Integrated Kurtosis-Based Algorithm for I-kaz™ Multilevel Coefficients Responses toward the Simultaneous Changes in Amplitude and Frequency of Signals

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Abstract: The response of Integrated Kurtosis-Based Algorithm for Z-Notch Filter (I-kaz™) Multilevel coefficients toward the simultaneous changes in amplitude and frequency of signals was unknown. This study presents the coefficients responses toward the synthetic signals which vary in amplitude and frequency simultaneously. The I-kaz™ Multilevel coefficients, 3rd order (\(Z^3\)), 4th order (\(Z^4\)), 5th order (\(Z^5\)), 6th order (\(Z^6\)), 7th order (\(Z^7\)) and 8th order (\(Z^8\)) were investigated by analyzing their response using three types of synthetic signals, FIAI, FIAD and FDAI. The responses of the I-kaz™ Multilevel coefficients were analyzed by either increasing or decreasing the amplitude and frequency in the synthetic signals simultaneously. This study indicated that the response of all orders of I-kaz™ Multilevel coefficients showed an increasing trend with respect to the increment in amplitude of FIAI and FDAI. All the coefficients however showed a decreasing trend in FIAD synthetic signal study regardless the increase in the frequency. The study also indicated that the change in amplitude has more influence than the change in frequency in the I-kaz™ Multilevel coefficients responses. The influence ratio of the amplitude to frequency change was estimated to be 86.96-13.06%, respectively. Recognizing the behaviour of I-kaz™ Multilevel coefficients toward the change in amplitude and frequency is important especially when analysing dynamic signals.

Key words: Statistical analysis, digital signal processing, I-kaz™ method, digital signal decomposition, synthetic signal, fast Fourier transform

INTRODUCTION

Statistical signal analysis is actually a mathematical science that involves data collection, analysis, interpretation and presentation (Nuawi et al., 2008a). The main objective for the statistical analysis is to provide an easy and a simple analysis of a complex random signal. Data at several levels could be revealed by the classification and interpretation of the signals (Chatfield and Collins, 1980). Common parameters such as the mean value, standard deviation value, the variance, the skewness, the kurtosis and the root mean square (rms) are used in statistical analysis (Pontuale et al., 2003; Abdullah, 2005).

Average value, standard deviation, variance, skewness, kurtosis and root mean square (rms) are the common Signal Features (SFs) that can be used for extraction from any time domain signal (Sick, 2002; Ghosh et al., 2007; Dong et al., 2006). In order a signal to be able to adequately described and maintained the relevant information, SF from the captured signal need to be properly derived (Teti et al., 2010). Previous works related to I-kaz™ method were mainly on the application of this method on analyzing dynamic signals. The main objective of this study on the other hand, is to investigate the I-kaz™ Multilevel coefficients responses toward the simultaneous variation of amplitude and frequency in synthetic signals.

The mean value \( \bar{x} \) for a signal with \( n \)-number of data points is mathematically defined through Eq. 1, where \( x_i \) is the value of the data point. The mean value is one of the most important and often used parameters in indicating the tendency of the data toward the center:

\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} (x_i)
\]

The standard deviation value is given by:

\[
s = \left( \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{1/2}
\]
where, \( x_i \) is the value of the data point and \( \bar{x} \) is the mean of the data. Base on Eq. 2, standard deviation value measures the spread of the data about the mean value. Variance is the square of the standard deviation as shown in Eq. 3:

\[
\sigma = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}
\]

(3)

Signal classification on the real-life signals base on mean and variance was not compatible mainly due to the signals contain outliers that can bring a noticeable shift in the actual value of both mean and variance (Pontuale et al., 2003).

Skewness is the measurement of the asymmetry from the normal distribution in a set of statistical data. The skewness \( S \), of a set of data is calculated base on the Eq. 4:

\[
S = \frac{1}{nS^2} \sum (x_i - \bar{x})^3
\]

(4)

where, \( x_i \) is the value of the data point and \( \bar{x} \) is the mean of the data and \( s \) is the standard deviation value. Azrullisham et al. (2012) in their study used skewness value to estimate the accelerated test model in fatigue life reliability evaluation of stub axle.

The signal 4th statistical moment Kurtosis \( K \), is an important global signal statistic that is very sensitive to the spikiness of the data. The value of Kurtosis \( K \), for discrete data sets is defined in Eq. 5:

\[
K = \frac{1}{nS^4} \sum (x_i - \bar{x})^4
\]

(5)

The normal or Gaussian distribution, the Kurtosis value is approximately 3.0. The presence of more extreme value or amplitude than should be found in a Gaussian distribution can be detected when the kurtosis value is higher than 3.0.

