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CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR

Malaysia France Institute

FINAL EXAMINATION

SEPTEMBER 2014 SESSION

SUBJECT CODE	:	**FWB22102**
SUBJECT TITLE	:	**FINITE ELEMENT ANALYSIS**
LEVEL	:	**BACHELOR**
TIME / DURATION	:	**2.00 PM – 5.00 PM (3.0 HOURS)**
DATE	:	**11 JANUARY 2015**

INSTRUCTIONS TO CANDIDATES

1. **Please read the instructions given in the question paper CAREFULLY.**
2. **This question paper is printed on both sides of the paper.**
3. **Please write your answers on the answer booklet provided.**
4. **Answer should be written in blue or black ink except for sketching, graphic and illustration.**
5. **This question paper consists of TWO (2) sections. Section A and B. Answer all questions in Section A. For Section B, answer THREE (3) questions only.**
6. **Answer all questions in English.**

THERE ARE 7 PAGES OF QUESTIONS AND 3 PAGES OF FORMULAE EXCLUDING THIS PAGE.

SECTION A (Total: 25 marks)

INSTRUCTION: Answer ALL questions.

Please use the answer booklet provided.

Question 1

(a) Write the process flow to simulate structure parts using Finite element software.

(3 marks)

(b) *Hermite shape function* is the shape function for beam element, which satisfy nodal value and slope continuity requirement, Draw **FOUR (4)** *Hermite shape function* pattern that have been stated.

(6 marks)

(c) Solve the following system of linear equation using Gaussian elimination. Show clearly all solution steps.

$$- X_1 + 3X_2 - 2X_3 = 2$$
$$2 X_1 - 4X_2 + 2X_3 = 1$$
$$4X_2 + X_3 = 3$$

(6 marks)

(d) Gives two square matrices as follow:

$$[A] = \begin{bmatrix} -6 & 2 & -9 & 7 \\ 6 & 12 & -8 & 4 \end{bmatrix}; \qquad [B] = \begin{bmatrix} -8 & 9 \\ -3 & -7 \end{bmatrix}$$

i. Find the product of $[A]^T [B]$;

ii. Determine the determinant of $[B]$

(5 marks)

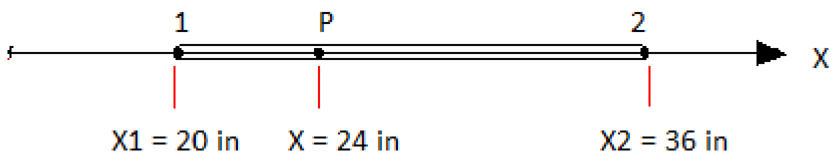

Figure 1

(e) **Figure 1** shows a genetic finite element model constructed using 1-D element

(a) Evaluate ξ, N_1 and N_2 at point P

(3 marks)

(b) If q_1 = 0.003 and q_2 = -0.005 in, determine the value of displacement u at point P.

(2 marks)

SECTION B (Total: 75 marks)

INSTRUCTION: Answer THREE (3) questions only.

Please use the answer booklet provided

Question 2

Consider the composite bar ABCD with 10 mm thickness consisting of a tapered steel (E = 200 GPa) section that is perfectly bonded to an aluminium (E = 70 GPa) section of uniform cross section and subjected to the load as shown in **Figure Q2.** There is a gap of 0.01 mm in between end D and the rigid wall E. The approximate solution for the bar can be obtained using the finite element method by employing bar elements.

(a) Transform the stepped–bar into finite element model using one-dimensional elements. Show clearly the element number, local and global node number, the global axis direction, the applied load and the constraint.

(3 marks)

(b) Determine the displacement of the midpoint of aluminium section.

(12 marks)

(c) Determine the maximum normal stress in the steel and aluminium section.

(5 marks)

(d) Determine the reaction force.

(5 marks)

(Note: Under the given loading, the free end D displaced more than 0.01 mm if there is no rigid wall E.)

