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SET A

# UNIVERSITI KUALA LUMPUR
## Malaysia France Institute

# FINAL EXAMINATION

# JANUARY 2014 SESSION

| | | |
|---|---|---|
| SUBJECT CODE | : | NMB 21104 |
| SUBJECT TITLE | : | SOLID MECHANICS |
| LEVEL | : | BACHELOR |
| TIME / DURATION | : | ( 3 HOURS ) |
| DATE | : | |

## INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. This question paper consists of only one (1) section. Answer four (4) questions only.
5. Answer all questions in English. Show all works.

**THERE ARE 6 PAGES OF QUESTIONS AND 2 PAGES OF APPENDIX, EXCLUDING THIS PAGE.**

**INSTRUCTION: ANSWER 4 (FOUR) QUESTIONS ONLY.**

**Please use the answer booklet provided.**

**Question 1**

A beam is loaded as shown in the Figure 1. It has a square cross section of 200 mm on each side.

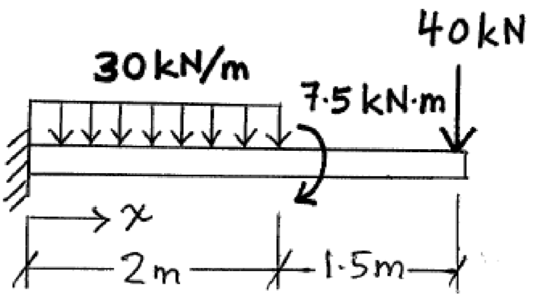



**Figure 1**

a) Draw the shear and bending moment diagrams. Use the graph paper provided if you prefer.

b) Determine the *x* location and value of absolute maximum bending (flexural) stress in the beam.

c) Determine the x location and value of absolute maximum shear stress in the beam.

[25 marks]

**Question 2**

The assembly shown in the Figure 2 consists of two A-36 steel suspender rods AC and BD pin connected to the 500 N uniform rigid beam AB. Determine the position, x, for the 1500 N loading so that the beam remains in a horizontal position both before and after the load is applied. Each rod has a diameter of 12 mm. Is the value of E necessary to find the answer? Use E = 200 GPa, if necessary.

[25 marks]

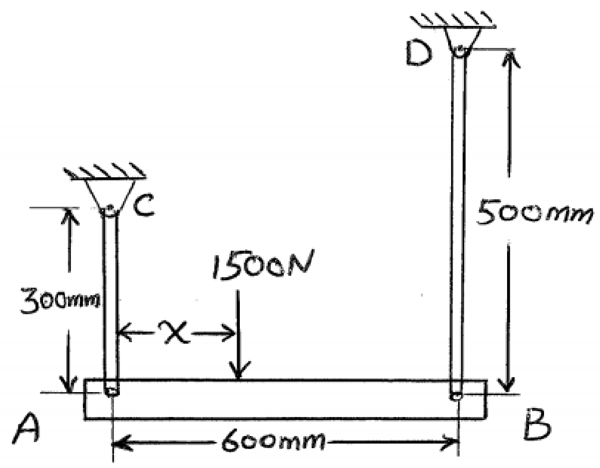



**Figure 2**

**Question 3**

The state of stress at a point on the lower surface of a student chair is shown in the Figure 3. Using an appropriate method that you prefer, determine:

a) The principal stresses.
b) The maximum in-plane shear stress and its associated normal stress.
c) And sketch the orientation of the element in each case (a) and (b) above.

[25 marks]

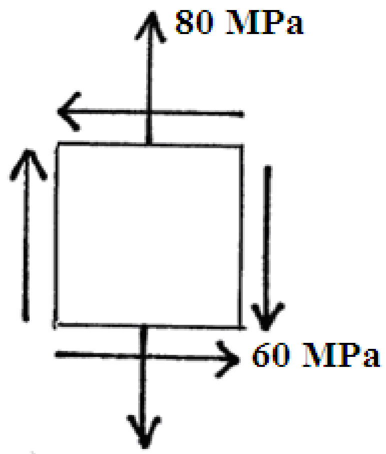



**Figure 3**

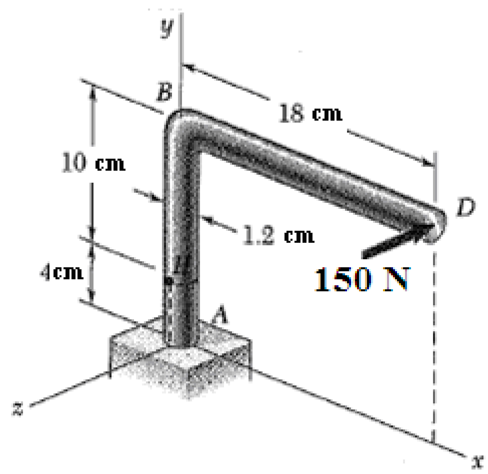
**Question 4**

A member ABD which is acted upon by the force 150 N and its resultant internal loadings at a cross section containing point H are shown in the Figure 4.

a) Find the value of N, V, M and T.

b) Find corresponding normal and shear stresses at point H.

