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SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2014 SESSION

SUBJECT CODE	:	**NCB 10103**
SUBJECT TITLE	:	**MATHEMATICS FOR ENGINEERS 1**
LEVEL	:	**BACHELOR**
TIME / DURATION	:	**9.00 am – 12.00 noon (3 HOURS)**
DATE	:	**27 MAY 2014**

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of SIX (6) questions. Answer FIVE (5) questions only.
6. Answer all questions in English.

THERE ARE 4 PAGES OF QUESTIONS AND 3 PAGES OF APPENDIX, EXCLUDING THIS PAGE.

INSTRUCTION : Answer FIVE (5) questions only.
Please use the answer booklet provided.

Question 1

(a) The matrix $A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ is an orthogonal such that $AA^T = I$ where I is an identity matrix. Show that

(i) $p^2 + q^2 = 1$

(2 marks)

(ii) $pr + qs = 0$

(1 mark)

(iii) $r^2 + s^2 = 1$

(1 mark)

(iv) Hence, using the results in (i) , (ii) and (iii) show that $|A|^2 = 1$.

(4 marks)

(b) Show that the matrix

$$A = \begin{pmatrix} 4 & -3 & 0 \\ 1 & 3 & 2 \\ -1 & 0 & 1 \end{pmatrix}$$

has only one real eigenvalue and determine the eigenvector corresponding to this real eigenvalue.

(12 marks)

Question 2

(a) The position vector r of a particle at time t is given by

$$r = (-\alpha \sin \omega t, \beta \cos \omega t, 0)$$

Where α, β and ω are positive constants with $\alpha < \beta$

i). Find the particle's velocity **v**.

(2 marks)

ii). Show that the speed is given by

$$\omega\sqrt{\beta^2 + (\alpha^2 - \beta^2)\cos^2 \omega t}$$

(3 marks)

iii). Find the acceleration, **f** ,of the particle.

(3 marks)

iv). Verify that the time $t = \dfrac{n\pi}{2\omega}$ when **f** is perpendicular to **v**.

(4 marks)

(b) Given that the vectors $\overrightarrow{X_1} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{X_2} = \begin{pmatrix} \lambda \\ 2 \\ \mu \end{pmatrix}$, $\lambda > 0$ are perpendicular to each other and the magnitude of $\overrightarrow{X_2}$ is $\sqrt{30}$. Determine the values of λ and μ.

(8 marks)

Question 3

(a) Evaluate $\dfrac{\left(-1 + j\sqrt{3}\right)^{15}}{(1 - j)^{20}} + \dfrac{\left(-1 - j\sqrt{3}\right)^{15}}{(1 + j)^{20}}$

(8 marks)

(b) Solve the equation $z^6 - 64 = 0$, obtaining the six distinct roots in **EXPONENTIAL FORM** . Also, plot the roots on an Argand diagram.

(12 marks)

Question 4

Given the function $f(x,y) = (x+y)^4 - x^2 - y^2 - 6xy$.

(a) Show that the function $f(x,y)$ has stationary points at $(0,0)$, $\left(\frac{1}{2},\frac{1}{2}\right)$ and $\left(-\frac{1}{2},-\frac{1}{2}\right)$.

(10 marks)

(b) Identify the type of all the stationary points of this function whether they are maxima, minima or saddle points.

(10 marks)

Question 5

(a) Given the function $z = \sin(xy) + x\cos y$.

(i) Determine the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(2 marks)

A change of variables is specified by means of the equation $x = u^2 + v^2$ and $y = uv$.

(ii) Use the Chain Rule to determine $\frac{\partial z}{\partial u}$ in terms of u and v .

(5 marks)

(iii) Calculate the value of this partial derivative when $u = 1$, $v = 0$

(3 marks)

(b) Evaluate the double integral $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{y^4+1}\,dy\,dx$ by reversing the order of integration. Sketch the region over which the integration takes place.

(10 marks)

Question 6

(a) Convert the integral $\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} \left(x^2+y^2+z^2\right) dz\,dx\,dy$ into spherical coordinates.

(11 marks)

(b) Hence, evaluate the integral.

(9 marks)

END OF QUESTION

APPENDIX 1 - Trigonometric Identities and Formulas

Fundamental Identities

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Formulas For Negatives

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\text{ta n}\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

Addition Formulas

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Subtraction Formulas

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A - B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Half-Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\sin\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$.......... = 1 - 2\sin^2\theta$$

$$.......... = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Product-To-Sum Formulas