In industries, statistical analysis using Kurtosis value were used frequently in detecting defect symptoms due to its sensitivity towards the existence of high amplitude (Pontuale et al., 2003). A proper maintenance can be conducted systematically and accurately base on the measurement of Kurtosis value.

Development of integrated kurtosis-based algorithm (I-kaz\textsuperscript{TM}): I-kaz\textsuperscript{TM} was formulated base on the concept of data distribution or scattering about its center points. It was developed with the purpose of giving descriptive and inferential statistics which is an advantage in comparison with other statistical methods that only rely on numerical value. I-kaz\textsuperscript{TM} coefficient, \( Z \) and the value was supported by a three dimensional graphical summarisations of frequency distribution (Nuawi et al., 2008a).

In the original I-kaz\textsuperscript{TM} coefficient calculation, a dynamic signal in time domain will be decomposed into three frequency band by following the 3rd order of the Daubechies concept in signal decomposition process (Daubechies, 1992). To calculate the sampling frequency of any signal in time domain, the Nyquist number must be 2 or greater than the maximum frequency in order to avoid the content of the sampling signal to be misinterpreted. Nyquist number in this calculation was chosen to be equal to 2 for the purpose of calculation simplification (Figliola and Beasley, 2000).

The sensitivity of the I-kaz\textsuperscript{TM} coefficient toward amplitude and frequency change in a signal was proven far better than the current statistical parameters (Nuawi et al., 2008a; Karim et al., 2011). With this advantage, the I-kaz\textsuperscript{TM} coefficient is very suitable in Signal Features (SF) translation. In previous researches, the structure-borne sound signal analyzed using I-kaz\textsuperscript{TM} which correlated with the internal pipe surface condition showed a high ability to differentiate between the smooth and rough pipe surface (Nuawi et al., 2009). Different study showed that the I-kaz\textsuperscript{TM} method was capable of improving the Taylor curve which was unable to exhibit the three typical wear curve for the cutting that use certain cutting speed (Nuawi et al., 2007). Ghani et al. (2011) in used the I-kaz\textsuperscript{TM} coefficient to analyze the flank wear during turning process for tool wear prediction purpose.

Development of I-kaz\textsuperscript{TM} multilevel coefficient (\( Z^L \)): The development of I-kaz\textsuperscript{TM} multilevel coefficient (\( Z^L \)) was inspired by the original I-kaz\textsuperscript{TM} (\( Z \)) which was pioneered by Nuawi et al. (2008b). The new symbol for I-kaz\textsuperscript{TM} Multilevel coefficient is defined as \( Z^L \) in which \( L \) is referring to the number of order of signal decomposition. The decomposition of signals in time domain into more frequency bands is to get a better coefficient response especially in the lower part of the frequency spectrum. The new developed coefficient (\( Z^L \)) is expected to have more sensitivity towards amplitude and frequency change in a signal. In I-kaz\textsuperscript{TM} Multilevel method, signal decomposition using \( L \)th order of Daubechies theorem will result in \( L \) number of frequency bands. This algorithm was summarized as presented in Fig. 1.

The frequency ranges of \( F_1 \), \( F_2 \), \( F_3 \) to \( F_L \) in Fig. 1 are depending on the value of \( n \) and \( f_{mc} \). For I-kaz\textsuperscript{TM} Multilevel with \( L \)-order of signal decomposition and for \( i = 1, 2, 3...L \), the frequency ranges are shown below (Karim et al., 2011):
Fig. 1: Flowchart of the I-kaz™ Multilevel method

\[ F_{i} = \frac{1}{2^{i-1}} \leq F_{i+1} \leq \frac{1}{2^{i-2}} \]

\[ F_{i} = \frac{1}{2^{i-1}} \leq F_{i+1} \leq \frac{1}{2^{i-2}} \]

\[ F_{i} = \frac{1}{2^{i-1}} \leq F_{i+1} \leq \frac{1}{2^{i-2}} \]

The related I-kaz™ Multilevel coefficient can be calculated as (Karim et al., 2011):

\[ I-kaz = \frac{1}{L} \sqrt{K_{1}S_{1}^{2} + K_{2}S_{2}^{2} + K_{3}S_{3}^{2} + \ldots + K_{L}S_{L}^{2}} \]

where, \( L \) indicates the order of signal decomposition.

Karim et al. (2012) in their study used the I-kaz™ Multilevel coefficient at level 7 of signal decomposition to correlate the wear rate of connecting rod bearing.

