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SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2010 SESSION

SUBJECT CODE	:	FAB 30703
SUBJECT TITLE	:	ROBOTICS
LEVEL	:	BACHELOR
TIME / DURATION	:	9.00am – 12.00pm (3 HOURS)
DATE	:	02 MAY 2010

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This questions paper consists of FIVE (5) questions. Answer FOUR (4) questions only.
6. Answer ALL questions in English.
7. Formula is appended.

THERE ARE 6 PAGES OF QUESTIONS AND 5 PAGES OF APPENDICES, EXCLUDING THIS PAGE.

Question 1

(a) Define each of the following common terminologies:

i. degrees-of-freedom (DOF)

(1.5 marks)

ii. manipulator

(1.5 marks)

iii. anthropomorphic

(1.5 marks)

iv. workspace

(1.5 marks)

v. kinematics

(1.5 marks)

vi. dynamics

(1.5 marks)

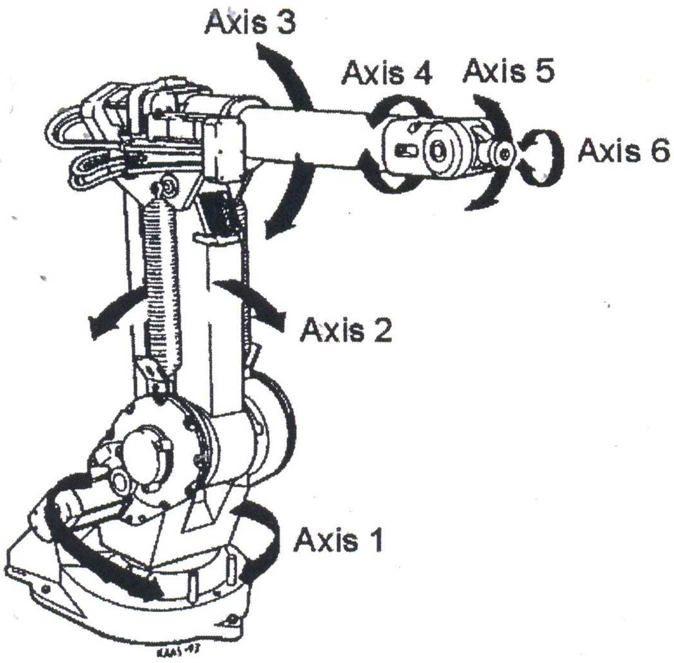

Figure 1: ABB Robot

(b) Draw the schematic diagram for this ABB IRB 1400 M2000 Robot using the revolute and prismatic convention for **Figure 1**.

(4 marks)

(c) Assign coordinate frames for the joints using the Denavit-Hartenberg algorithm.

(6 marks)

(d) Suppose that a robot is made of a Cartesian and RPY combination of joints. Find the necessary RPY angles to achieve the following transformation:

$$T = \begin{bmatrix} 0.527 & -0.574 & 0.628 & 4 \\ 0.369 & 0.819 & 0.439 & 6 \\ -0.766 & 0 & 0.643 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(6 marks)

Question 2

(a) Define 'Inverse Kinematics'

(2 marks)

(b) Explain the difficulties of solving the inverse kinematics.

(6 marks)

(c) In a 6 degrees-of-freedom (DOF) robotics manipulator, three of the joints are for providing position. Describe the function for the rest of the joints.

(2 marks)

(d) **Figure 2** shows a spherical arm with two rotary joints and a prismatic joint. The corresponding arm parameters and the forward kinematics are given in **Table 1** and **Equation 1**. Find the equation of inverse kinematics of d_3, θ_1 and θ_2.

