Document No : UniKL MFI_SD_AC41
Revision No: 02
Effective Date: 01 December 2008

CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
SEPTEMBER 2014 SESSION

SUBJECT CODE	:	**NCB10103**
SUBJECT TITLE	:	**MATHEMATICS FOR ENGINEERS 1**
LEVEL	:	**BACHELOR**
TIME / DURATION	:	**9.00 AM – 12.00 PM (3 HOURS)**
DATE	:	**30 DECEMBER 2014**

INSTRUCTIONS TO CANDIDATES

1. **Please read the instructions given in the question paper CAREFULLY.**
2. **This question paper is printed on both sides of the paper.**
3. **Please write your answers on the answer booklet provided.**
4. **Answer should be written in blue or black ink except for sketching, graphic and illustration.**
5. **This question paper consists of FIVE (5) questions. You are required to answer ALL questions in English.**
6. **Formula is appended.**

THERE ARE 8 PAGES OF QUESTIONS, INCLUDING THIS PAGE.

(Total : 100 marks)

Answer ALL questions.

Please use the answer booklet provided.

Question 1

(a) Solve the following and write the answer in a standard complex number form:

$$\frac{3+2i}{6+7i}+\frac{2+3i}{1+2i}$$

[8 marks]

(b) A copper cable for telecommunication purposes has the following parameter constants:

Resistance, *R*	100 Ω
Inductance, *L*	0.2 m*H*
Conductance, *G*	2μ siemens
Capacitance, *C*	3n *F*

For $\omega = 1000$rad/s, evaluate the characteristic impedance, Z_0 where $Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$

and write the answer in polar form.

[12 marks]

Question 2

(a) An astronaut measures the percentage of ore contents of a planet Mars based on three collected Martian ores. The ores consist of Silicon, Pyroxene and Iron. The results are shown below:

	Ore 1	Ore 2	Ore 3
Silicon	0	1	1
Pyroxene	2	0	3
Iron	1	1	1

If the ores are mixed together so there are *x* kg of Ore 1, *y* kg of Ore 2, and *z* kg of Ore 3, the amount of each material in the mixture can be computed as shown in the system of linear equation below:

$$(0)x+(1)y+(1)z=2$$
$$(2)x+(0)y+(3)z=5$$
$$(1)x+(1)y+(1)z=3$$

Solve *x*, *y* and *z* (the weight of each element) using Gaussian elimination method

[8 marks]

(b) Given the linear system as shown below:

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

Evaluate

i. the eigenvalues

ii. the eigenvectors corresponding to each eigenvalue

[12 marks]

Question 3

(a) Let $A=(0,4,1)$, $B=(-4,6,7)$, $C=(3,2,-4)$, $D=(5,0,-8)$ be four points in space.

(i) Prove than the lines AB and CD intersect. (7 marks)

(ii) Hence, find the intersection point between the two lines. (3 marks)

(b) If $\underset{\sim}{u}=\begin{pmatrix}-3\\3\\3\end{pmatrix}$ and $\underset{\sim}{v}=\begin{pmatrix}1\\5\\-1\end{pmatrix}$, write down the components of $\underset{\sim}{u}+\underset{\sim}{v}$ and $\underset{\sim}{u}-\underset{\sim}{v}$.

Hence show that $\underset{\sim}{u}+\underset{\sim}{v}$ and $\underset{\sim}{u}-\underset{\sim}{v}$ are perpendicular. (5 marks)

(c) In an orienteering race, you walk 100 m due east and then walk N70°E for 60 m. How far are you from your starting position, and at what bearing?

(5 marks)

Question 4

(a) Calculate

$$\lim_{(x,y)\to(1,1)} \frac{x^2y-y^2x}{\sqrt{x}-\sqrt{y}}$$

[4 marks]

(b) Let $w=x^2y+y^2z^3$, where $x=r\cos s$, $y=r\sin s$ and $z=re^s$. Determine the value of $\frac{\partial w}{\partial s}$ when $r=1$ and $s=0$.

[6 marks]

(c) Given $f(x,y)=x^2+y^2-xy+9x$

i. Determine the critical point of $f(x,y)$.

[4 marks]

ii. Classify the critical points as a maximum, minimum or saddle point using the second derivative test.