**MATERIALS AND METHODS**

Creating synthetic signals FIAI, FIAD and FDAI: Three different synthetic signals, Frequency Increase and Amplitude Increase (FIAI), Frequency Increase and Amplitude Decrease (FIAD) and Frequency Decrease and Amplitude Increase (FDAI) were created with the same initial specifications. All signals were created by using MATLAB® and were defined with 512 data points and sampled at 1000 Hz \((F_{s} = 1000 \text{ Hz})\). The synthetic signals originally consist of 10, 25, 40, 55, 70, 100, 140, 200, 300 and 350 Hz sinusoidal waves. The plots of the signals in time and frequency domain are shown in Fig. 2a and b. The unit used in time domain for y axis is volt (V). The

Fast Fourier Transform (FFT) method was used to transform the signal in time domain to frequency domain (Nuawi et al., 2008b).

The amplitude and frequency of the synthetic signals were increased by 10, 20, 30, 40% and 10, 20, 30 and 10 Hz, respectively. For each pair of incremental value, the higher order of I-kaz™ Multilevel coefficients were calculated and compared.

Creating synthetic signals FRAI and FIAI: Another two types of synthetic signals were created in order to identify the influence ratio of the amplitude and frequency in I-kaz™ Multilevel coefficient response. Frequency Remain and Amplitude Increase (FRAI) signal was used to investigate how much the I-kaz™ Multilevel coefficient response towards amplitude change. Whereas, Frequency Increase and Amplitude Remain (FIAI) signal was used to investigate how much the I-kaz™ Multilevel coefficient response towards frequency change. The initial specification of both FRAI and FIAI signals were the same as the specification in signal FIAI, FIAD and FDAI.
The amplitude of signal FRAM was increased by 40% while the frequencies were kept constant. For the amplitude increment, the higher order of I-kaz$^{TM}$ coefficients, $Z^3$, $Z^2$, $Z^1$, $Z^0$ and $Z^{-1}$ were calculated and compared.

In FIAD signal, the frequency was increased by the incremental of 40 Hz while the amplitude was kept constant. Similarly, for the frequency increment, the higher order of I-kaz$^{TM}$ coefficients, $Z^3$, $Z^2$, $Z^1$, $Z^0$ and $Z^{-1}$ were calculated and compared.

RESULTS AND DISCUSSION

The I-kaz$^{TM}$ method was applied in various field of study, such as automotive engine performance monitoring (Nuawi et al., 2008b), fatigue analysis (Abdullah et al., 2007; Putra et al., 2010) and machining condition monitoring (Nuawi et al., 2007; Jabarah et al., 2009). Previous works related to I-kaz$^{TM}$ method were mainly on the application of this method on analyzing dynamic signals. There is no specific work on the investigation of the I-kaz$^{TM}$ Multilevel coefficients responses toward the simultaneous variation of amplitude and frequency in synthetic signals. The most related works related to this study is the study on the I-kaz$^{TM}$ multilevel coefficient response toward the change in amplitude and frequency when one of them was increased and one of them was kept constant (Karim et al., 2011; Nuawi et al., 2008b). The results from these studies showed that the I-kaz$^{TM}$ Multilevel coefficient and the normal I-kaz$^{TM}$ coefficient response were mainly due the change in amplitude of signals. These results was in accordance with the result in this study.

I-kaz$^{TM}$ multilevel coefficient response towards the FIAD signal: Four types of signals from original FIAD signal were created by increasing its amplitude and frequency from 10-40% by 10% incremental. At the same time, the signal frequency was changed by 10-40 Hz by 10 Hz incremental. Figure 3a-b and 4a-b show the sample plot of FIAD signals in time and frequency domain after 20 and 40% amplitude increment and 20 and 40 Hz frequency increment, respectively.

The results of I-kaz$^{TM}$ ($Z^3$) and I-kaz$^{TM}$ Multilevel coefficients ($Z^2$, $Z^1$, $Z^0$) toward different level of amplitude and frequency for the FIAD signal are presented in Fig. 5 and Table 1.

The I-kaz$^{TM}$ Multilevel coefficient values increase linearly with the increase in amplitude and frequency of the FIAD signal. The superscript number on the top left of letter $Z$ represents the level of signal decomposition. The sensitivity of all coefficients toward the amplitude change can be seen clearly from Table 1. The higher the order of the I-kaz$^{TM}$ Multilevel coefficients, the more sensitive it responded to the amplitude and frequency change. For this type of particular synthetic signal, the sensitivity of the I-kaz$^{TM}$ Multilevel coefficient saturated at the 7th order ($Z^7$).

