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SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2014 SESSION

SUBJECT CODE	:	FKB20203 / FKB13202 / FKB14202 / FKB23302 / FKB24302
SUBJECT TITLE	:	ENGINEERING TECHNOLOGY MATHEMATICS 2
LEVEL	:	BACHELOR
TIME / DURATION	:	9.00 am - 12.00 noon (3 HOURS)
DATE	:	3 1 MAY 2014

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of SIX (6) questions. Answer five (5) questions only.
6. Answer all questions in English.
7. Trigonometric Formulas, Table of Differentiation and Integration are appended.

THERE ARE 4 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

INSTRUCTION: Answer FIVE questions only (Total: 100 marks)
Show all the necessary steps. A correct answer with no relevant work may received no credit while an incorrect answer with some correct work may received partial credit.

Question 1

(a) Given two functions, $h(x)=\dfrac{2}{x+3}$ and $g(x)=\dfrac{x}{5-x}$

(i) Determine the domain of the function, $A(x)$ where $A(x)=h(x)+g(x)$

(3 Marks)

(ii) Determine algebraically whether the function $A(x)$ is even, odd or neither even nor odd.

(2 Marks)

(iii) Determine $\lim_{x\to -3} A(x)$. What conclusion can you make based on the result found?

(3 Marks)

(iv) Determine $\lim_{x\to \infty} A(x)$. What conclusion can you make based on the result found?

(3 Marks)

(v) Calculate $h(3)+g(2)-A(0)$.

(4 Marks)

(b) A function is defined as $f(x)=\sqrt{2x+3}$.

Determine it's derivative, $\dfrac{df}{dx}$ by applying the <u>definition of derivative</u>.

(5 Marks)

Question 2

(a) A piecewise function is defined as

$$f(x)=\begin{cases} x+6 & \text{if} \quad -6<x<-4 \\ 2 & \text{if} \quad -4\le x\le -2 \\ -2 & \text{if} \quad -2<x<2 \\ 2 & \text{if} \quad x=2 \\ 3x-8 & \text{if} \quad 2<x\le 4 \end{cases}$$

(i) Draw the graph of the piecewise function in the provided graph paper. (Marks will be given based on the correct graph, the endpoints as well as the axes)

(6 Marks)

(ii) State the domain of the function, $f(x)$ by using the interval notation.

(1 Mark)

(iii) From the graph, is the function $f(x)$ even, odd or neither even nor odd? Justify your answer.

(1 Mark)

(iv) Does $\lim_{x\to -2} f(x)$ exist? Justify your answer.

(3 Marks)

(v) By using the definition of continuity at a point, explain why the function is discontinuous at $x=2$ exist? State the type of discontinuity.

(4 Marks)

(b) Apply the Substitution Method and show that

$$\int_0^1 x^2(x^3+1)^4\,dx=\frac{31}{15}$$

(5 Marks)

Question 3

(a) A function with two variables is defined as $f(x,y) = 9 - 2x + 4y - x^2 - 4y^2$

(i) Determine the critical point (stationary point) of the function. Show that this point is a maximum point.

(13 Marks)

(ii) Determine the maximum value.

(2 Marks)

(b) Verify that the function $u(t,x) = e^{-\alpha^2 k^2 t} \sin kx$ is a solution of the heat conduction equation, $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.

(5 Marks)

Question 4

(a) The value of the following integral has a physical meaning, called **moment of inertia**.

$$\int_0^{\pi^{\frac{1}{3}}} \int_{-\sin(x^3)}^{\sin(x^3)} x^2 \; dy\,dx$$

Determine the value of the moment of inertia by applying a suitable method of integration.

(9 Marks)

(b) Determine the integral of $\int_0^1 \int_2^3 \int_1^2 xyz \; dz\,dy\,dx$

(6 Marks)

(c) Let $S = \int \frac{\ln x}{x^5} dx$. Apply the <u>Integration by Parts</u> to determine S

(5 Marks)

Question 5

(a) The general solution of a first order differential equation, $\frac{1}{(2y+3)}\frac{dy}{dx} = \frac{-x}{(x^2+1)}$

where $y(0) = 0$ is given by $Ax^2y + By + Cx^2 = 0$.

Determine the value of A , B and C

(15 Marks)

(b) Given that $\frac{x}{x^2-4x-5} = \frac{A}{(x-5)} + \frac{B}{(x+1)}$. Determine the value of A and B.