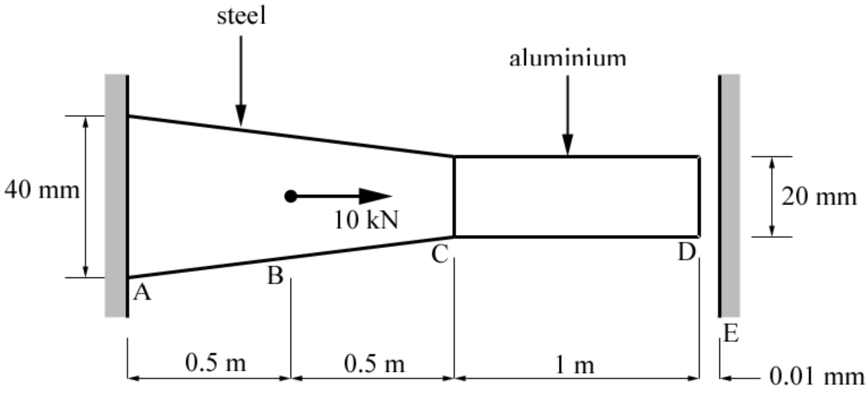

Figure Q2

Question 3

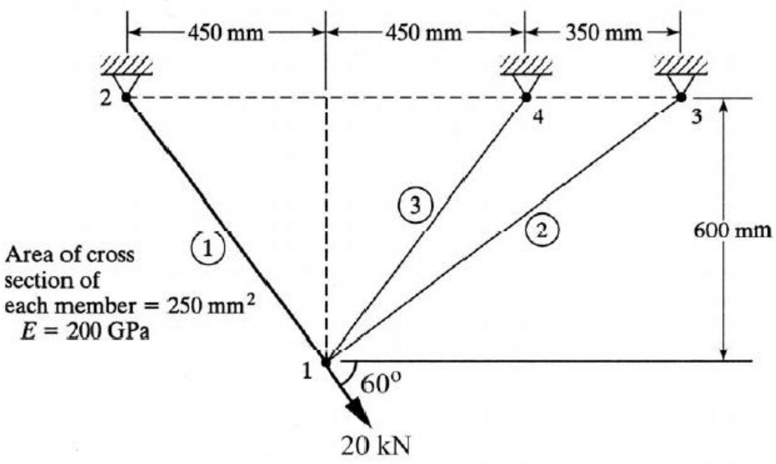

Figure Q3

A three-bar truss as shown in **Figure 3** is loaded with a point load of 20 kN. The cross sectional area and the modulus of elasticity are as shown in the figure. The nodes and elements numbering as shown in the figure are to be used in the solution. The nodes coordinate and the elements connectivity are as shown in the following tables:

NODE COORDINATES		
NODE	x (mm)	y (mm)
1	0	0
2	-450	600
3	800	600
4	450	600

ELEMENT CONNECTIVITY		
ELEMENT	1st NODE	2nd NODE
1	1	2
2	1	3
3	1	4

Based on the nodes coordinate and elements connectivity given above, the stiffness matrices for elements 2 and 3 are:

$$\left[k^{(2)}\right] = 5\times10^{7}\begin{bmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{bmatrix} \quad \text{(N/m)}$$

$$\left[k^{(3)}\right] = 6.67\times10^{7}\begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 \\ 0.48 & 0.64 & -0.48 & -0.64 \\ -0.36 & -0.48 & 0.36 & 0.48 \\ -0.48 & -0.64 & 0.48 & 0.64 \end{bmatrix} \quad \text{(N/m)}$$

Determine the:

(a) Stiffness matrix for element 1

(5 marks)

(b) Global stiffness matrix for the whole structure, [K]

(5 marks)

(c) Global load vector, {F}

(5 marks)

(d) Reduced system of linear equations

(5 marks)

(e) Displacements of node 1

(5 marks)

Question 4

A propped cantilever beam is loaded as shown in **Figure Q4.** The beam is fixed at A and roller supported at C. The cross-section of the beam is circular with 3 cm diameter and 2 cm diameter for part Ab and part BC, respectively. The whole beam is made of steel with Young modulus of 200 GPa. By using point A, B, and C as node 1, 2, and 3, respectively, and the elements connectivity as shown in Table 1:

(a) Sketch the finite element model for the beam, label each node and element and also show clearly the degree of freedom

(2 marks)

(b) Calculate the stiffness matrix for each element

(4 marks)

(c) Write the global stiffness matrix for the whole beam

(4 marks)

(d) Calculate the equivalent nodal force and moment due to the uniformly distributed Load.

