# UNIVERSITI KUALA LUMPUR 

Malaysia France Institute

## FINAL EXAMINATION

## SEPTEMBER 2013 SESSION

| SUBJECT CODE | $:$ FAB21203 |
| :--- | :--- |
| SUBJECT TITLE | $:$ LINEAR SYSTEMS AND SIGNALS 1 |
| LEVEL | $:$ BACHELOR |
| TIME I DURATION | $: 3$ HOURS |
| DATE | $:$ |

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of five (5) questions. Answer four (4) questions only.
6. Answer all questions in English.
7. Formula is appended.

## Question 1

a) Determine whether the signal $x(t)=5+\cos (2 \pi t)+5 \sin (4 \pi t)-7 \cos (7 \pi t)$ is periodic. If it is periodic, find the fundamental period.
b) For the signal, $x(t)$ that is shown in Figure 1, do the followings:
i) Represent the signal $x(t)$ in terms of sum of unit step only.
ii) Sketch the derivative of $x(t)$ i.e $\frac{d x(t)}{d t}$
iii) Sketch the integral of $y(t)$ i.e $\int_{-\infty}^{\infty} x(t) d t$
(3 marks)
iv) Determine whether the signal, $x(t)$ is energy signal, power signal or neither energy nor power signal. Justify your answer.
v) Sketch the signal $-1+x\left(-\frac{t}{2}+\frac{1}{2}\right)$.


Figure 1: The signal $x(t)$
b) State whether the following system is with memory or memoryless, causal or noncausal, time variant or time invariant, linear or nonlinear, given that $x(t)$ and $y(t)$ denote the system's input and output respectively. Justify your answer.

$$
y(t)=4 e^{-3 t} x\left(\frac{t}{2}\right)
$$

## Question 2

a) Consider the electric circuit shown in Figure 2, where $i(t)$ is the input current source and $y(t)$ is the output voltage.
i) Derive the differential equation of the circuit. (4 marks)
ii) Determine $y(t)$ for $t>0^{-}$if $i(t)=-\cos (0.001 t)$ and $y\left(0^{-}\right)=0$ with $R=5 k \Omega$ and $L=1 \mathrm{mH}$.
(8 marks)


Figure 2: The electric circuit
b) Consider a system with the following differential equation:

$$
\frac{d^{2} y(t)}{d t^{2}}+6 \frac{d y(t)}{d t}+8 y(t)=e^{-4 t}
$$

If the initial conditions are $y\left(0^{-}\right)=1$ and $\frac{d y\left(0^{-}\right)}{d t}=0$, by using the classical method (undetermined coefficient), determine:
i) The zero input response, $y_{z i}(t)$.
ii) The zero state response, $y_{z s}(t)$. (7 marks)
iii) The total response, $y(t)$.

## Question 3

a) Refer to a periodic signal, $x(t)$ in Figure $\mathbf{3}$ and determine the following:
i) The fundamental period, $T_{0}$, angular frequency, $\omega_{0}$ and symmetric property.
$\begin{array}{ll}\text { ii) The trigonometric Fourier series of } x(t) & \text { (2 marks) } \\ \text { iii) The complex exponential Fourier series of } x(t) & \text { (8 marks) } \\ & \end{array}$


Figure 3: A periodic signal, $x(t)$
b) Sketch the frequency spectrum of the following trigonometric Fourier series of the signal, $f(t)$ for $n=0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5$.

$$
\begin{equation*}
f(t)=\frac{1}{2}-\sum_{n=1}^{\infty} \frac{2}{n \pi} \sin (n \pi t) \tag{7marks}
\end{equation*}
$$

c) Given $y(t)$ is a periodic continuous time signal with period $\pi \mathrm{sec}$ and having even half-wave symmetric property. Write down the equations for the trigonometric Fourier series coefficients taking advantage of the symmetric effect. DO NOT SOLVE IT.

## Question 4

a) If the unilateral Laplace transform is given by:

$$
L\{x(t)\}=\int_{0^{-}}^{\infty} x(t) e^{-s t} d t
$$

Determine the Laplace transform of $x(t)=\left(4+3 e^{-4 t}\right) u(t)$ using direct integration.
(4 marks)
b) From the unilateral Laplace transform table in Appendix, determine the Laplace transform of the followings:
i) $\quad f(t)=t u(t)-2(t-2) u(t-2)+(t-3) u(t-3)$
ii) $\quad v(t)=e^{-10 t} \cos (100 t-10) u(t-0.1)$
c) A system is described by a differential equation,

$$
\frac{d^{2} y(t)}{d t^{2}}+7 \frac{d y(t)}{d t}+12 y(t)=6 x(t)
$$

where $x(t)$ is the input, $y(t)$ is the output.
i) Determine the transfer function of the system.
(3 marks)
ii) Draw the poles and zeros plot and determine the stability of the system stability.
iii) Solve for $y(t)$ using Laplace transform method if $x(t)=u(t), y\left(0^{-}\right)=0$ and

$$
\frac{d y\left(0^{-}\right)}{d t}=-2 .
$$

(6 marks)
iv) Determine the steady state value of $y(t)$ using Final Value Theorem.

## Question 5

a) Find the transfer function of the RC network shown in Figure 4 using Laplace transform method.
(8 marks)


Figure 4: The RC network
b) Determine the inverse Laplace transform, $x(t)$ for the following equation:

$$
X(s)=\frac{8 s+10}{(s+1)(s+2)^{3}}
$$

(10 marks)
c) Given that $G(s)$ denote the forward path transfer function and $H(s)$ denote the feedback path transfer function of a unity negative feedback closed-loop system. Let $R(s)$ and $Y(s)$ be the input signal and the output signal respectively. Answer the followings:
i) Sketch the overall block diagram of the system.
ii) Show that the equivalent closed-loop transfer function is given by:

$$
T(s)=\frac{G(s)}{1+G(s) H(s)}
$$

iii) State one (1) condition for the closed-loop system to be stable.

| APPENDIX <br> Laplace Transform Table |  |  |
| :---: | :---: | :---: |
| Item no. | $f(t)$ | $F(s)$ |
| 1. | $\delta(t)$ | 1 |
| 2. | $u(t)$ | $\frac{1}{s}$ |
| 3. | $t u(t)$ | $\frac{1}{s^{2}}$ |
| 4. | $t^{n} u(t)$ | $\frac{n!}{s^{n+1}}$ |
| 5. | $e^{-a t} u(t)$ | $\frac{1}{s+a}$ |
| 6. | $\sin \omega t u(t)$ | $\frac{\omega}{s^{2}+\omega^{2}}$ |
| 7. | $\cos \omega t u(t)$ | $\frac{s}{s^{2}+\omega^{2}}$ |