(15 marks)

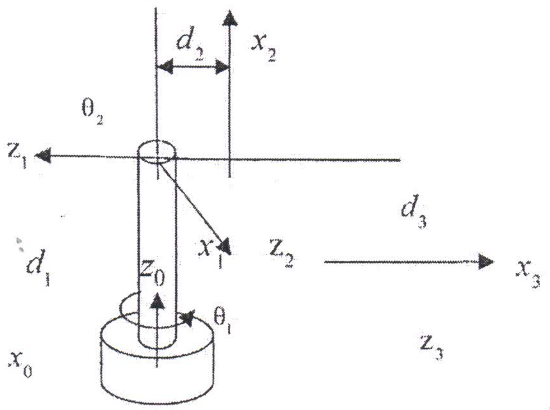

Figure 2: A Spherical Arm

Table 1: The arm parameters

Link	θ_i	a_i	α_i	d_i
1	θ_1	0	90°	d_1
2	θ_2	0	90°	d_2
3	-90°	0	0°	d_3

$$H_0^3 = \begin{bmatrix} -s_1 & c_1c_2 & c_1s_2 & d_3c_1s_2 + d_2s_1 \\ c_1 & s_1c_2 & s_1s_2 & d_3s_1s_2 - d_2c_1 \\ 0 & s_2 & -c_2 & d_1 - d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ - **Equation 1**

Question 3

(a) Consider the two-link robot arm shows in **Figure 3**.

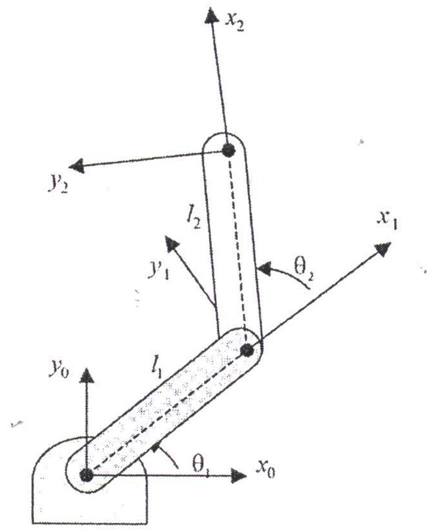

Figure 3: The two link robot arm

i. Find the ${}^0J$, the Jacobian matrix relative to the base frame (4 marks)

ii. Find the ${}^2J$, the Jacobian matrix relative to the frame $x_2y_2z_2$. (4 marks)

iii. Show that Jacobian ${}^0J$ and ${}^2J$, satisfy ${}^0J = \begin{bmatrix} R_0^n & 0 \\ 0 & R_0^n \end{bmatrix} {}^nJ$ with $n = 2$. (5 marks)

(b) A spherical wrist with three rotary joints is shown in **Figure 4**, where the joint axes Z_3, Z_4 and Z_5 intersect at one point. The arm parameters are given in **Table 2**. Compute the Jacobian matrix J.

(12 marks)

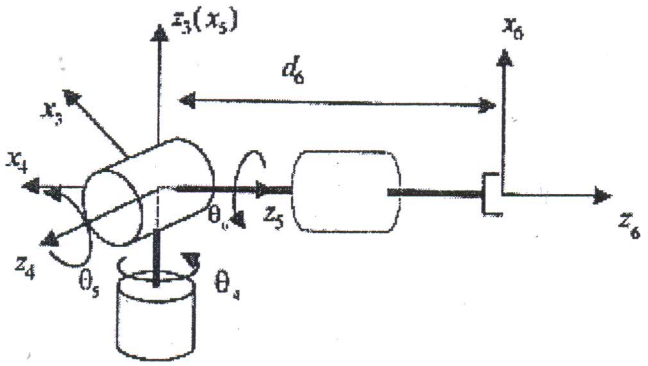

Figure 4: A spherical wrist with three rotary joints

Table 2: The arm parameters

Link	θ_i	a_i	α_i	d_i
4	θ_4	0	-90°	0
5	θ_5	0	90°	0
6	θ_6	0	0°	d_6

Question 4

(a) Explain the difference between point-to-point control and continuous control in robot path planning.

