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CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR

MALAYSIA FRANCE INSTITUTE

FINAL EXAMINATION

SEPTEMBER 2014 SESSION

SUBJECT CODE	:	**FAB 20703 / FAB 30703**
SUBJECT TITLE	:	**ROBOTICS**
LEVEL	:	**BACHELOR**
DURATION	:	**9.00 AM – 12.00 PM (3 HOURS)**
DATE / TIME	:	**9 JANUARY 2015**

INSTRUCTIONS TO CANDIDATES

1. **Please read the instructions given in the question paper CAREFULLY.**
2. **This question paper is printed on both sides of the paper.**
3. **Please write your answers on the answer booklet provided.**
4. **Answer should be written in blue or black ink except for sketching, graphic and illustration.**
5. **This question paper consists of TWO (2) sections. Section A and B. Answer all questions in Section A. For Section B, answer three (3) questions only.**
6. **Answer all questions in English.**
7. **Fomulais appended.**

THERE ARE 8PRINTED PAGES OF QUESTIONS, AND 5 PAGES OF FOMULA EXCLUDING THIS PAGE.

SECTION A (Total:40 marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question 1

a) Most simple grippers are ___________ of freedom devices. (2 marks)

b) A spherical manipulator arm consists of ____________ revolute pairs. (3 marks)

c) The maximum horizontal distance from the center of the robot base to the end of its wrist is known as _______________. (2 marks)

d) State **five** (5) technical criteria commonly used in selecting a suitable robot for an industrial application. (7 marks)

e) Define the difference between forward kinematics and inverse kinematics of a robot. (2 marks)

f) Define the difference between the Roll-Pitch-Yaw rotation and the Euler Angle rotation. (4 marks)

Question 2

a) A frame represented by the matrix $\begin{bmatrix} 0.527 & -0.574 & 0.628 & 6 \\ 0.369 & 0.819 & 0.439 & 4 \\ -0.766 & 0 & 0.643 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ has been moved 6 units along the x-axis and 4 units along the z-axis. Find the representation of the new frame after the two movements. (10 marks)

b) A point $P = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}^T$ is attached to a rotating frame. The frame rotates 90^0 about the X-axis of the reference frame. Find the coordinates of the point relative to the reference frame after the rotation. (10 marks)

SECTION B(Total: 60 marks)

INSTRUCTION: Answer THREE (3) questions only.
Please use the answer booklet provided.

Question 3

a) The **Figure 1** shows a 2-DOF planar elbow-arm robot, sometimes called as the R-R manipulator. The non-zero link and joint parameters are :

$$a_1 = 6, a_2 = 9, \theta_1 = 60^0, \theta_2 = 45^0.$$

Obtain the transformation matrices and find the solution for the forward kinematics problem.

(10 marks)

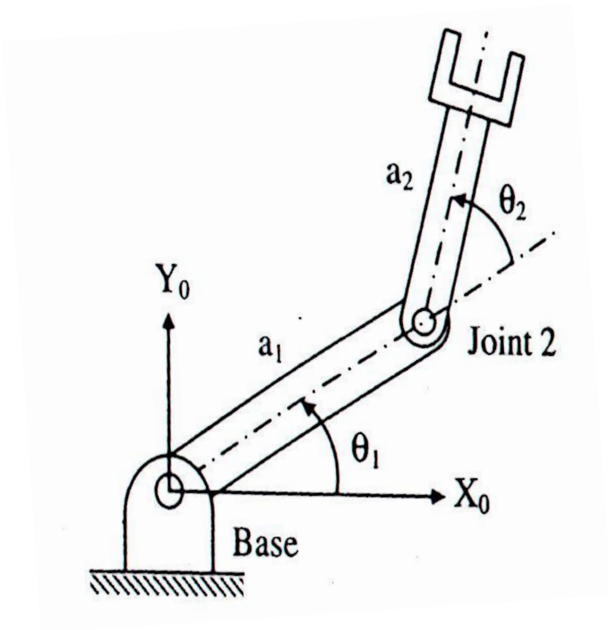

Figure 1: Two-Link Planar R-R Manipulator

b) Compute the manipulator transformation matrix for the 3-DOF-manipulator arm with Cartesian (PPP) configuration. Three-prismatic joints are perpendicular to each other and a possible frame assignment is given in **Figure 2 (a) and (b).**

(10 marks)