[6 marks]

Question 5

(a) Evaluate the $\iint_R xy\, dA$, where R is the region in the first quadrant that lies between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

[11 marks]

(b) Solve $\iiint_R z\, \mathrm{dV}$, where R is the solid in the first octant bounded by the graphs of $z = 1 - x^2$ and $y = x$. [9 marks]

Appendix

FUNDAMENTAL IDENTITIES	FORMULAS FOR NEGATIVES
$\csc\theta = \dfrac{1}{\sin\theta}$	$\sin(-\theta) = -\sin\theta$
$\sec\theta = \dfrac{1}{\cos\theta}$	$\cos(-\theta) = \cos\theta$
$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$	$\tan(-\theta) = -\tan\theta$
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\csc(-\theta) = -\csc\theta$
$\sin^2\theta + \cos^2\theta = 1$	$\sec(-\theta) = \sec\theta$
$1 + \tan^2\theta = \sec^2\theta$	$\cot(-\theta) = -\cot\theta$
$1 + \cot^2\theta = \csc^2\theta$	

ADDITION FORMULAS	SUBTRACTION FORMULAS
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$	$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

HALF-ANGLE FORMULAS	DOUBLE-ANGLE FORMULAS
$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$	$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ .... $= 1 - 2\sin^2\theta$ .... $= 2\cos^2\theta - 1$
$\tan\dfrac{\theta}{2} = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$	$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

PRODUCT-TO-SUM FORMULAS	SUM-TO-PRODUCT FORMULAS
$\sin\alpha\cos\beta = \dfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$	$\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$
$\cos\alpha\sin\beta = \dfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$	$\sin\alpha - \sin\beta = 2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$
$\cos\alpha\cos\beta = \dfrac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$	$\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$
$\sin\alpha\sin\beta = \dfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$	$\cos\alpha - \cos\beta = -2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \cos x\, dx = \sin x + c$	$\int \cos f(x)\, dx = \frac{\sin f(x)}{f'(x)} + c$
$\int \sin x\, dx = -\cos x + c$	$\int \sin f(x)\, dx = \frac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x\, dx = \tan x + c$	$\int \sec^2 f(x)\, dx = \frac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x\, dx = \sec x + c$	$\int \sec f(x) \tan f(x)\, dx = \frac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x\, dx = -\csc x + c$	$\int \csc f(x) \cot f(x)\, dx = \frac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x\, dx = -\cot x + c$	$\int \csc^2 f(x)\, dx = \frac{-\cot f(x)}{f'(x)} + c$

$M_y = \iint_R x\sigma(x,y)dA$	$M_x = \iint_R y\sigma(x,y)dA$
$\bar{x} = \frac{M_y}{m}$	$\bar{y} = \frac{M_x}{m}$
$\bar{x} = \frac{1}{AreaR}\iint_R xdA$	$\bar{y} = \frac{1}{AreaR}\iint_R ydA$
$I_x = \iint_R y^2\sigma(x,y)dA$	$I_y = \iint_R x^2\sigma(x,y)dA$

$G(a,b) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix}$	$\frac{d}{dt}\left[F(t) \bullet G(t)\right] = F'(t) \bullet G(t) + F(t) \bullet G'(t)$
$L = \int_a^b \lvert v \rvert dt$	$\frac{d}{dt}\left[F(t) \times G(t)\right] = F'(t) \times G(t) + F(t) \times G'(t)$
$s(t) = \int_{t_0}^{t} \lvert v(t) \rvert dt$	$\frac{d}{dt}\left[F(f(t))\right] = f'(t)F'(f(t))$
$T = \frac{dF/dt}{ds/dt} = \frac{v}{\lvert v \rvert}$	$N = \frac{dT/dt}{\lvert dT/dt \rvert}$
$\tau = \frac{\lvert dB/dt \rvert}{\lvert dr/dt \rvert}$	$k = \frac{d\underline{T}/dt}{\lvert d\underline{r}(t)/dt \rvert}$

Coordinate Conversion Formulas

CYLINDRICAL TO RECTANGULAR	SPHERICAL TO RECTANGULAR	SPHERICAL TO CYLINDRICAL
$x = r\cos\theta$	$x = \rho\sin\phi\cos\theta$	$r = \rho\sin\phi$
$y = r\sin\theta$	$y = \rho\sin\phi\sin\theta$	$z = \rho\cos\phi$
$z = z$	$z = \rho\cos\phi$	$\theta = \theta$

Corresponding formulas for dV in triple integrals:

$$\begin{aligned} dV &= dx\,dy\,dz \\ &= dz\,r\,dr\,d\theta \\ &= \rho^2\sin\phi\,d\rho\,d\phi\,d\theta \end{aligned}$$