I-kaz$^{TM}$ multilevel coefficient response toward the FIAD signal: Four types of signals from original FIAD signal
were created by increasing its sinusoidal frequencies by 10 Hz incremental and decreasing its amplitude by 10% decremental as shown in Fig. 6a and b. Figure 7a and b show the sample plot of FIAD signal in time and frequency domain after 40 Hz frequency increment and 40% amplitude decrement of the FIAD.

The result of I-kaz\textsuperscript{TM} (Z\textsuperscript{+}) and I-kaz\textsuperscript{TM} Multilevel coefficients (Z\textsuperscript{−}) toward different degree of the FIAD signal are presented in Fig. 8 and Table 2.

The I-kaz\textsuperscript{TM} Multilevel coefficient values decrease linearly when frequency is increased and amplitude is decreased. At this stage, the change in amplitude has more influence on the response of I-kaz\textsuperscript{TM} Multilevel values. From Table 2, it was found that the most sensitive I-kaz\textsuperscript{TM} Multilevel value was found at level 3.

I-kaz\textsuperscript{TM} multilevel coefficient response toward the FDAI signal: Four types of signals from original FDAI signal were created by decreasing its sinusoidal frequencies by 10 Hz decremental and increasing its amplitude by 10% incremental. Figure 9a and b show the sample plot of FDAI signal in time and frequency domain after 10 Hz
Fig. 7(a-b): The 40% amplitude decrease and 40 Hz frequency increase of FIAD signal (a) Time-domain (b) Frequency domain

Fig. 9(a-b): The 10% amplitude increase and 10 Hz frequency decrease of FDAI signal (a) Time-domain (b) Frequency domain

Table 2: The response of each coefficient in deviation percentage for FIAD signal

<table>
<thead>
<tr>
<th>Amplitude (%)</th>
<th>-10</th>
<th>-20</th>
<th>-30</th>
<th>-40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
<td>+10</td>
<td>+20</td>
<td>+30</td>
<td>+40</td>
</tr>
<tr>
<td>'Z'</td>
<td>-20.1258</td>
<td>-35.8491</td>
<td>-52.8302</td>
<td>-66.6667</td>
</tr>
<tr>
<td>'Z'</td>
<td>-18.1818</td>
<td>-35.5372</td>
<td>-51.2397</td>
<td>-63.6364</td>
</tr>
<tr>
<td>'Z'</td>
<td>-15.0943</td>
<td>-30.1887</td>
<td>-44.3396</td>
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<tr>
<td>'Z'</td>
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<td>-26.0000</td>
<td>-41.0000</td>
<td>-56.0000</td>
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<tr>
<td>'Z'</td>
<td>-10.1010</td>
<td>-25.2525</td>
<td>-40.4040</td>
<td>-55.5556</td>
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<tr>
<td>'Z'</td>
<td>-10.1010</td>
<td>-25.2525</td>
<td>-40.4040</td>
<td>-55.5556</td>
</tr>
</tbody>
</table>

'Z', 'Z', 'Z', 'Z', and 'Z': Value of the coefficient calculated when the FIAD signal decomposed to 3, 4, 5, 6, 7 and 8 levels, respectively.

Fig. 8: The response of I-kaz™ Multilevel coefficient in FIAD signal frequency increment and 10% amplitude reduction of FDAI signal. Figure 10a and b show the sample plot of FDAI signal in time and frequency domain after 40 Hz frequency increment and 40% amplitude reduction of FDAI signal.

The result of I-kaz™ (Z') and I-kaz™ Multilevel coefficients ("Z") toward different degree of the FDAI signal are presented in Fig. 11 and Table 3.

The I-kaz™ Multilevel value increase linearly when the amplitude is increased and the frequency is decreased.
Fig. 10(a-b): The 40% amplitude increase and 40 Hz frequency decrease of FDAQ signal (a) Time-domain (b) Frequency domain

<table>
<thead>
<tr>
<th>Amplitude (%)</th>
<th>-10</th>
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<th>+10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
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<td>0</td>
</tr>
<tr>
<td>32&quot;</td>
<td>22.500</td>
<td>43.333</td>
<td>58.333</td>
</tr>
<tr>
<td>&quot;Z&quot;</td>
<td>23.1579</td>
<td>50.5286</td>
<td>82.1035</td>
</tr>
<tr>
<td>&quot;Z&quot;</td>
<td>21.4286</td>
<td>29.5918</td>
<td>48.9796</td>
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<tr>
<td>&quot;Z&quot;</td>
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<td>26.5306</td>
<td>36.7347</td>
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<tr>
<td>&quot;Z&quot;</td>
<td>21.4286</td>
<td>26.5306</td>
<td>36.7347</td>
</tr>
</tbody>
</table>

Table 3: The response of each coefficient in deviation percentage for FDAQ signal