(5 marks)

(b) Consider a single-link robot manipulator with rotary joint. The robot joint is required to perform a trajectory with the following two cubic segments: The first segment connects the initial angular position $\theta(0) = 25^0$ to the via point $\theta(1) = 15^0$, and the second segment connects the via point $\theta(1) = 15^0$ to the final angular position $\theta(2) = 60^0$. The designed trajectory should have zero initial velocity and zero final velocity. Also, at the via point $\theta(1) = 15^0$, the trajectory should have continuous velocity and acceleration.

i. Determine the constraint equations for the trajectory based on the above constraints given.

(5 marks)

ii. Derive the polynomial parameters for the trajectory.

(10 marks)

iii. Determine the position, velocity and acceleration equation for the trajectory.

(5 marks)

Question 5

(a) Explain the difference between the Lagrange-Euler approach and the Newton-Euler approach in computing the dynamics of a robot.

(4 marks)

(b) Consider the point masses, m_1 and m_2 at the distal end of links of the R - θ robot manipulator with one prismatic and one rotary joints as shown in **Figure 5**. Derive the followings:

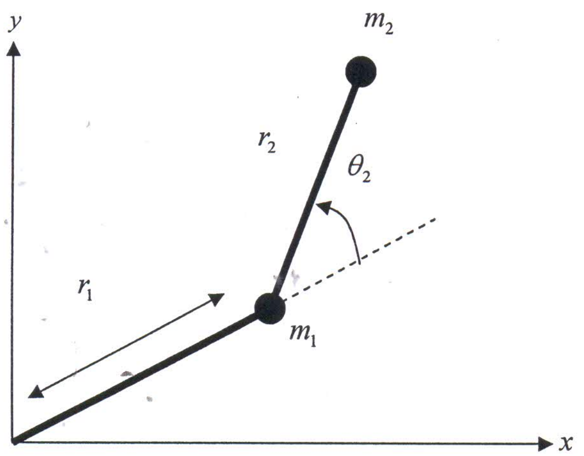

Figure 5: The R - θ robot manipulator with one prismatic and one rotary joints

i. The Kinetic and Potential Energy equation for the manipulator

(6 marks)

ii. The Lagrange equation of the manipulator.

(6 marks)

iii. The Torque equation for joint 2.

(9 marks)

END OF QUESTION

APPENDIX

Matrix Functions

Rotation transformation:

$$R_x(\theta)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{bmatrix};\quad R_y(\theta)=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix};\quad R_z(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}.$$

The D-H transformation matrix from the frame $x_{i-1}y_{i-1}z_{i-1}$ to the frame $x_iy_iz_i$ is:

$$H_{i-1}^{i}=H(\theta_i)Tran(d_i)Tran(a_i)H(\alpha_i)$$

$$=\begin{bmatrix}C\theta_i & -C\alpha_iS\theta_i & S\alpha_iS\theta_i & a_iC\theta_i\\ S\theta_i & C\alpha_iC\theta_i & -S\alpha_iC\theta_i & a_iS\theta_i\\ 0 & S\alpha_i & C\alpha_i & d_i\\ 0 & 0 & 0 & 1\end{bmatrix}$$

The forward kinematics solution for an n-linked robot can be expressed as:

$$H_0^n=\begin{bmatrix}\mathbf{n} & \mathbf{o} & \mathbf{a} & \mathbf{P}\\ 0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}n_x & o_x & a_x & P_x\\ n_y & o_y & a_y & P_y\\ n_z & o_z & a_z & P_z\\ 0 & 0 & 0 & 1\end{bmatrix}$$

The inverse of a homogeneous transformation matrix H can be expressed as:

$$H^{-1}=\begin{bmatrix}R^T & -R^TP\\ 000 & 1\end{bmatrix}=\begin{bmatrix}n_x & n_y & n_z & -n^TP\\ o_x & o_y & o_z & -o^TP\\ a_x & a_y & a_z & -a^TP\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Trigonometry Functions

