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SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
SEPTEMBER 2014 SESSION

SUBJECT CODE	:	**FKB20203 / FKB24302 / FKB23302**
SUBJECT TITLE	:	**ENGINEERING TECHNOLOGY MATHEMATICS 2**
LEVEL	:	**BACHELOR**
TIME / DURATION	:	**9.00 AM – 12.00 PM (3 HOURS)**
DATE	:	**6 JANUARY 2015**

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of SIX (6) questions. Answer five (5) questions only.
6. Answer all questions in English.
7. Trigonometric Formulas, Table of Differentiation and Integration are appended.

THERE ARE 5 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

INSTRUCTION: Answer FIVE questions only (Total: 100 marks)
Show all the necessary steps. A correct answer with no relevant work may received no credit while an incorrect answer with some correct work may received partial credit.

Question 1

(a) A rational function is defined as follows: $f(x) = \dfrac{x^2 + x - 2}{x^2 - x - 6}$

(i) Determine the domain of $f(x)$. (use the interval notation)

(3 Marks)

(ii) Calculate the following limits:

(1) $\lim\limits_{x \to -2} \dfrac{x^2 + x - 2}{x^2 - x - 6}$ **(3 Marks)**

(2) $\lim\limits_{x \to 3} \dfrac{x^2 + x - 2}{x^2 - x - 6}$, what conclusion can you make?

(3 Marks)

(3) $\lim\limits_{x \to \infty} \dfrac{x^2 + x - 2}{x^2 - x - 6}$, what conclusion can you make?

(4 Marks)

(iii) Determine whether $f(x)$ is even, odd or neither even nor odd

(2 Marks)

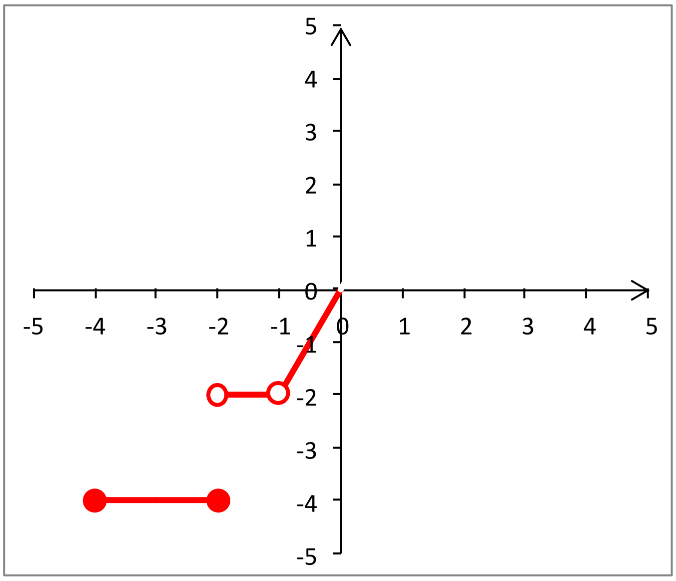
(b) A function is defined as $f(x) = \dfrac{x+1}{2-x}$.

Determine it's derivative, $\dfrac{df}{dx}$ by applying the **definition of derivative**.

(5 Marks)

Question 2

(a) The graph of a piecewise function $g(x)$ is given below.



(i) **Copy** the graph of $g(x)$ above onto **APPENDIX 4**. Complete the graph if the function $g(x)$ is an **ODD** function.

(2 Marks)

(ii) Based on your answer in part (a)-i, determine the missing part of the piecewise function below.

$$g(x) = \begin{cases} \_\_\_\_ & \text{if} \quad -4 \le x \le -2 \\ \_\_\_\_ & \text{if} \quad -2 < x < -1 \\ \_\_\_\_ & \text{if} \quad \_\_\_\_\_\_\_\_\_\_ \\ 2 & \text{if} \quad \_\_\_\_\_\_\_\_\_\_ \\ 4 & \text{if} \quad \_\_\_\_\_\_\_\_\_\_ \end{cases}$$

(3 Marks)

(iii) Determine the domain of $g(x)$. Use the interval notation.

(2 Marks)

(iv) Determine $\lim_{x \to 2^-} g(x)$ and $\lim_{x \to 2^+} g(x)$. What can you say about $\lim_{x \to 2} g(x)$?

(3 Marks)

(v) By using the **definition of continuity at a point**, explain why the function $g(x)$ is discontinuous at $x = -1$

(3 Marks)

(vi) Determine the type of discontinuity at $x = -2$ and $x = -1$

(2 Marks)

(b) Integrate the following by using **Substitution Method**

$$\int \frac{\sin(\ln x)}{x}\, dx$$

(5 Marks)

Question 3

(a) A function with two variables is defined as $f(x,y) = x^3 + y^2 - 6xy + 9x + 5y + 2$

Show that there are two critical points. Classify these points as maximum, minimum or saddle point.

