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SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2014 SESSION

SUBJECT CODE	:	**FAB21203**
SUBJECT TITLE	:	**LINEAR SYSTEMS AND SIGNALS 1**
LEVEL	:	**BACHELOR**
TIME / DURATION	:	**3 HOURS**
DATE	:	

INSTRUCTIONS TO CANDIDATES

1. **Please read the instructions given in the question paper CAREFULLY.**
2. **This question paper is printed on both sides of the paper.**
3. **Please write your answers on the answer booklet provided.**
4. **Answer should be written in blue or black ink except for sketching, graphic and illustration.**
5. **This question paper consists of five (5) questions. Answer four (4) questions only.**
6. **Answer all questions in English.**
7. **Formula is appended.**

THERE ARE 5 PAGES OF QUESTIONS AND 1 PAGE OF APPENDIX, EXCLUDING THIS PAGE.

Question 1

a) Determine whether the signal $x(t) = \sin(2(t - \frac{\pi}{3})) + 2\cos(4t) - 7\sin(10t)$ is periodic. If it is periodic, find the fundamental period. (4 marks)

b) The piecewise continuous function of a signal, $x(t)$ is given as:

$$x(t) = \begin{cases} -2t - 2 & ; \quad -2 < t < -1 \\ t + 1 & ; \quad -1 < t < 0 \\ 1 & ; \quad 0 < t < 2 \\ 0 & ; \quad elsewhere \end{cases}$$

i) Sketch the signal $x(t)$ (3 marks)

ii) Represent the signal $x(t)$ in terms of sum of unit step only. (3 marks)

iii) Sketch the derivative of $x(t)$ i.e $\dfrac{dx(t)}{dt}$ (3 marks)

iv) Determine whether the signal, $x(t)$ is energy signal, power signal or neither energy nor power signal. Justify your answer.

(5 marks)

c) Consider the signal $y(t)$ in **Figure 1** below:

i) Sketch the integral of $y(t)$ i.e. $\int_{-\infty}^{\infty} y(t)dt$ (3 marks)

ii) Sketch the signal $z(t) = y(1 - \frac{1}{2}t)$ (4 marks)

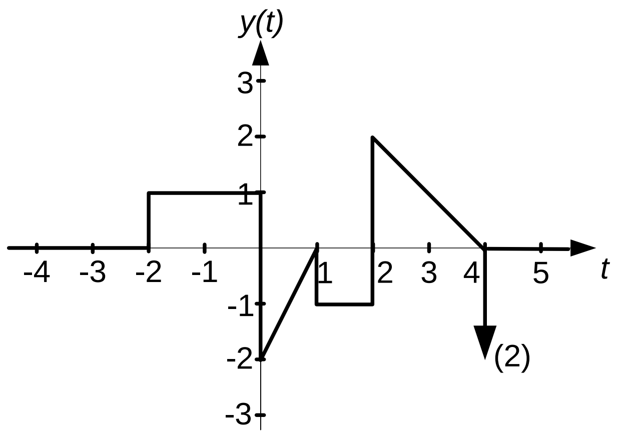

Figure 1: The signal $y(t)$

Question 2

a) List the **four (4)** possibilities of the second order differential equation characteristic roots and define the general form of homogeneous solution for each root respectively.

(8 marks)

b) Show that the electrical system shown in **Figure 2** where, $v_{in}(t)$ is the input and $v_{C_2}(t)$ is the output, can be represented by the following differential equation:

$$\frac{d^2 v_{c2}(t)}{dt^2} + \left(\frac{R}{L}\right)\frac{dv_{c2}(t)}{dt} + \left(\frac{C_1 + C_2}{LC_1C_2}\right)v_{c2}(t) = \left(\frac{1}{LC_2}\right)v_{in}(t)$$

(6 marks)

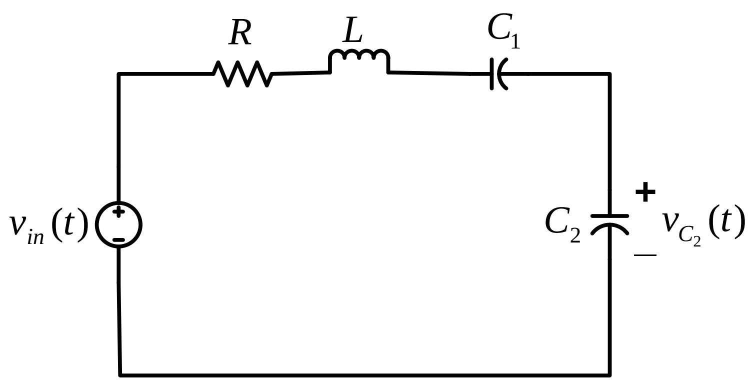

Figure 2: The RLC circuit

c) Consider a system with the following differential equation:

$$\frac{d^2 y(t)}{dt^2} + 9\frac{dy(t)}{dt} + 18y(t) = 9t^2 + 5e^{-5t}$$

If the initial conditions are $y(0^-) = 1$ and $\frac{dy(0^-)}{dt} = 0$, determine the $y(t)$ by using the classical method (undetermined coefficient).