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\frac{\partial}{\partial\theta_A}\left(\cos(A+B)\right)=-\sin(A+B)$$

$$\frac{\partial}{\partial\theta_B}\left(\cos(A+B)\right)=-\sin(A+B)$$

$$\frac{\partial}{\partial\theta_A}\left(\sin(A+B)\right)=\cos(A+B)$$

$$\frac{\partial}{\partial\theta_B}\left(\sin(A+B)\right)=\cos(A+B)$$

Equation	Solution
(a) $\sin\theta = a$	$\theta = A\tan 2\left(a, \quad \pm\sqrt{1-a^2}\right)$
(b) $\cos\theta = b$	$\theta = A\tan 2\left(\pm\sqrt{1-b^2}, \quad b\right)$
(c) $\begin{cases}\sin\theta = a \\ \cos\theta = b\end{cases}$	$\theta = A\tan 2(a,b)$
(d) $a\cos\theta - b\sin\theta = 0$	$\theta^{(1)} = A\tan 2(a, \quad b)$ $\theta^{(2)} = A\tan 2(-a, \quad -b) = \pi + \theta^{(1)}$
(e) $a\cos\theta + b\sin\theta = c$	$\theta^{(1)} = A\tan 2\left(c, \quad \sqrt{a^2+b^2-c^2}\right) - A\tan 2(a, \quad b)$ $\theta^{(2)} = A\tan 2\left(c, \quad -\sqrt{a^2+b^2-c^2}\right) - A\tan 2(a, \quad b)$
(f) $\begin{cases}a\cos\theta - b\sin\theta = c \\ a\sin\theta + b\cos\theta = d\end{cases}$	$\theta = A\tan 2(ad - bc, \quad ac + bd)$
(g) $\begin{cases}\sin\alpha\sin\beta = a \\ \cos\alpha\sin\beta = b \\ \cos\beta = c\end{cases}$	$\begin{cases}\alpha^{(1)} = A\tan 2(a, \quad b) \\ \beta^{(1)} = A\tan 2\left(\sqrt{a^2+b^2}, \quad c\right)\end{cases}$ $\begin{cases}\alpha^{(2)} = A\tan 2(-a, \quad -b) = \pi + \alpha^{(1)} \\ \beta^{(2)} = A\tan 2\left(-\sqrt{a^2+b^2}, \quad c\right)\end{cases}$

The Atan2 function can be defined as follows:

$$A\tan 2(p_x, p_y) = \begin{cases} \arctan\left(\dfrac{p_y}{p_x}\right) & p_x > 0 \\ \arctan\left(\dfrac{p_y}{p_x}\right) + \pi & p_x < 0 \\ \dfrac{\pi}{2} & p_x = 0 \quad \& \quad p_y > 0 \\ \dfrac{-\pi}{2} & p_x = 0 \quad \& \quad p_y < 0 \end{cases}$$

Jacobian Function.

The Del Operator, ∇ for small motion about a fixed world coordinate frame can be given as follows:-

$$\Delta T \approx \begin{bmatrix} 0 & -\delta_z & \delta_y & dx \\ \delta_z & 0 & -\delta_x & dy \\ -\delta_y & \delta_x & 0 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet T \approx \nabla \bullet T$$

where,

$$\nabla \approx \begin{bmatrix} 0 & -\delta_z & \delta_y & dx \\ \delta_z & 0 & -\delta_x & dy \\ -\delta_y & \delta_x & 0 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Del Operator for small motion with respect to its own frame ${}^T\nabla$ can be defined as follows:

$${}^T\nabla = \begin{bmatrix} 0 & -{}^T\delta_z & {}^T\delta_y & {}^Tdx \\ {}^T\delta_z & 0 & -{}^T\delta_x & {}^Tdy \\ -{}^T\delta_y & {}^T\delta_x & 0 & {}^Tdz \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where,

$${}^Tdx = \delta \bullet (\overrightarrow{d \times n}) + \vec{d}_p \bullet \vec{n} \qquad {}^T\delta x = \delta \bullet \vec{n}$$

$${}^Tdy = \delta \bullet (\overrightarrow{d \times n}) + \vec{d}_p \bullet \vec{o} \qquad {}^T\delta y = \delta \bullet \vec{o}$$

$${}^Tdz = \delta \bullet (\overrightarrow{d \times n}) + \vec{d}_p \bullet \vec{a} \qquad {}^T\delta x = \delta \bullet \vec{a}$$