(3 marks)

(e) Assemble the global force vector

(2 marks)

(f) Write the system of linear equation

(2 marks)

(g) Write the reduce system of linear equations

(2 marks)

(h) Solve for the unknown degree of freedom

(2 marks)

(i) Calculate the reaction at A and C

(2 marks)

(j) Calculate the maximum bending stress at B

(2 marks)

Note: Second moment of area for circular cross section I= $\pi d^4/64$, and neglect any stress concentration)

Table 1

ELEMENT	1^{ST} NODE	2^{ND} NODE
1	1	2
2	2	3

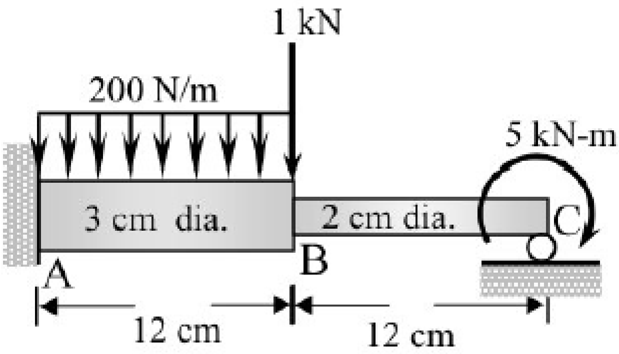

Figure Q4

Question 5

A 20 mm x 20 mm silicon chip is 4 mm thick and is attached to a 10 mm thick aluminium substrate. The chip and the substrate are separated by a 0.5 mm thick epoxy joint, as illustrated in **Figure Q5.** The upper surface of the chip is exposed to an ambient air which is at a temperature of 25 ^{o}C, and provides a convection coefficient of 150 W/m^2 ^{o}C. A uniform heat generation, Q = 10 x 10^6 W/m^3 occurs in the silicon chip. At steady-state condition, the temperature at the bottom surface of the substrate is found to be uniform at 30 ^{o}C. Model the assembly using three one-dimensional heat transfer element and assume one-dimensional heat flow in y-direction.

Determine the:

(a) Temperature distribution within the assembly

(b) Amount of heat dissipated by the aluminium substrate

Data: Thermal conductivities for the silicon chip, epoxy and aluminium substrate are 110, 5 and 230 W/m ^{o}C respectively

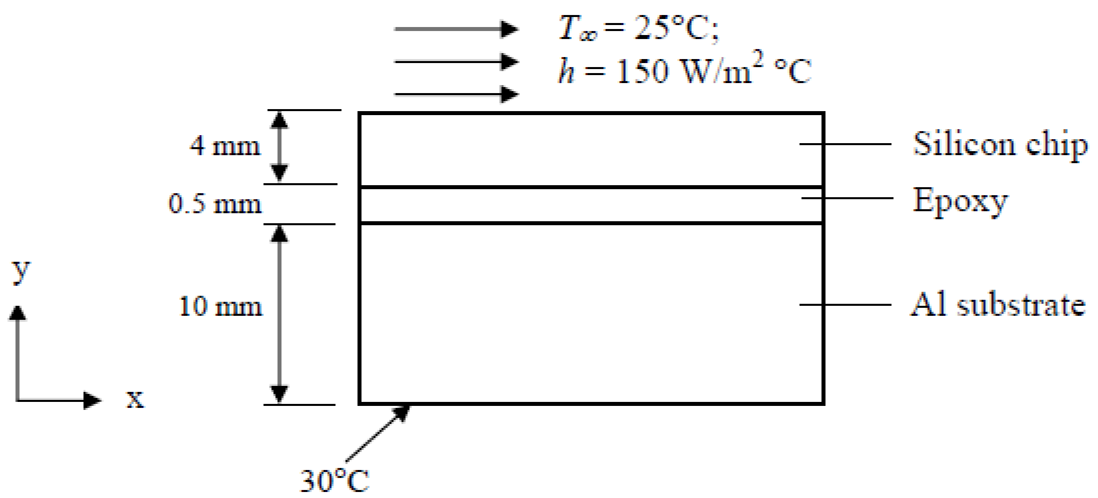

Figure Q5

END OF QUESTION

FORMULAE

1. Natural or intrinsic coordinate system,

$$\xi = \frac{2}{(x_2 - x_1)}(x - x_1) - 1$$

2. Interpolar function

$$N_1(\xi) = \frac{1-\xi}{2} \quad \text{and} \quad N_2(\xi) = \frac{1+\xi}{2}$$

3. Stress-strain relation

$$\sigma = \frac{E}{l_e}\begin{bmatrix} -1 & 1 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

