CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR

FINAL EXAMINATION
JANUARY 2010 SESSION

SUBJECT CODE	:	WQD10202
SUBJECT TITLE	:	TECHNICAL MATHEMATICS II
LEVEL	:	DIPLOMA
TIME / DURATION	:	9.00 am – 11.00 am (2 HOURS)
DATE	:	27 APRIL 2010

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of THREE (3) parts. Part A, B and C. Answer all questions in Part A and B. For Part C, answer two (2) questions only.
6. Answer all questions in English.
7. Formula Sheet is appended.

THERE ARE 8 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

PART A (Total: 15 marks)

MULTIPLE CHOICE QUESTIONS
INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

1. Determine the type of relation in **FIGURE 1** below.

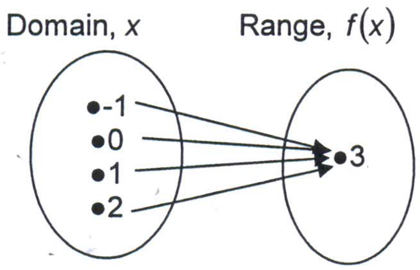

FIGURE 1

A. One to One
B. Many to Many
C. One to Many
D. Many to One

2. Let $f(x)=\begin{cases} 2 & \text{for } x<-2 \\ x^2+2x & \text{for } -2\le x<1 \\ 4\text{-}x & \text{for } x\ge 1 \end{cases}$.

$f(0)$ equals to

A. 4
B. 2
C. 0
D. -2

3. Compute the limit of $\lim_{x\to 0}\left(2x^2+x-1\right)$.

A. 2
B. $\frac{1}{2}$
C. 0
D. -1

4. Given that $y = \frac{1}{x^6}$, determine $\frac{dy}{dx}$

A. $-\frac{7}{x^5}$

B. $\frac{7}{x^7}$

C. $-\frac{6}{x^5}$

D. $-\frac{6}{x^7}$

5. Which of the following methods can be used to differentiate $y = \frac{x^2}{\cos 3x}$?

A. Quotient Rule

B. Power Rule

C. Product Rule

D. Chain Rule

6. Determine the value of $\frac{dy}{dx}$ for the function $y = x^{\frac{1}{3}}$ at $x = 8$?

A. 2

B. -6

C. -4

D. $\frac{1}{12}$

7. The differentiation of $\cos 4x$ with respect to x is

A. $\cos 4x$

B. $4\sin 4x$

C. $-4\sin 4x$

D. $-\sin 4x$

8. Given $y = x^3 - x^2 - 2x + 3$. Determine $\frac{d^2y}{dx^2}$.

A. $3x^2 - 2x - 2$

B. $6x - 2$

C. 6

D. 0

9. Which of the following is **NOT** a technique of integration?

A. Integration by substitution

B. Integration by part

C. Integration by partial fraction

D. Integration by chain rule

10. Choose the suitable pair to solve the following function using **integration by part** method: $\int 2xe^{2x}\,dx$

A. $u = x^2$, $v = e^{2x}dx$

B. $u = 2x$, $dv = e^{2x}dx$

C. $u = x^2$, $v = e^{x}dx$

D. $u = 2x$, $dv = e^{x}dx$

11. Evaluate $\int_0^2 (4x^3 + 5)dx$.

A. 26

B. 20

C. 8

D. 48

12. Determine the value for A and B by using **partial fractions method** if

$$1=\frac{A}{x+1}+\frac{B}{x-1}.$$

A. $A=-\frac{1}{2}$ and $B=\frac{1}{2}$

B. $A=\frac{1}{2}$ and $B=\frac{1}{2}$

C. $A=-2$ and $B=2$

D. $A=-1$ and $B=1$

13. The **conjugate** of $Z=-1+j7$ is

A. $1-j7$

B. $1+j7$

C. $-1-j7$

D. $-1+j7$

14. Simplify j^{99}

A. -1

B. 1

C. $-j$

D. j

15. Let $Z=-2-j$. Calculate the **arg(**Z**)**.

A. 333.435°

B. 26.565°

C. 153.435°

D. 206.565°

PART B (Total: 35 marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question 1

Consider the following functions; $f(x) = \dfrac{1}{1+x}$ and $g(x) = \sqrt{x^3 - 4}$. Determine $(f \circ g)(x)$.