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

Sum-To-Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

APPENDIX 2 – Table of Differentiation

Trigonometric Functions

$$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$$

$$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$$

$$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$$

$$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$$

$$\frac{d}{dx}(\sec f(x)) = f'(x)\csc f(x)\tan f(x)$$

$$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$$

Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1}U) = \frac{1}{\sqrt{1-U^2}}\frac{dU}{dx}, \quad |U|<1$$

$$\frac{d}{dx}(\cos^{-1}U) = \frac{-1}{\sqrt{1-U^2}}\frac{dU}{dx}, \quad |U|<1$$

$$\frac{d}{dx}(\tan^{-1}U) = \frac{1}{1+U^2}\frac{dU}{dx}$$

$$\frac{d}{dx}(\csc^{-1}U) = \frac{-1}{|U|\sqrt{U^2-1}}\frac{dU}{dx}, \quad |U|>1$$

$$\frac{d}{dx}(\sec^{-1}U) = \frac{1}{|U|\sqrt{U^2-1}}\frac{dU}{dx}, \quad |U|>1$$

$$\frac{d}{dx}(\cot^{-1}U) = \frac{-1}{1+U^2}\frac{dU}{dx}$$

Hyperbolic Functions

$$\frac{d}{dx}(\sinh U) = \cosh U\frac{dU}{dx}$$

$$\frac{d}{dx}(\cosh U) = \sinh U\frac{dU}{dx}$$

$$\frac{d}{dx}(\tanh U) = \operatorname{sech}^2 U\frac{dU}{dx}$$

$$\frac{d}{dx}(\operatorname{csch} U) = -\operatorname{csch} U\coth U\frac{dU}{dx}$$

$$\frac{d}{dx}(\operatorname{sech} U) = -\operatorname{sech} U\tanh U\frac{dU}{dx}$$

$$\frac{d}{dx}(\coth U) = -\operatorname{csch}^2 U\frac{dU}{dx}$$

Inverse Hyperbolic Functions

$$\frac{d}{dx}(\sinh^{-1}U) = \frac{1}{\sqrt{1+U^2}}\frac{dU}{dx}$$

$$\frac{d}{dx}(\cosh^{-1}U) = \frac{1}{\sqrt{U^2-1}}\frac{dU}{dx}, \quad U>1$$

$$\frac{d}{dx}(\tanh^{-1}U) = \frac{1}{1-U^2}\frac{dU}{dx}, \quad |U|<1$$

$$\frac{d}{dx}(\operatorname{csch}^{-1}U) = \frac{-1}{|U|\sqrt{1+U^2}}\frac{dU}{dx}, \quad U\neq 0$$

$$\frac{d}{dx}(\operatorname{sech}^{-1}U) = \frac{-1}{U\sqrt{1-U^2}}\frac{dU}{dx}, \quad 0<U<1$$

$$\frac{d}{dx}(\coth^{-1}U) = \frac{1}{1-U^2}\frac{dU}{dx}, \quad |U|>1$$

Exponential Function

$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$$

Natural Logarithmic Function

$$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)}f'(x)$$

APPENDIX 3 – Table of Integration

Trigonometric Functions
Where $f(x) = ax + b$

$$\int \cos f(x)dx = \frac{\sin f(x)}{f'(x)} + C$$

$$\int \sin f(x)dx = -\frac{\cos f(x)}{f'(x)} + C$$

$$\int \sec^2 f(x)dx = \frac{\tan f(x)}{f'(x)} + C$$

$$\int \csc^2 f(x)dx = -\frac{\cot f(x)}{f'(x)} + C$$

$$\int \sec f(x)\tan f(x)dx = \frac{\sec f(x)}{f'(x)} + C$$

$$\int \csc f(x)\cot f(x)dx = -\frac{\csc f(x)}{f'(x)} + C$$

Inverse Trigonometric Functions

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad , \quad |x| < a$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + C \quad , \quad |x| < a$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{|x|\sqrt{x^2 - a^2}} dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C \quad , \quad |x| > a$$

$$\int \frac{1}{|x|\sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C \quad , \quad |x| > a$$

$$\int \frac{-1}{a^2 + x^2} dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$$

Hyperbolic Functions
Where $f(x) = ax + b$

$$\int \cosh f(x)dx = \frac{\sinh f(x)}{f'(x)} + C$$

$$\int \sinh f(x)dx = \frac{\cosh f(x)}{f'(x)} + C$$

$$\int \text{sech}^2 f(x)dx = \frac{\tanh f(x)}{f'(x)} + C$$

$$\int \text{csch}^2 f(x)dx = -\frac{\coth f(x)}{f'(x)} + C$$

$$\int \text{sech}\, f(x)\tanh f(x)dx = \frac{-\text{sech}\, f(x)}{f'(x)} + C$$

$$\int \text{csch}\, f(x)\coth f(x)dx = -\frac{\text{csch}\, f(x)}{f'(x)} + C$$

Inverse Hyperbolic Functions

$$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \sinh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad a > 0$$

$$\int \frac{-1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad x > a$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C \ , \ |x| < a$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C \ , \ |x| > a$$

$$\int \frac{1}{x\sqrt{a^2 + x^2}} dx = -\frac{1}{a}\text{csch}^{-1}\left(\frac{x}{a}\right) + C \quad , \quad 0 < x <$$

$$\int \frac{1}{x\sqrt{a^2 - x^2}} dx = -\frac{1}{a}\text{sech}^{-1}\left(\frac{x}{a}\right) + C \quad , \quad 0 < x <$$

Exponential Function
Where $f(x) = ax + b$

$$\int e^{f(x)} dx = \frac{e^{f(x)}}{f'(x)} + C$$

Form $\frac{1}{f(x)}$, where $f(x) = ax + b$

$$\int \frac{1}{f(x)} dx = \frac{\ln|f(x)|}{f'(x)} + C$$