Hence, determine $\int \frac{x}{x^2-4x-5}\,dx$.

(5 Marks)

Question 6

(a) A nonhomogeneous second order differential equation is given as

$$y'' + y' - 12y = 4e^{2x}$$

(i) Determine the general solution of the equation

(6 Marks)

(ii) Determine the particular solution given the initial conditions $y(0) = 7$ and $y'(0) = 0$

(9 Marks)

(Note that $y'' = \frac{d^2y}{dx^2}$, $y' = \frac{dy}{dx}$)

(b) Apply Logarithmic Differentiation to determine the derivative, $\frac{dy}{dx}$ of the following function.

$$y = (x+3)^3(4-x)^4(x+5)^5$$

(5 Marks)

END OF QUESTION

APPENDIX 1 - Trigonometric Identities and Formulas

Fundamental Identities

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Formulas for Negatives

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

Addition Formulas

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Subtraction Formulas

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Half Angle Formulas

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2} = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}$$

Double Angle Formulas

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

Product to Sum Formulas

$$\sin A\cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$

$$\cos A\sin B = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$$

$$\cos A\cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$

$$\sin A\sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

Sum to Product Formulas

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

APPENDIX 2 – Table of Differentiation

(Trigonometric functions) General Form	(Inverse Trigonometric functions) General Form
$\frac{d}{dx}(\sin U)=\cos U\frac{dU}{dx}$	$\frac{d}{dx}(\sin^{-1}U)=\frac{1}{\sqrt{1-U^2}}\frac{dU}{dx}$
$\frac{d}{dx}(\cos U)=-\sin U\frac{dU}{dx}$	$\frac{d}{dx}(\cos^{-1}U)=\frac{-1}{\sqrt{1-U^2}}\frac{dU}{dx}$
$\frac{d}{dx}(\tan U)=\sec^2U\frac{dU}{dx}$	$\frac{d}{dx}(\tan^{-1}U)=\frac{1}{1+U^2}\frac{dU}{dx}$
$\frac{d}{dx}(\csc U)=-\csc U\cot U\frac{du}{dx}$	$\frac{d}{dx}(\csc^{-1}U)=\frac{-1}{\lvert U\rvert\sqrt{U^2-1}}\frac{dU}{dx}$
$\frac{d}{dx}(\sec U)=\sec U\tan U\frac{dU}{dx}$	$\frac{d}{dx}(\sec^{-1}U)=\frac{1}{\lvert U\rvert\sqrt{U^2-1}}\frac{dU}{dx}$
$\frac{d}{dx}(\cot U)=-\csc^2U\frac{dU}{dx}$	$\frac{d}{dx}(\cot^{-1}U)=\frac{-1}{1+U^2}\frac{dU}{dx}$

(Hyperbolic functions) General Form	(Inverse Hyperbolic functions) General Form
$\frac{d}{dx}(\sinh U)=\cosh U\frac{dU}{dx}$	$\frac{d}{dx}(\sinh^{-1}U)=\frac{1}{\sqrt{1+U^2}}\frac{dU}{dx}$
$\frac{d}{dx}(\cosh U)=\sinh U\frac{dU}{dx}$	$\frac{d}{dx}(\cosh^{-1}U)=\frac{1}{\sqrt{U^2-1}}\frac{dU}{dx}$
$\frac{d}{dx}(\tanh U)=\text{sech}^2U\frac{dU}{dx}$	$\frac{d}{dx}(\tanh^{-1}U)=\frac{1}{1-U^2}\frac{dU}{dx}$
$\frac{d}{dx}(\text{csch}U)=-\text{csch}U\coth U\frac{du}{dx}$	$\frac{d}{dx}(\text{csch}^{-1}U)=\frac{-1}{\lvert U\rvert\sqrt{1+U^2}}\frac{dU}{dx}$
$\frac{d}{dx}(\text{sech}U)=-\text{sech}U\tanh U\frac{dU}{dx}$	$\frac{d}{dx}(\text{sech}^{-1}U)=\frac{-1}{U\sqrt{1-U^2}}\frac{dU}{dx}$
$\frac{d}{dx}(\cot U)=-\csc^2U\frac{dU}{dx}$	$\frac{d}{dx}(\coth^{-1}U)=\frac{1}{1-U^2}\frac{dU}{dx}$