4. Element stiffness matrix

$$[k] = \frac{A_e E_e}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

5. Element force vector due to traction force, f_b

$$\{f\} = \frac{A_e \, l_e \, E_e}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

6. Element force vector due to body force, T

$$\{T\} = \frac{T l_e}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

7. Formula for Evaluating *l* and *m*

$$l = \cos\theta = \frac{x_2 - x_1}{l_e}$$

$$m = \sin\theta = \frac{y_2 - y_1}{l_e}$$

$$l_e = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$$

8. Element Stiffness Matrix (Trusses)

$$[k]^e = \frac{E_e A_e}{l_e} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ lm & -m^2 & lm & m^2 \end{bmatrix}$$

9. Stress Calculations

$$\sigma = \frac{E}{l_e}[-l \ \ -m \ \ l \ \ m] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

10. Beam

$$\mathbf{k}^e = \frac{EI}{\ell_e^3} \begin{bmatrix} 12 & 6\ell_e & -12 & 6\ell_e \\ 6\ell_e & 4\ell_e^2 & -6\ell_e & 2\ell_e^2 \\ -12 & -6\ell_e & 12 & -6\ell_e \\ 6\ell_e & 2\ell_e^2 & -6\ell_e & 4\ell_e^2 \end{bmatrix};$$

$$H_1 = \tfrac{1}{4}(1-\xi)^2(2+\xi)$$
$$H_2 = \tfrac{1}{4}(1-\xi)^2(\xi+1)$$
$$H_3 = \tfrac{1}{4}(1+\xi)^2(2-\xi)$$
$$H_4 = \tfrac{1}{4}(1+\xi)^2(\xi-1)$$

$$\{f\}^e = \left[\frac{pl_e}{2}, \quad \frac{pl_e^2}{12}, \quad \frac{pl_e}{2}, \quad -\frac{pl_e^2}{12}\right]^T;$$

$$v(\xi) = H_1 v_1 + \frac{l_e}{2} H_2 v_1' + H_3 v_2 + \frac{l_e}{2} H_4 v_2'$$

$$M = \frac{EI}{l_e^2}\left[6\xi q_1 + (3\xi - 1)l_e q_2 - 6\xi q_3 + (3\xi + 1)l_e q_4\right]; \qquad \sigma = \frac{M}{I} \cdot y$$

11. Constant Strain Triangles

$$[B] = \frac{1}{\det[J]} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \qquad [J] = \begin{bmatrix} x_{13} & y_{13} \\ x_{23} & y_{23} \end{bmatrix}$$

$$A_e = \frac{1}{2}\left|\det[J]\right| \qquad \{\sigma\}^e = [D][B]\{q\}^e$$

$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \qquad [D] = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} (1-\nu) & \nu & 0 \\ \nu & (1-\nu) & 0 \\ 0 & 0 & (\frac{1}{2}-\nu) \end{bmatrix}$$

$$[k]^e = t_e A_e [B]^T [D][B] \qquad \{f_b\}^e = \frac{t_e A_e}{3}\left[f_x \quad f_y \quad f_x \quad f_y \quad f_x \quad f_y\right]^T$$

$$\{T\}^e = \frac{t_e l_{1-2}}{6}\left[(2T_{x1} + T_{x2}) \quad (2T_{y1} + T_{y2}) \quad (T_{x1} + 2T_{x2}) \quad (T_{y1} + 2T_{y2})\right]^T$$

12. Element Conductivity matrix

$$[k_T] = \frac{k_e}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

13. Fourier's Law

$$q = -k \, . \, \frac{1}{l_e}\,[-1 \quad 1\,]\begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

14. Newton's Law of cooling

$$q = h(T_{wall} \;-\; T_\infty)$$