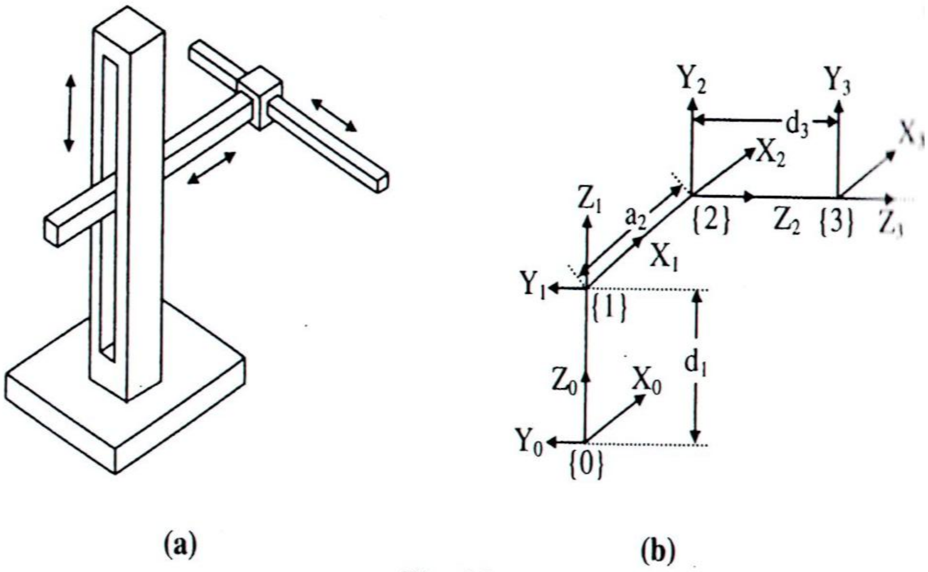

Figure 2 : Cartesian (PPP) manipulator

Question 4

a) When a robot arm is moving with a certain velocity, there will be velocity effects at the end of the robot arm and joints of the robot. What are the different types of velocities experienced at the end of the robot arm and joints.

(4 marks)

b) The **Figure 3** below shows a spherical RRR robot,

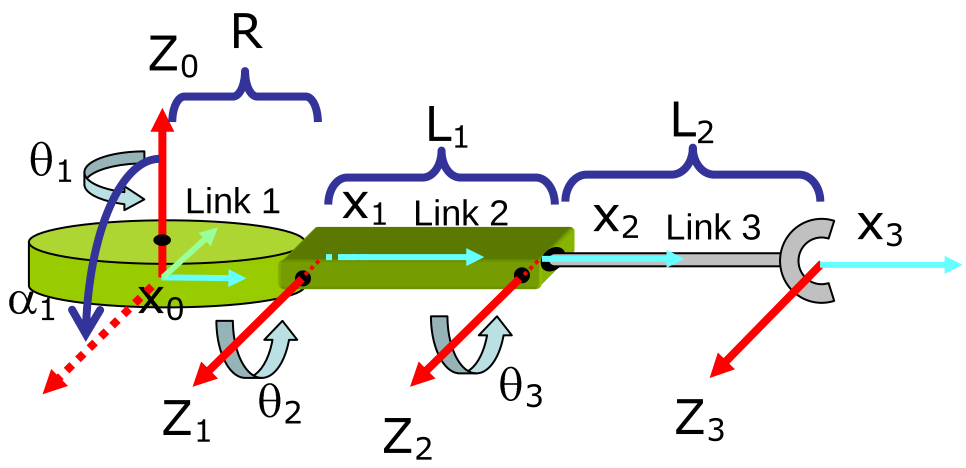

Figure 3: RRR Robot

With the DH parameter as shown in **Table 1** below:

Table 1: DH Parameter for RRR Robot

Link	Var	θ	d	α	a
1	θ_1	θ_1	0	90^o	R
2	θ_2	θ_2	0	0	L_1
3	θ_3	θ_3	0	0	L_2

The forward kinematics of the robot is as follows:

$$H_0^1 = \begin{bmatrix} c_1 & 0 & s_1 & Rc_1 \\ s_1 & 0 & -c_1 & Rs_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^2 = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & L_1c_1c_2 + Rc_1 \\ s_1c_2 & s_1s_2 & -c_1 & L_1s_1c_2 + Rs_1 \\ 0 & c_2 & 0 & L_1s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & c_1(L_2c_{23} + L_1c_2 + R \\ s_1c_{23} & -s_1s_{23} & -c_1 & s_1(L_2c_{23} + L_1c_2 + R \\ s_{23} & c_{23} & 0 & L_2s_{23} + L_1c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i) Find the linear velocity for the tool frame of the robot. (6 marks)

ii) Find the Jacobian matrix for the RRR robot. (10 marks)

Question 5

The **Figure 4** below shows a relationship between 3 coordinate systems operating in the world coordinate frame, WC. In this robotics workcell the Robot Gripper is trying to track the Object Part as seen by the Camera.