[2 marks]

Question 2

For the function $f(x)$ graphed in **FIGURE 2**, determine each of the following;

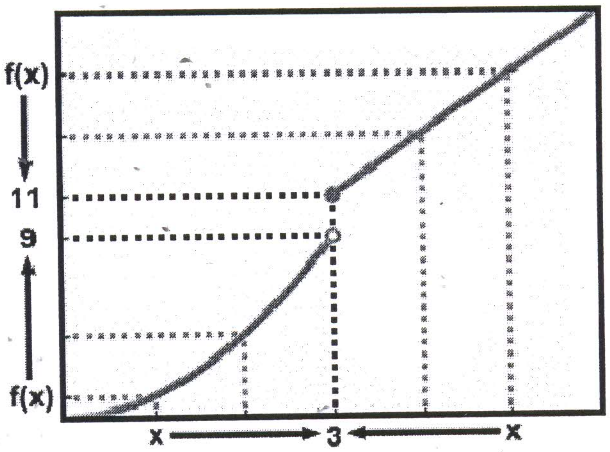

FIGURE 2

a) $\lim\limits_{x \to 3^-} f(x)$.

[1 mark]

b) $\lim\limits_{x \to 3^+} f(x)$

[1 mark]

c) $\lim\limits_{x \to 3} f(x)$

[1 mark]

Question 3

Use **implicit differentiation** to determine $\frac{dy}{dx}$ of $x^3 - y^3 = 6xy$.

[6 marks]

Question 4

The width of a rectangle is increasing at a rate of 2cm/sec and its length is increasing at a rate of 3cm/sec. At what rate is the area of the rectangle increasing when its width is 4cm and its length is 5cm?

[5 marks]

Question 5

Verify that $\int_1^2 \left[\frac{1}{2u^2} - \frac{3}{2u^3} + \frac{9}{2u^4}\right] du = 1$.

[7 marks]

Question 6

Integrate $\int \frac{3}{x^2 - x - 6} dx$ by using the **partial fractions method**.

[7 marks]

Question 7

If $Z_1 = 2 - 5j$ and $Z_2 = 2 + 7j$, compute $\frac{\overline{Z_1}}{Z_2}$.

[5 marks]

PART C (Total: 30 marks)

INSTRUCTION: Answer TWO questions.

Please use the answer booklet provided.

Question 1

a) Functions f and g are defined as $f : x \rightarrow x^2 + 1$ and $g :\rightarrow (x+2)^2$, determine;

i) $(f+g)(2)$.

[2 marks]

ii) $(g-f)(1)$.

[1 mark]

b) Determine the limit of $\lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h}$.

[4 marks]

c) Given that $W_1 = -1 + \sqrt{3}j$ and $W_2 = 2 + 2j$

(i) sketch W_1 and W_2 on the Argand diagram,

(ii) determine the **modulus** and **argument** of W_1, and

(iii) express W_1 in trigonometric form.

[8 marks]

Question 2

a) The first derivative of $f(x) = \frac{2+x^3}{1+x^2}$ is given by $f'(x) = \frac{(a+b)x^4 + cx^2 + bx}{(1+x^2)^2}$, determine the values of a, b and c.

[9 marks]

b) Determine the derivative of $y = \frac{e^{2x}\sqrt{x+3}}{(2x+5)^4}$ by using logarithmic differentiation.

[6 marks]

Question 3

a) Given $\int_{-3}^{4} f(x)\,dx = 4$, determine;

(i) $\int_{-3}^{4} 2f(x)\,dx$

[1 mark]

(ii) $\int_{-3}^{4} [f(x) - 3]\,dx$

[3 marks]

b) Determine $\int \cos x \sin^2 x\,dx$ by using substitution method.

[4 marks]

c) Given the area of the region enclosed by the curves $y = x^2$ and $y = 4x$ shown in **FIGURE 3** below.