(15 Marks)

(b) Let $Z = x^2y - y^2$ where x and y are defined as $x = t^2$ and $y = 2t$. Determine $\frac{dZ}{dt}$ by applying the multivariable **chain rule**.

(5 Marks)

Question 4

(a) Evaluate $\int_1^2 \frac{1}{x^2 - 2x + 2}\,dx$. (HINT: use completing the square)

(7 Marks)

(b) Show that $\int_4^6 \frac{x-5}{(x-2)(x-3)}\,dx = \ln\left(\frac{8}{9}\right)$

(8 Marks)

(c) Show that $\int_{-1}^{2}\int_{0}^{3} xe^{xy}\,dy\,dx = \frac{e^6}{3} - \frac{e^{-3}}{3} - 3$

(5 Marks)

Question 5

(a) Show that the solution of $y \tan x \dfrac{dy}{dx} = (4 + y^2)\sec^2 x$ is $y^2 = A\tan^2 x - 4$.

(15 Marks)

(b) By using **integration by parts**, show that $\int x e^{3x}\,dx = \dfrac{1}{9}(3x - 1)e^{3x} + C$

(5 Marks)

Question 6

(a) A nonhomogeneous second order differential equation is given as

$$y'' + 4y' + 5y = 2e^{-2x}$$

(i) Determine the **general solution** of the equation

(6 Marks)

(ii) Determine the **particular solution** given the initial conditions $y(0) = 1$ and $y'(0) = -2$

(9 Marks)

(Note that $y'' = \dfrac{d^2y}{dx^2}$, $y' = \dfrac{dy}{dx}$)

(b) Apply **Logarithmic Differentiation** to determine the derivative, $\dfrac{dy}{dx}$ of the following function.

$$y = \frac{(x + 1)(x + 5)^3}{(x^2 + 4x - 3)^2}$$

(5 Marks)

END OF QUESTION

APPENDIX 1 - Trigonometric Identities and Formulas

Fundamental Identities	Formulas for Negatives
$\csc A = \frac{1}{\sin A}$ $\sec A = \frac{1}{\cos A}$ $\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A}$ $\tan A = \frac{\sin A}{\cos A}$ $\sin^2 A + \cos^2 A = 1$ $1 + \tan^2 A = \sec^2 A$ $1 + \cot^2 A = \csc^2 A$	$\sin(-A) = -\sin A$ $\cos(-A) = \cos A$ $\tan(-A) = -\tan A$ $\csc(-A) = -\csc A$ $\sec(-A) = \sec A$ $\cot(-A) = -\cot A$
Addition Formulas	**Subtraction Formulas**
$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$	$\sin(A-B) = \sin A \cos B - \cos A \sin B$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
Half Angle Formulas	**Double Angle Formulas**
$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$ $\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$ $\tan\left(\frac{A}{2}\right) = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}$	$\sin 2A = 2\sin A \cos A$ $\cos 2A = \cos^2 A - \sin^2 A$ $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$
Product to Sum Formulas	**Sum to Product Formulas**
$\sin A \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$ $\cos A \sin B = \frac{1}{2}(\sin(A+B) - \sin(A-B))$ $\cos A \cos B = \frac{1}{2}(\cos(A+B) + \cos(A-B))$ $\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$	$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

Logarithmic Properties	
$\ln(MN) = \ln M + \ln N$ $\ln\left(\frac{M}{N}\right) = \ln M - \ln N$	$\ln M^K = k\ln M$ $e^{\ln x} = x$ $\ln e^x = x$

APPENDIX 2 – Table of Differentiation (General Form)

