a function of *x*. Given that $\theta(0) = \theta_1$, $\theta(t) = \theta_2$ *and* $Q = -kA\frac{d\theta}{dx}$, show that $Q = -kA\frac{(\theta_2 - \theta_1)}{t}$

(7 marks)

Question 2

Consider the following initial-value problem (IVP):

$$y'' - 2y' + 5y = 10\sin x, \quad y(0) = 2, \quad y'(0) = 1$$

a) By solving the auxiliary equation, find the complementary function, y_c.

(5 marks)

b) Find the solution of the IVP, i.e. $y(x) = y_c + y_p$, by using the suitable particular solution, y_p, for the given ODE.

(15 marks)

Question 3

a) The unit step function $u(t)$ is defined as

$$u(t) = \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$$

Show from the definition of Laplace transform that

$$\mathcal{L}\{u(t)\} = \frac{1}{s^2}\left(1 - e^{-s}\right)$$

Hint: Use $L\{u(t)\} = \int_0^{\infty} e^{-st} u(t)\,dt$

(5 marks)

b) The temperature T(*x*) of a fin of a perimeter P with a cross sectional area A is given by

$\frac{d^2T}{dx^2} - \frac{hP}{kA}T = 0$ where h is the convection heat transfer coefficient, *k* is the thermal conductivity and *x* is the distance along the fin. Given that $T(0) = x_0$, $T'(0) = x_1$, Use Laplace Transform to find the expression for T in terms of *x*.

(15 marks)

Question 4

A certain RLC series circuit gives a differential equation in this form :

$\frac{d^2i}{dt^2} + 3\frac{di}{dt} + 2i = 5e^{-3t}$. Solve this differential equation for the initial conditions, when $t = 0$, both $i = 0$ and $\frac{di}{dt} = 0$

(20 marks)

Question 5

a) Determine whether the following function is either even, odd, or neither:

(i) $f(x) = 3x^4 - 4x^3$

(ii) $g(x) = 6x^5 - x^3$

(6 marks)

b) Given the function $f(t)$ of period 2π which is described as

$$f(t) = \begin{cases} 0, & -\pi < t < 0 \\ \pi - t, & 0 \le t < \pi \end{cases}$$

over the interval $-\pi < t < \pi$.

(i) Find the Fourier series of $f(t)$.

(ii) Sketch a graph of $f(t)$ over the interval $[-3\pi, 3\pi]$.

(14 marks)

Question 6

a) Consider the function $f(x)=\begin{cases}0 & -\pi<x<0\\ x & 0<x<\pi\end{cases}$

The Fourier Series for this function on $(-\pi,\pi)$ is :

$$f(x)=\frac{\pi}{4}-\frac{2}{\pi}\left(\cos x+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+....\right)+\left(\sin x-\frac{\sin 2x}{2}+\frac{\sin 3x}{3}+.....\right)$$

Use the series to show that : $\sum_{\substack{n=1\\ n\,odd}}\frac{1}{n^2}=\frac{\pi^2}{8}, when\ f(0)=0$

(6 marks)

b) Given the function $g(t)$ which is described as

$$g(t)=\begin{cases}-t, & -\pi\le t<0\\ t, & 0\le t<\pi\end{cases}$$

and $g(t)=g(t+2\pi)$.

(i) Sketch the waveform of the periodic function above for 3 cycles.

(ii) Obtain the first 4 terms of the Fourier trigonometric series for $g(t)$, using half-range series.

(14 marks)

END OF QUESTIONS

APPENDIX 1

Table of Differentiation

Trigonometric Functions – GENERAL FORM
$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$
$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$
$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$

Exponential Function – GENERAL FORM
$\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$

Logarithmic Function – GENERAL FORM
$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)}f'(x)$

APPENDIX 2

Table of Integration

Trigonometric Functions - GENERALFORM Where : $f(x) = ax + b$
$\int \cos f(x)\,dx = \frac{\sin f(x)}{f'(x)} + C$
$\int \sin f(x)\,dx = \frac{-\cos f(x)}{f'(x)} + C$
$\int \sec^2 f(x)\,dx = \frac{\tan f(x)}{f'(x)} + C$
$\int \sec f(x) \tan f(x)\,dx = \frac{\sec f(x)}{f'(x)} + C$
$\int \csc f(x) \cot f(x)\,dx = \frac{-\csc f(x)}{f'(x)} + C$
$\int \csc^2 f(x)\,dx = \frac{-\cot f(x)}{f'(x)} + C$

Exponential Function - GENERALFORM Where : $f(x) = ax + b$
$\int e^{f(x)}\,dx = \frac{e^{f(x)}}{f'(x)} + C$

Logarithmic Function - GENERALFORM Where : $f(x) = ax + b$
$\int \frac{1}{f(x)}\,dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + C$

APPENDIX 3

Trigonometric Identities and Formulas

FUNDAMENTAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

FORMULAS FOR NEGATIVES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

ADDITION FORMULAS

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

SUBTRACTION FORMULAS

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

HALF-ANGLE FORMULAS

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

DOUBLE-ANGLE FORMULAS

$$\sin 2\theta = 2\sin\theta\sin\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\ldots\ldots\ldots = 1 - 2\sin^2\theta$$

$$\ldots\ldots\ldots = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

PRODUCT-TO-SUM FORMULAS

$$\sin\alpha\sin\alpha c = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos\alpha\cos\alpha s = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

$$\cos\alpha\cos\alpha c = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$\sin\alpha\sin\alpha s = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