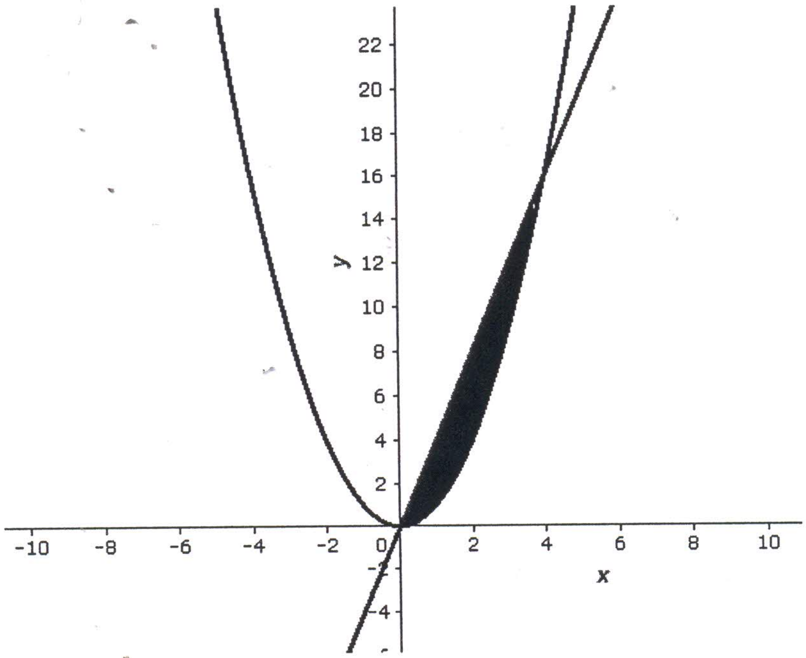

FIGURE 3

(i) Based on **FIGURE 3**, state the intersection point of the curves.

[1 mark]

(ii) Evaluate the volume of the solid generated if the shaded region is rotated 360^0 about the x-axis.

[6 marks]

END OF QUESTION

FORMULA SHEET

TRIGONOMETRY IDENTITIES

FUNDAMENTAL IDENTITIES	FORMULAS FOR NEGATIVES
$\csc\theta = \frac{1}{\sin\theta}$	$\sin(-\theta) = -\sin\theta$
$\sec\theta = \frac{1}{\cos\theta}$	$\cos(-\theta) = \cos\theta$
$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$	$\tan(-\theta) = -\tan\theta$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\csc(-\theta) = -\csc\theta$
$\sin^2\theta + \cos^2\theta = 1$	$\sec(-\theta) = \sec\theta$
$1 + \tan^2\theta = \sec^2\theta$	$\cot(-\theta) = -\cot\theta$
$1 + \cot^2\theta = \csc^2\theta$	

ADDITION FORMULAS	SUBTRACTION FORMULAS
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$	$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

HALF-ANGLE FORMULAS	DOUBLE-ANGLE FORMULAS
$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$	$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $.......... = 1 - 2\sin^2\theta$ $.......... = 2\cos^2\theta - 1$
$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$	$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

PRODUCT-TO-SUM FORMULAS	SUM-TO-PRODUCT FORMULAS
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$	$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$	$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$	$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$	$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

DIFFERENTIATION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc x) = -\csc x\cot x$	$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec x) = \sec x\tan x$	$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$

INTEGRATION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \cos x\, dx = \sin x + c$	$\int \cos f(x) dx = \dfrac{\sin f(x)}{f'(x)} + c$
$\int \sin x\, dx = -\cos x + c$	$\int \sin f(x) dx = \dfrac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x\, dx = \tan x + c$	$\int \sec^2 f(x) dx = \dfrac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x\, dx = \sec x + c$	$\int \sec f(x) \tan f(x) dx = \dfrac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x\, dx = -\csc x + c$	$\int \csc f(x) \cot f(x) dx = \dfrac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x\, dx = -\cot x + c$	$\int \csc^2 f(x) dx = \dfrac{-\cot f(x)}{f'(x)} + c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int e^x dx = e^x + c$	$\int e^{f(x)} dx = \dfrac{e^{f(x)}}{f'(x)} + c$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$	$\int \dfrac{1}{f(x)} dx = \dfrac{\ln\lvert f(x)\rvert}{f'(x)} + c$