[25 marks]



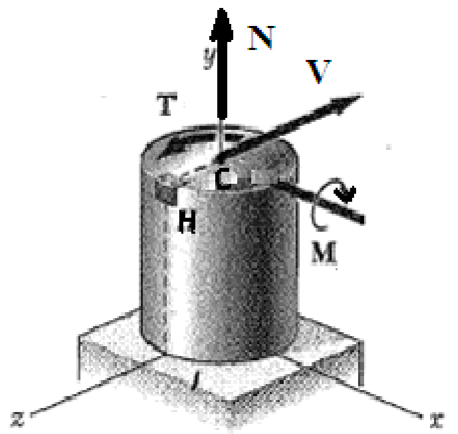




**Figure 4**

**Question 5**

Determine the maximum deflection of the beam shown in the Figure 5. The material has an elastic modulus of 200 GPa.

[25 marks]

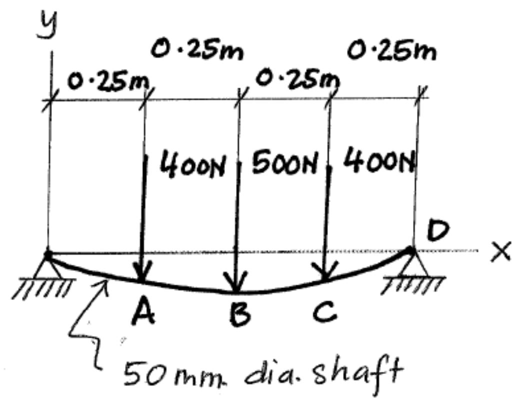



**Figure 5**

**Question 6**

A viewing platform in a zoo is supported by a row of aluminum pipes having length of L = 3.25m and outer diameter d = 100mm. The bases of the columns are set in concrete footings and the tops of the columns are pinned to the platform. The columns are being designed to support compressive loads of P = 100kN.

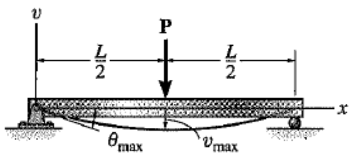
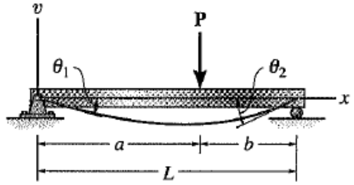
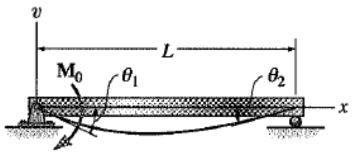
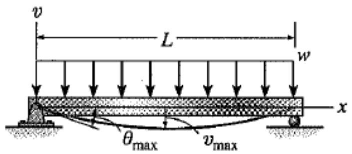
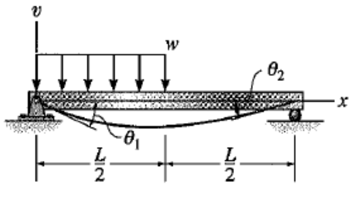
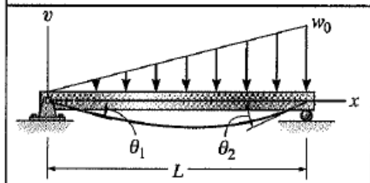
Determine the minimum required thickness "t" of the columns if a safety factor n = 3 is required with respect to Euler buckling. (For aluminum, use 72 GPa for the modulus of elasticity and use 480 MPa for the yield strength or proportional limit)

[25 marks]

**END OF QUESTIONS**

# APPENDIX

## Simply Supported Beam Slopes and Deflections

| Beam | Slope | Deflection | Elastic Curve |
|---|---|---|---|
|  | $\theta_{max} = \dfrac{-PL^2}{16EI}$ | $v_{max} = \dfrac{-PL^3}{48EI}$ | $v = \dfrac{-Px}{48EI}(3L^2 - 4x^2)$ <br> $0 \le x \le L/2$ |
|  | $\theta_1 = \dfrac{-Pab(L+b)}{6EIL}$ <br> $\theta_2 = \dfrac{Pab(L+a)}{6EIL}$ | $v\Big\rvert_{x=a} = \dfrac{-Pba}{6EIL}(L^2 - b^2 - a^2)$ | $v = \dfrac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ <br> $0 \le x \le a$ |
|  | $\theta_1 = \dfrac{-M_0L}{3EI}$ <br> $\theta_2 = \dfrac{M_0L}{6EI}$ | $v_{max} = \dfrac{-M_0L^2}{\sqrt{243}EI}$ | $v = \dfrac{-M_0x}{6EIL}(x^2 - 3Lx + 2L^2)$ |
|  | $\theta_{max} = \dfrac{-wL^3}{24EI}$ | $v_{max} = \dfrac{-5wL^4}{384EI}$ | $v = \dfrac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$ |
|  | $\theta_1 = \dfrac{-3wL^3}{128EI}$ <br> $\theta_2 = \dfrac{7wL^3}{384EI}$ | $v\Big\rvert_{x=L/2} = \dfrac{-5wL^4}{768EI}$ <br> $v_{max} = -0.006563\dfrac{wL^4}{EI}$ <br> at $x = 0.4598L$ | $v = \dfrac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ <br> $0 \le x \le L/2$ <br> $v = \dfrac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ <br> $L/2 \le x < L$ |
|  | $\theta_1 = \dfrac{-7w_0L^3}{360EI}$ <br> $\theta_2 = \dfrac{w_0L^3}{45EI}$ | $v_{max} = -0.00652\dfrac{w_0L^4}{EI}$ <br> at $x = 0.5193\ L$ | $v = \dfrac{-w_0x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$ |