<table>
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<tr>
<th>Amplitude (%)</th>
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<th>+50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
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<td>+50</td>
</tr>
<tr>
<td>32&quot;</td>
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<tr>
<td>&quot;Z&quot;</td>
<td>0.0210</td>
<td>0.0134</td>
</tr>
<tr>
<td>&quot;Z&quot;</td>
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<td>0.0134</td>
</tr>
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<td>&quot;Z&quot;</td>
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<td>&quot;Z&quot;</td>
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<td>0.0134</td>
</tr>
<tr>
<td>Average</td>
<td>0.0114</td>
<td>0.0135</td>
</tr>
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</table>

Table 4: The response of I-kaz™ multilevel coefficient in FDAQ signal

I-kaz™ multilevel coefficient response toward the FRAI and FIAR signal: Synthetic signals FRAI and FIAR were created to investigate the percentage contribution of frequency and amplitude toward the change in I-kaz™ Multilevel coefficient value. Table 4 shows the value of I-kaz™ Multilevel coefficients calculated on FIAR signal when the frequency was increased to 50 Hz while the amplitude was kept constant. In Table 5, FRAI synthetic signals were used to calculate the value of I-kaz™ Multilevel coefficients in which the amplitude of this was increased to 50% while the frequency was kept constant.

The above results also clearly indicate that the amplitude change has more influence than the frequency change in FRAI and FIAR signals, respectively for the I-kaz™ Multilevel coefficient value. A very similar results
**I-kaz™ multilevel representation for FIAI signals:** The original I-kaz™ technique provides not only the coefficient value, but also a 3-D graphical illustration. The higher value of "Z" refers to the bigger space scattering of I-kaz™ Multilevel representation.

By using Eq. 6-8, for the Nyquist number equal to 2, frequency span equal to 1000 Hz, f_{max} equal to 500 Hz and 1 equal to 3, the frequency ranges of the I-kaz™ Multilevel representation can be summarized as follows:

- **x-axis:** Low Frequency (LF) range of 0-0.25 f_{max}
- **y-axis:** High Frequency (HF) range of 0.25 f_{max}-0.5 f_{max}
- **z-axis:** Very high frequency (VF) range of 0.5 f_{max}-f_{max}

The I-kaz™ Multilevel 3D representation and their coefficients values are shown in Fig. 12 and 13 for the original FIAI signal and the FIAI signal after 40% amplitude and 40 Hz 000011 frequency increase, respectively. The data distribution of the I-kaz™ Multilevel 3D representation for the Z-notch filtered signal in Fig. 13 was spread compared to the I-kaz™ Multilevel 3D representation in Fig. 12. Thus, the judgment of the existence of different signals can be based on the I-kaz™ Multilevel coefficient and also the I-kaz™ Multilevel 3D representation. A similar single I-kaz™ 3D scattering pattern was reported by Nuawi et al. (2008b) when studying for the filtered and the unfiltered signal in machining.

**CONCLUSION**

This study discussed the response of I-kaz™ Multilevel coefficients toward the simultaneous change in amplitude and frequency of signals. This new I-kaz™ Multilevel method was proven to be very sensitive and detects very well in amplitude and frequency changes of measured signals.

In FIAI and FDAl signal study, all level of I-kaz™ Multilevel coefficients were noted to be increasing with the increase in amplitude regardless of the frequency condition, either increasing or decreasing. For both types of signals, the optimized value of I-kaz™ was found to be at the highest order of signal decomposition which is "Z". The increasing rate of I-kaz™ Multilevel coefficients in response with FIAI signal is greater compare to the response in FDAl signal.

The study of FIAD showed that the I-kaz™ Multilevel coefficients were decreasing with the reduction in amplitude regardless of the increasing of the frequency in the signal. The optimized value of I-kaz™ Multilevel coefficient was found to be at the lowest order of signal decomposition which is "Z".
In FIAI, FIAD and FDAI signal study, the response of I-kaz¥ Multilevel coefficients are greatly depending on the amplitude change in the signals. The increasing or decreasing of the I-kaz¥ Multilevel coefficients would follow the trend in the amplitude of the measured signals. The influence of amplitude and frequency in the I-kaz¥ Multilevel coefficients response can be estimated in the FRAI and FIAR signals study. From this study, the influence ratio of amplitude and frequency is 86.96 and 13.04%, respectively. The study of this five different synthetic signals showed that the I-kaz¥ Multilevel coefficients saturated at the highest level of signal decomposition except in the response of FIAD signal. Recognizing and understanding the behaviour of I-kaz¥ Multilevel coefficients toward the change in amplitude and frequency is important especially when analysing dynamic signals.

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