Trigonometric Functions	Hyperbolic Functions	Exponential Functions
$\frac{d}{dx}(\sin u)=\cos u\frac{du}{dx}$	$\frac{d}{dx}(\sinh u)=\cosh u\frac{du}{dx}$	$\frac{d}{dx}(e^u)=e^u\frac{du}{dx}$
$\frac{d}{dx}(\cos u)=-\sin u\frac{du}{dx}$	$\frac{d}{dx}(\cosh u)=\sinh u\frac{du}{dx}$	$\frac{d}{dx}(a^u)=a^u\ln a\frac{du}{dx}$
$\frac{d}{dx}(\tan u)=\sec^2 u\frac{du}{dx}$	$\frac{d}{dx}(\tanh u)=\operatorname{sec h}^2 u\frac{du}{dx}$	
$\frac{d}{dx}(\csc u)=-\csc u\cot u\frac{du}{dx}$	$\frac{d}{dx}(\operatorname{csc h} u)=-\operatorname{csc h} u\coth u\frac{du}{dx}$	
$\frac{d}{dx}(\sec u)=\sec u\tan u\frac{du}{dx}$	$\frac{d}{dx}(\operatorname{sec h} u)=-\operatorname{sec h} u\tanh u\frac{du}{dx}$	
$\frac{d}{dx}(\cot u)=-\csc^2 u\frac{du}{dx}$	$\frac{d}{dx}(\coth u)=-\operatorname{csc h}^2 u\frac{du}{dx}$	
Inverse Trigonometric Functions	**Inverse Hyperbolic Functions**	**Logarithmic Functions**
$\frac{d}{dx}(\sin^{-1}u)=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$	$\frac{d}{dx}(\sinh^{-1}u)=\frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$	$\frac{d}{dx}(\ln u)=\frac{1}{u}\frac{du}{dx}$
$\frac{d}{dx}(\cos^{-1}u)=\frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$	$\frac{d}{dx}(\cosh^{-1}u)=\frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$	$\frac{d}{dx}(\log_a u)=\frac{1}{u\ln a}\frac{du}{dx}$
$\frac{d}{dx}(\tan^{-1}u)=\frac{1}{1+u^2}\frac{du}{dx}$	$\frac{d}{dx}(\tanh^{-1}u)=\frac{1}{1-u^2}\frac{du}{dx}$	
$\frac{d}{dx}(\csc^{-1}u)=\frac{-1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$	$\frac{d}{dx}(\operatorname{csc h}^{-1}u)=\frac{-1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx}$	
$\frac{d}{dx}(\sec^{-1}u)=\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$	$\frac{d}{dx}(\operatorname{sec h}^{-1}u)=\frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx}$	
$\frac{d}{dx}(\cot^{-1}u)=\frac{-1}{1+u^2}\frac{du}{dx}$	$\frac{d}{dx}(\coth^{-1}u)=\frac{1}{1-u^2}\frac{du}{dx}$	

Differentiation Rules

$\frac{d}{dx}(x^n)=nx^{n-1}$, $\frac{d}{dx}(ax^n)=anx^{n-1}$

$\frac{d}{dx}(C)=0$, where C is a constant

$\frac{d}{dx}(u\pm v)=\frac{du}{dx}\pm\frac{dv}{dx}$

$\frac{d}{dx}(u.v)=v\frac{du}{dx}+u\frac{dv}{dx}$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

$\frac{d}{dx}(u\circ v)=\frac{d}{dx}(u(v))=\frac{du}{dv}\times\frac{dv}{dx}$

APPENDIX 3 – Table of Integration (General Form)

Trigonometric Functions (where u=ax+b)	Inverse Trigonometric Functions
$\int \cos u\,dx = \frac{\sin u}{u'} + C$	$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \sin u\,dx = -\frac{\cos u}{u'} + C$	$\int \frac{-1}{\sqrt{a^2 - x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \sec^2 u\,dx = \frac{\tan u}{u'} + C$	$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \csc u \cot u\,dx = -\frac{\csc u}{u'} + C$	$\int \frac{-1}{\lvert x \rvert \sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \sec u \tan u\,dx = \frac{\sec u}{u'} + C$	$\int \frac{1}{\lvert x \rvert \sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \csc^2 u\,dx = -\frac{\cot u}{u'} + C$	$\int \frac{-1}{a^2 + x^2}\,dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$
Hyperbolic Functions (where u=ax+b)	**Inverse Hyperbolic Functions**
$\int \cosh u\,dx = \frac{\sinh u}{u'} + C$	$\int \frac{1}{\sqrt{a^2 + x^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad a > 0$
$\int \sinh u\,dx = \frac{\cosh u}{u'} + C$	$\int \frac{-1}{\sqrt{x^2 - a^2}}\,dx = \cosh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad x > a$
$\int \operatorname{sech}^2 u\,dx = \frac{\tanh u}{u'} + C$	$\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \operatorname{csch} u \coth u\,dx = -\frac{\operatorname{csch} u}{u'} + C$	$\int \frac{1}{x\sqrt{a^2 + x^2}}\,dx = -\frac{1}{a}\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + C \quad , \quad 0 < x < a$
$\int \operatorname{sech} u \tanh u\,dx = \frac{-\operatorname{sech} u}{u'} + C$	$\int \frac{1}{x\sqrt{a^2 - x^2}}\,dx = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right) + C \quad , \quad 0 < x < a$
$\int \operatorname{csch}^2 u\,dx = -\frac{\coth u}{u'} + C$	$\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
Exponential Functions (where u=ax+b)	**Form of 1/u Functions (where u=ax+b)**
$\int e^u\,dx = \frac{e^u}{u'} + C$	$\int \frac{1}{u}\,dx = \frac{\ln u}{u'} + C$
Other form of integration	
$\int \frac{1}{z^2 - A^2}\,dz = \frac{1}{2A}\ln\left(\frac{z - A}{z + A}\right) + C$	
$\int \frac{1}{A^2 - z^2}\,dz = \frac{1}{2A}\ln\left(\frac{A + z}{A - z}\right) + C$	

APPENDIX 4 – Graph paper

(Please detach this page and attach it to your answer booklet)

Name: ___________________________________ Student ID: ____________________