# APPENDIX

### Material Property Relations

*Poisson's ratio*

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$

*Generalized Hooke's Law*

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}, \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}, \quad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

*where*

$$G = \frac{E}{2(1+\nu)}$$

### Relations Between $w$, $V$, $M$

$$\frac{dV}{dx} = -w(x), \qquad \frac{dM}{dx} = V$$

### Elastic Curve

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$EI\frac{d^4v}{dx^4} = -w(x)$$

$$EI\frac{d^3v}{dx^3} = V(x)$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

### Buckling

*Critical axial load*

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

*Critical stress*

$$\sigma_{\text{cr}} = \frac{\pi^2 E}{(KL/r)^2}, \; r = \sqrt{I/A}$$

*Secant formula*

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

### Energy Methods

*Conservation of energy*

$$U_e = U_i$$

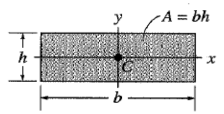
*Strain energy*

$$U_i = \frac{N^2L}{2AE} \quad \textit{constant axial load}$$

$$U_i = \int_0^L \frac{M^2dx}{EI} \quad \textit{bending moment}$$

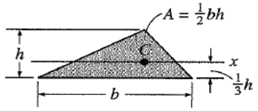
$$U_i = \int_0^L \frac{f_sV^2dx}{2GA} \quad \textit{transverse shear}$$

$$U_i = \int_0^L \frac{T^2dx}{2GJ} \quad \textit{torsional moment}$$



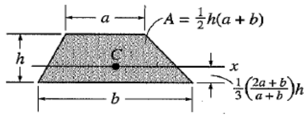
Rectangular area

$I_x = \frac{1}{12}bh^3$
$I_y = \frac{1}{12}hb^3$

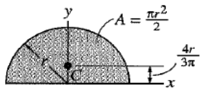


Triangular area

$I_x = \frac{1}{36}bh^3$

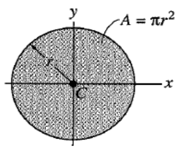


Trapezoidal area



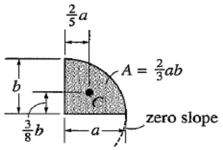
Semicircular area

$I_x = \frac{1}{8}\pi r^4$
$I_y = \frac{1}{8}\pi r^4$

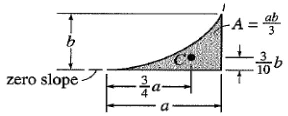


Circular area

$I_x = \frac{1}{4}\pi r^4$
$I_y = \frac{1}{4}\pi r^4$
$J = I_x + I_y$
$= \frac{1}{2}\pi r^4$



Semiparabolic area



Exparabolic area

### Axial Load

*Normal Stress*

$$\sigma = \frac{P}{A}$$

*Displacement*

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E}$$

$$\delta = \Sigma\frac{PL}{AE}$$

$$\delta_T = \alpha\,\Delta T L$$

### Torsion

*Shear stress in circular shaft*

$$\tau = \frac{T\rho}{J}$$

*where*

$$J = \frac{\pi}{2}c^4 \text{ solid cross section}$$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4) \text{ tubular cross section}$$

*Power*

$$P = T\omega = 2\pi f T$$

*Angle of twist*

$$\phi = \int_0^L \frac{T(x)dx}{J(x)G}$$

$$\phi = \Sigma\frac{TL}{JG}$$

*Average shear stress in a thin-walled tube*

$$\tau_{\text{avg}} = \frac{T}{2tA_m}$$

*Shear Flow*

$$q = \tau_{\text{avg}}t = \frac{T}{2A_m}$$

### Bending

*Normal stress*

$$\sigma = \frac{My}{I}$$

*Unsymmetric bending*

$$\sigma = -\frac{M_zy}{I_z} + \frac{M_yz}{I_y}, \qquad \tan\alpha = \frac{I_z}{I_y}\tan\theta$$

### Shear

*Average direct shear stress*

$$\tau_{\text{avg}} = \frac{V}{A}$$

*Transverse shear stress*

$$\tau = \frac{VQ}{It}$$

*Shear flow*

$$q = \tau t = \frac{VQ}{I}$$

### Stress in Thin-Walled Pressure Vessel

*Cylinder*

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$

*Sphere*

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

### Stress Transformation Equations

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

*Principal Stress*

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

*Maximum in-plane shear stress*

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}$$

*Absolute maximum shear stress*

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

$$\sigma_{\text{avg}} = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$