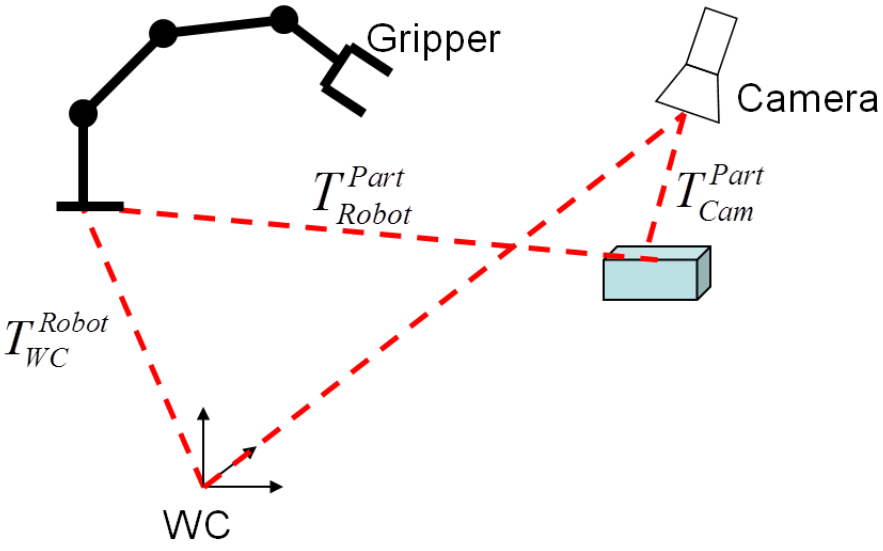

Figure 4: Coordinate Frame relationship between robot gripper, camera and object part operating in the world coordinate space.

a) Draw the "Motion Mapping" for the transformation of the Gripper frame to the Part frame.

(5 marks)

b) Derive the Del Operator equation matrix for ${}^{Robot}\nabla$, the motion of the gripper about its own space.

(15 marks)

Question 6

a) Describe the function of 'Jacobian Matrix' in the study of 'Motion Kinematics' of Robotics ? (5 marks)

b) Consider the SCARA robot arm with two revolute joints and a prismatic joint in **Figure 5**. Derive the expression of the inertia matrix *D(q)* given the following variables and parameters. (15 marks)

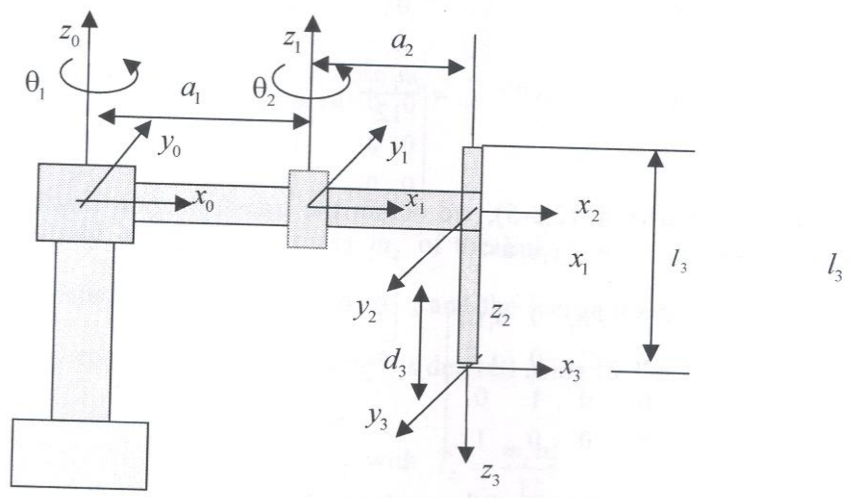

Figure 5 : SCARA Robot (RRP)

The arm parameters of the three-joint SCARA robot arm are given as **Table 2** below:

Table 2: Arm parameter for the three-joint SCARA robot.