(Exponential functions) **General Form**	(Logarithmic functions) **General Form**
$\frac{d}{dx}(e^U) = e^U \frac{dU}{dx}$ $\frac{d}{dx}(a^U) = a^U \ln a \frac{dU}{dx}$	$\frac{d}{dx}(\ln U) = \frac{1}{U}\frac{dU}{dx}$ $\frac{d}{dx}(\log_a U) = \frac{1}{U \ln a}\frac{dU}{dx}$

Rules in Differentiation	**Properties of Logarithm**
$\frac{d}{dx}(x^n) = nx^{n-1}$ $\frac{d}{dx}(ax^n) = anx^{n-1}$ $\frac{d}{dx}(C) = 0$ $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$ $\frac{d}{dx}(u \cdot v) = v\frac{du}{dx} + u\frac{dv}{dx}$ $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ $\frac{d}{dx}(u \circ v) = \frac{d}{dx}(u(v)) = \frac{du}{dv} \times \frac{dv}{dx}$	$\ln(MN) = \ln M + \ln N$ $\ln\left(\frac{M}{N}\right) = \ln M - \ln N$ $\ln M^k = k \ln M$ $e^{\ln x} = x$ $\ln e^x = x$

APPENDIX 3 – Table of Integration

Trigonometric Functions Where $f(x)=ax+b$
$\int \cos f(x)dx = \frac{\sin f(x)}{f'(x)} + C$
$\int \sin f(x)dx = -\frac{\cos f(x)}{f'(x)} + C$
$\int \sec^2 f(x)dx = \frac{\tan f(x)}{f'(x)} + C$
$\int \csc^2 f(x)dx = -\frac{\cot f(x)}{f'(x)} + C$
$\int \sec f(x)\tan f(x)dx = \frac{\sec f(x)}{f'(x)} + C$
$\int \csc f(x)\cot f(x)dx = -\frac{\csc f(x)}{f'(x)} + C$

Inverse Trigonometric Functions
$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x\rvert < a$
$\int \frac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x\rvert < a$
$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{-1}{\lvert x\rvert\sqrt{x^2-a^2}}dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x\rvert > a$
$\int \frac{1}{\lvert x\rvert\sqrt{x^2-a^2}}dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x\rvert > a$
$\int \frac{-1}{a^2+x^2}dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$

Hyperbolic Functions Where $f(x)=ax+b$
$\int \cosh f(x)dx = \frac{\sinh f(x)}{f'(x)} + C$
$\int \sinh f(x)dx = \frac{\cosh f(x)}{f'(x)} + C$
$\int \text{sech}^2 f(x)dx = \frac{\tanh f(x)}{f'(x)} + C$
$\int \text{csch}^2 f(x)dx = -\frac{\coth f(x)}{f'(x)} + C$
$\int \text{sech} f(x)\tanh f(x)dx = \frac{-\text{sech} f(x)}{f'(x)} + C$
$\int \text{csch} f(x)\coth f(x)dx = -\frac{\text{csch} f(x)}{f'(x)} + C$

Inverse Hyperbolic Functions
$\int \frac{1}{\sqrt{a^2+x^2}}dx = \sinh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad a > 0$
$\int \frac{-1}{\sqrt{x^2-a^2}}dx = \cosh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad x > a$
$\int \frac{1}{a^2-x^2}dx = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C \ , \ \lvert x\rvert < a$
$\int \frac{1}{a^2-x^2}dx = \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C \ , \ \lvert x\rvert > a$
$\int \frac{1}{x\sqrt{a^2+x^2}}dx = -\frac{1}{a}\text{csch}^{-1}\left(\frac{x}{a}\right) + C \quad , \quad 0 < x < a$
$\int \frac{1}{x\sqrt{a^2-x^2}}dx = -\frac{1}{a}\text{sech}^{-1}\left(\frac{x}{a}\right) + C \quad , \quad 0 < x < a$

Exponential Function Where $f(x)=ax+b$
$\int e^{f(x)}dx = \frac{e^{f(x)}}{f'(x)} + C$

Form $\frac{1}{f(x)}$, where $f(x)=ax+b$
$\int \frac{1}{f(x)}dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + C$

APPENDIX 4 – Graph paper

(Please detach this page and attach it to your answer booklet)

Name: ______________________________ Student ID: __________________