SUM-TO-PRODUCT FORMULAS

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

APPENDIX 4

Fundamental Hyperbolic Identities	
$\cosh x + \sinh x = e^{x}$ $\cosh x - \sinh x = e^{-x}$	$\sinh(-x) = -\sinh x$ $\cosh(-x) = \cosh x$
$\operatorname{csch} x = \dfrac{1}{\sinh x}$ $\operatorname{sech} x = \dfrac{1}{\cosh x}$	$\tanh x = \dfrac{\sinh x}{\cosh x}$ $\coth x = \dfrac{1}{\tanh x} = \dfrac{\cosh x}{\sinh x}$
$\cosh^2 x - \sinh^2 x = 1$ $1 - \tanh^2 x = \operatorname{sech}^2 x$ $\coth^2 x - \operatorname{csch}^2 x = 1$	$\sinh 2x = 2\sinh x \cosh x$ $\cosh 2x = \cosh^2 x + \sinh^2 x$ $\cosh 2x = 2\sinh^2 x + 1$ $\cosh 2x = 2\cosh^2 x - 1$ $\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$

$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$
$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$
$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$
$\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$
$\tanh(x + y) = \dfrac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$

APPENDIX 5

TABLE OF LAPLACE TRANSFORMS					
	$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$		$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	$t^n,\ \ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\ \ n=1,2,3,\ldots$	$\frac{1.3.5\ldots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at},\ \ n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	e^{-cs}
27.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	28.	$u_c(t)g(t)$	$e^{-\varpi}\mathcal{L}\{g(t+c)\}$
29.	$e^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\ \ n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)du$	32.	$\int_0^t f(\nu)d\nu$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)dt}{1-e^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\ldots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

APPENDIX 6

<table>
<tr><th colspan="4">FIRST ORDER LINEAR DIFFERENTIAL EQUATION</th></tr>
<tr><td>1.</td><td>General Form of 1st order ODE nonhomogeneous</td><td colspan="2">$\frac{dy}{dx} + P(x)y = Q(x)$</td></tr>
<tr><td>2.</td><td>Integrating Factor</td><td colspan="2">$V(x) = e^{\int P(x)\,dx}$</td></tr>
<tr><td>3.</td><td>General Solution</td><td colspan="2">$V(x)y = \int V(x)\,Q(x)\,dx$</td></tr>
<tr><th colspan="4">SECOND ORDER LINEAR DIFFERENTIAL EQUATION</th></tr>
<tr><td>1.</td><td>General Form of 2nd order ODE nonhomogeneous</td><td colspan="2">$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = Q(x)$</td></tr>
<tr><td rowspan="3">2.</td><td rowspan="3">Complementary Function</td><td>$y_c = Ae^{\alpha x} + Be^{\beta x}$</td><td>2 distinct real roots</td></tr>
<tr><td>$y_c = (A + Bx)e^{ax}$</td><td>2 equal (repeated) real roots</td></tr>
<tr><td>$y_c = e^{ax}(A\sin\beta x + B\cos\beta x)$</td><td>Complex roots</td></tr>
<tr><td rowspan="6">3.</td><td rowspan="6">Particular Solution,</td><td>$Q(x)$</td><td>y_p</td></tr>
<tr><td>x^n
ax^n
$ax^n + bx^{n-1} + \cdots$</td><td>$A + Bx + Cx^2 + \cdots + kx^n$</td></tr>
<tr><td>ae^{bx}</td><td>Ae^{bx}</td></tr>
<tr><td>axe^{bx}</td><td>$Ae^{bx} + Bxe^{bx}$</td></tr>
<tr><td>$a\cos bx$
$a\sin bx$</td><td>$A\sin bx + B\cos bx$</td></tr>
<tr><td>$ax\cos bx$
$ax\sin bx$</td><td>$A\sin bx + B\cos bx + Cx\cos bx + Ex\sin bx$</td></tr>
<tr><td>4.</td><td>General Solution</td><td colspan="2">$y = y_c + y_p$</td></tr>
</table>

APPENDIX 7

FOURIER SERIES

$$f(x)=\frac{a_o}{2}+\sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi x}{L}+b_n\sin\frac{n\pi x}{L}\right],\ -L<x<L,\ T=2L.$$

where the Fourier coefficients, a_o, a_n, b_n, as following:

$$a_o=\frac{1}{L}\int_{-L}^{L}f(x)dx$$

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}dx$$

FOURIER COSINE SERIES

The Fourier series of an **even** function on the interval $-L<x<L$ is the **cosine series**

$$f(x)=\frac{a_o}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}$$

where the Fourier coefficients, a_o, a_n, b_n, as following:

$$a_o=\frac{2}{L}\int_{0}^{L}f(x)dx$$

$$a_n=\frac{2}{L}\int_{0}^{L}f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n=0$$

FOURIER SINE SERIES

The Fourier series of an **odd** function on the interval $-L<x<L$ is the **sine series**

$$f(x)=\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$$

where the Fourier coefficients, a_o, a_n, b_n, as following:

$$a_o=0$$

$$a_n=0$$

$$b_n=\frac{2}{L}\int_{0}^{L}f(x)\sin\frac{n\pi x}{L}dx$$