(11 marks)

Question 3

a) The trigonometric Fourier series of an even symmetric signal, $x(t)$ consists of dc value and cosine terms only while an odd symmetric, $f(t)$ consists of sine terms only. Express the general form of trigonometric Fourier series of $x(t)$ and $f(t)$.

(3 marks)

b) Refer to a periodic signal, $y(t)$ in **Figure 3** and determine the following:

i) The fundamental period, T_0, angular frequency, ω_0 and symmetric property.

(2 marks)

ii) The trigonometric Fourier series of $y(t)$

(8 marks)

iii) The exponential Fourier series of $y(t)$

(5 marks)

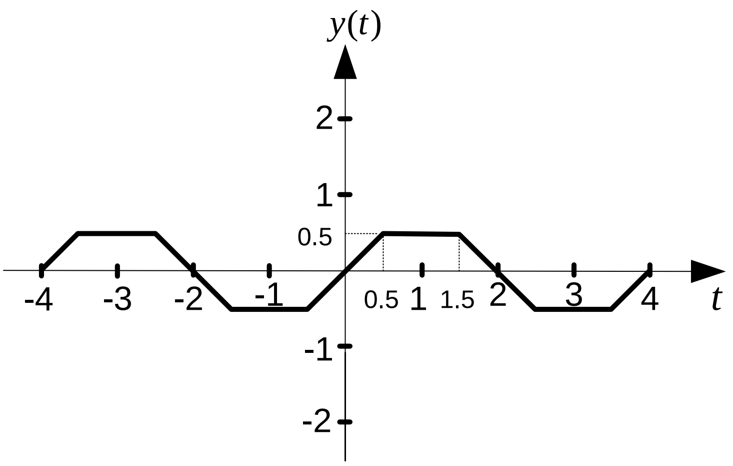

Figure 3: A periodic signal, $y(t)$

c) Sketch the frequency spectrum of the following trigonometric Fourier series of the signal, $f(t)$ for $n = 0, \pm1, \pm2, \pm3, \pm4, \pm5$.

$$f(t) = \frac{2}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{1-4n^2}$$

(7 marks)

Question 4

a) Prove that the shifting in s-domain property of unilateral Laplace transform is given by:

$$L\{e^{-\alpha t}x(t)u(t)\} = X(s+\alpha)$$

Hence, determine the $X(s)$ if given that $x(t) = 3e^{-5t}\cos(2t)u(t)$

(4 marks)

b) Determine the unilateral Laplace transform for the signal, $y(t)$ in **Figure 4** by applying appropriate theorems.

(7 marks)

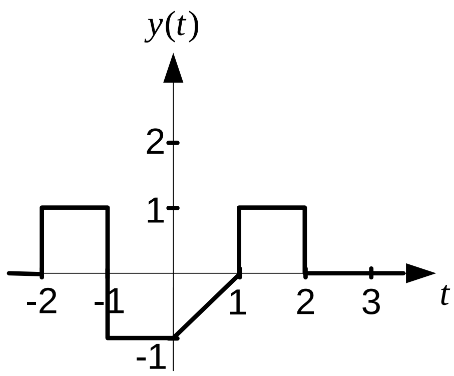

Figure 4: The signal, $y(t)$

c) A system is described by a differential equation,

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} = x(t)$$

where $x(t)$ is the input, $y(t)$ is the output and all initial conditions are zero.

i) Determine the transfer function of the system.

(3 marks)

ii) Draw the poles and zeros plot and determine the stability of the system stability.

(3 marks)

iii) Solve for $y(t)$ using Laplace transform method if $x(t) = e^{-2t}u(t)$

(6 marks)

iv) Determine the steady state value of $y(t)$ using Final Value Theorem

(2 marks)

Question 5

a) Given a system that is described by the following equation:

$$y(t) = x(t-1) + tx(t) + 10$$

where $x(t)$ is the input and $y(t)$ is the output. Determine and justify whether the system is:

i) Static and dynamic
(2 marks)

ii) Causal or non – causal
(2 marks)

iii) Time-variant or time-invariant
(5 marks)

iv) Linear or non-linear
(5 marks)

b) Determine the inverse Laplace transform, $x(t)$ for the following equation:

$$X(s) = \frac{s^2 + 6s + 7}{s^2 + 3s + 2}$$

(6 marks)

c) Explain the role of feedback gain, $H(s)$ in order to stabilize the following unstable first-order continuous-time system, $G(s)$:

$$G(s) = \frac{b}{s-a} \text{ with feedback gain, } H(s) = K$$

Conclude your explanation to the condition in which the similar system is also referred as proportional feedback system.

(5 marks)

END OF QUESTION

APPENDIX

Laplace Transform Table

Item no.	f(t)	F(s)
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$