Link	θ_i	d_i	a_i	α_i
1	θ_1	0	a_1	0^0
2	θ_2	0	a_2	180^0
3	0^0	l_3	0	0^0

END OF QUESTION

APPENDIX

Matrix Functions

Rotation transformation:

$$R_x(\theta)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{bmatrix} \quad ; \quad R_y(\theta)=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix} \quad ; \quad R_z(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} \; .$$

The D-H transformation matrix from the frame $x_{i-1}y_{i-1}z_{i-1}$ to the frame $x_iy_iz_i$ is:

$$H_{i-1}^{i} = H \;\; \theta_i \;\; Tran \;\; d_i \;\; Tran \;\; a_i \;\; H \;\; \alpha_i$$

$$=\begin{bmatrix}C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i\\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i\\ 0 & S\alpha_i & C\alpha_i & d_i\\ 0 & 0 & 0 & 1\end{bmatrix}$$

The forward kinematics solution for an n-linked robot can be expressed as:

$$H_0^n=\begin{bmatrix}\mathbf{n} & \mathbf{o} & \mathbf{a} & \mathbf{P}\\ 0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}n_x & o_x & a_x & P_x\\ n_y & o_y & a_y & P_y\\ n_z & o_z & a_z & P_z\\ 0 & 0 & 0 & 1\end{bmatrix}$$

The inverse of a homogeneous transformation matrix H can be expressed as:

$$H^{-1}=\begin{bmatrix}R^T & -R^T P\\ 000 & 1\end{bmatrix}=\begin{bmatrix}n_x & n_y & n_z & -n^T P\\ o_x & o_y & o_z & -o^T P\\ a_x & a_y & a_z & -a^T P\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Trigonometry Functions

$$\sin(A \pm B)=\sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B)=\cos A\cos B \mp \sin A\sin B$$

$$\frac{\partial}{\partial\theta_A}\cos(A+B) = -\sin(A+B)$$

$$\frac{\partial}{\partial\theta_B}\cos(A+B) = -\sin(A+B)$$

$$\frac{\partial}{\partial\theta_A}\sin(A+B) = \cos(A+B)$$

$$\frac{\partial}{\partial\theta_B}\sin(A+B) = \cos(A+B)$$

Equation	Solution
(a) $\sin\theta = a$	$\theta = A\tan 2\ \ a,\quad \pm\sqrt{1-a^2}$
(b) $\cos\theta = b$	$\theta = A\tan 2\ \ \pm\sqrt{1-b^2},\quad b$
(c) $\begin{cases} \sin\theta = a \\ \cos\theta = b \end{cases}$	$\theta = A\tan 2(a,b)$
(d) $a\cos\theta - b\sin\theta = 0$	$\theta^{(1)} = A\tan 2\ \ a,\quad b$ $\theta^{(2)} = A\tan 2\ \ -a,\quad -b\ \ = \pi + \theta^{(1)}$
(e) $a\cos\theta + b\sin\theta = c$	$\theta^{(1)} = A\tan 2\ \ c,\quad \sqrt{a^2+b^2-c^2}\ \ - A\tan 2\ \ a,\quad b$ $\theta^{(2)} = A\tan 2\ \ c,\quad -\sqrt{a^2+b^2-c^2}\ \ - A\tan 2\ \ a,\quad b$
(f) $\begin{cases} a\cos\theta - b\sin\theta = c \\ a\sin\theta + b\cos\theta = d \end{cases}$	$\theta = A\tan 2\ \ ad - bc,\quad ac + bd$
(g) $\begin{cases} \sin\alpha\sin\beta = a \\ \cos\alpha\sin\beta = b \\ \cos\beta = c \end{cases}$	$\begin{cases} \alpha^{(1)} = A\tan 2\ \ a,\quad b \\ \beta^{(1)} = A\tan 2\ \ \sqrt{a^2+b^2},\quad c \end{cases}$ $\begin{cases} \alpha^{(2)} = A\tan 2\ \ -a,\quad -b\ \ = \pi + \alpha^{(1)} \\ \beta^{(2)} = A\tan 2\ \ -\sqrt{a^2+b^2},\quad c \end{cases}$

The Atan2 function can be defined as follows:

$$A\tan 2(p_x, p_y) = \begin{cases} \arctan\left(\dfrac{p_y}{p_x}\right) & p_x > 0 \\ \arctan\left(\dfrac{p_y}{p_x}\right) + \pi & p_x < 0 \\ \dfrac{\pi}{2} & p_x = 0 \quad \& \quad p_y > 0 \\ \dfrac{-\pi}{2} & p_x = 0 \quad \& \quad p_y < 0 \end{cases}$$

Jacobian Function.