Note:
d, **n**, **o** & **a** vectors are extracts from the *T* Matrix
d_p is the translation vector in $\forall$
δ is the rotational effects in $\forall$

The Jacobian matrix for an n-linked robot with position vector $\mathbf{P}$ at the end of the robot arm and joint variables $\mathbf{q}$ where $P = [p_x \quad p_y \quad p_z]^T$ and $\mathbf{q} = [q_1 \quad q_2 \quad \cdots \quad q_n]^T$, can be defined as follows:

$$p_x = f_1(q_1, q_2, \ldots, q_n)$$
$$p_y = f_2(q_1, q_2, \ldots, q_n)$$
$$p_z = f_3(q_1, q_2, \ldots, q_n)$$

$$v_x = \frac{dp_x}{dt} = \frac{\partial f_1}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial f_1}{\partial q_2}\frac{dq_2}{dt} + \cdots + \frac{\partial f_1}{\partial q_n}\frac{dq_n}{dt}$$
$$v_y = \frac{dp_y}{dt} = \frac{\partial f_2}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial f_2}{\partial q_2}\frac{dq_2}{dt} + \cdots + \frac{\partial f_2}{\partial q_n}\frac{dq_n}{dt}$$
$$v_z = \frac{dp_z}{dt} = \frac{\partial f_3}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial f_3}{\partial q_2}\frac{dq_2}{dt} + \cdots + \frac{\partial f_3}{\partial q_n}\frac{dq_n}{dt}$$

where,

$$\frac{dq_i}{dt} = \dot{q}_i$$

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \cdots & \frac{\partial f_1}{\partial q_n} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \cdots & \frac{\partial f_2}{\partial q_n} \\ \frac{\partial f_3}{\partial q_1} & \frac{\partial f_3}{\partial q_2} & \cdots & \frac{\partial f_3}{\partial q_n} \\ \eta_1 R^0_{0(3col)} & \eta_2 R^1_{0(3col)} & \cdots & \eta_n R^{n-1}_{0(3col)} \end{bmatrix}$$

where,

$\eta_1 = 1$ for revolute joint

$\eta_1 = 0$ for prismatic joint

In vector form, the Jacobian matrix can also be defined as follows:

i) For Revolute Joint:

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} \overrightarrow{Z_{i-1}} \times \left(\overrightarrow{O_n} - \overrightarrow{O_{i-1}}\right) \\ \overrightarrow{Z_{i-1}} \end{bmatrix}$$

ii) For Prismatic Joint:

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} \overrightarrow{Z_{i-1}} \\ \vec{0} \end{bmatrix}$$

where, Z_{i-1}'s and O_{i-1}'s are the frame coordinates for the *i-1*

- Z_{i-1} is the 3rd column of the T_0^{i-1} (= A_1*... *A_{i-1})
- O_{i-1} is 4th column of the T_0^{i-1} (= A_1*... *A_{i-1})
- O_n is 4th column Of T_0^n (the FKS!)
- NOTE: when we extract the columns we only need the first 3 rows !!!.

Vector Function.

Cross Product of two vectors **A** and **B**. The end product is a vector **C**.

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i + \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

$$= \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix} = (a_2b_3 - a_3b_2)i + (a_3b_1 - a_1b_3)j + (a_1b_2 - a_2b_1)k$$

Dot Product of two vectors A and B. The end product is a scalar c.

$$c = A \bullet B = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = (a_1b_1 + a_2b_2 + a_3b_3)$$