The Del Operator ∇ for small motion about a fixed world coordinate frame can be given as follows:-

$$\Delta T \approx \begin{bmatrix} 0 & -\delta_z & \delta_y & dx \\ \delta_z & 0 & -\delta_x & dy \\ -\delta_y & \delta_x & 0 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet T \approx \nabla \bullet T$$

where,

$$\nabla \approx \begin{bmatrix} 0 & -\delta_z & \delta_y & dx \\ \delta_z & 0 & -\delta_x & dy \\ -\delta_y & \delta_x & 0 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Del Operator for small motion with respect to its own frame ${}^T\nabla$ can be defined as follows:

$${}^T\nabla = \begin{bmatrix} 0 & -{}^T\delta_z & {}^T\delta_y & {}^Tdx \\ {}^T\delta_z & 0 & -{}^T\delta_x & {}^Tdy \\ -{}^T\delta_y & {}^T\delta_x & 0 & {}^Tdz \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where,

$${}^Tdx = \delta \bullet (\overrightarrow{d \times n}) + \vec{d}_p \bullet \vec{n} \qquad {}^T\delta x = \delta \bullet \vec{n}$$

$${}^Tdy = \delta \bullet (\overrightarrow{d \times o}) + \vec{d}_p \bullet \vec{o} \qquad {}^T\delta y = \delta \bullet \vec{o}$$

$${}^Tdz = \delta \bullet (\overrightarrow{d \times a}) + \vec{d}_p \bullet \vec{a} \qquad {}^T\delta x = \delta \bullet \vec{a}$$

Note:
d, **n**, **o** & **a** vectors are extracts from the *T* Matrix
d_p is the translation vector in ∀
δ is the rotational effects in ∀

The Jacobian matrix for an n-linked robot with position vector $\mathbf{P}$ at the end of the robot arm and joint variables $\mathbf{q}$ where $P = [p_x \quad p_y \quad p_z]^T$ and $\mathbf{q} = [q_1 \quad q_2 \quad \cdots \quad q_n]^T$, can be defined as follows:

$$p_x = f_1(q_1, q_2, ..., q_n)$$
$$p_y = f_2(q_1, q_2, ..., q_n)$$
$$p_z = f_3(q_1, q_2, ..., q_n)$$

$$v_x = \frac{dp_x}{dt} = \frac{\partial f_1}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial f_1}{\partial q_2}\frac{dq_2}{dt} + \cdots + \frac{\partial f_1}{\partial q_n}\frac{dq_n}{dt}$$
$$v_y = \frac{dp_y}{dt} = \frac{\partial f_2}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial f_2}{\partial q_2}\frac{dq_2}{dt} + \cdots + \frac{\partial f_2}{\partial q_n}\frac{dq_n}{dt}$$
$$v_z = \frac{dp_z}{dt} = \frac{\partial f_3}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial f_3}{\partial q_2}\frac{dq_2}{dt} + \cdots + \frac{\partial f_3}{\partial q_n}\frac{dq_n}{dt}$$

where,

$$\frac{dq_i}{dt} = \dot{q}_i$$

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \cdots & \frac{\partial f_1}{\partial q_n} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \cdots & \frac{\partial f_2}{\partial q_n} \\ \frac{\partial f_3}{\partial q_1} & \frac{\partial f_3}{\partial q_2} & \cdots & \frac{\partial f_3}{\partial q_n} \\ \eta_1 R^0_{0(3col)} & \eta_2 R^1_{0(3col)} & \cdots & \eta_n R^{n-1}_{0(3col)} \end{bmatrix}$$

where,
$\eta_1 = 1$ for revolute joint
$\eta_1 = 0$ for prismatic joint

In vector form, the Jacobian matrix can also be defined as follows:

i) For Revolute Joint:

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} \overrightarrow{\overrightarrow{Z_{i-1}} \times \overrightarrow{O_n} - \overrightarrow{O_{i-1}}} \\ \overrightarrow{Z_{i-1}} \end{bmatrix}$$

ii) For Prismatic Joint:

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} \overrightarrow{Z_{i-1}} \\ \vec{0} \end{bmatrix}$$

where, Z_{i-1}'s and O_{i-1}'s are the frame coordinates for the $i\text{-}1^{th}$ robot joint given by:

- Z_{i-1} is the 3rd column of the T_0^{i-1} (=A_1*… *A_{i-1})
- O_{i-1} is 4th column of the T_0^{i-1} (=A_1*… *A_{i-1})
- O_n is 4th column Of T_0^n (the FKS!)
- NOTE: when we extract the columns we only need the first 3 rows !!!.

Vector Function.

Cross Product of two vectors **A** and **B**. The end product is a vector **C**.

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i + \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

$$= \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix} = (a_2b_3 - a_3b_2)i + (a_3b_1 - a_1b_3)j + (a_1b_2 - a_2b_1)k$$

Dot Product of two vectors A and B. The end product is a scalar c.

$$c = A \bullet B = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = (a_1b_1 + a_2b_2 + a_3b_3)$$