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CONFIDENTIAL

SET B

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2010 SESSION

SUBJECT CODE	:	FKB 24302
SUBJECT TITLE	:	ENGINEERING MATHEMATICS
LEVEL	:	BACHELOR
TIME / DURATION	:	8.00pm – 10.00pm (2 HOURS)
DATE	:	03 MAY 2010

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of FIVE (5) questions. Answer any FOUR (4) questions only.
6. Answer ALL questions in English.
7. *Formula is appended.*

THERE ARE 9 PAGES OF QUESTIONS AND 1 PAGE OF APPENDIX, EXCLUDING THIS PAGE.

(Total : 60 marks)

INSTRUCTION : Answer FOUR (4) questions only.
Please use the answer booklet provided.

Question 1

(a) In the transformation of two dimensional space, find the matrix operator for a reflection on the line $3y = -4x.$

(5 marks)

(b) The transformation **V** maps the point $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ to the point $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$ where

$$\begin{aligned} p &= x + 4y + 5z \\ q &= -4x \quad + 2z \\ r &= \quad\quad 3z \end{aligned}$$

(i) State **V** ,the matrix operator of this transformation.

(2 marks)

The transformation **W** has matrix operator $W = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 1 \\ -3 & -1 & 1 \end{pmatrix}.$

Given that U is the compound transformation consisting of W followed by V.

(ii) Find the matrix operator , U for this compound transformation.

(5 marks)

(iii) Determine the image of point **M** under the transformation U when

$\text{M} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}.$

(3 marks)

Question 2

(a) Given $T = 10p^3 - \frac{3}{p}$, find $\frac{dT}{dq}$ when $p = 1$ given that for that value of p, $\frac{dp}{dq} = -4$

(4 marks)

(b) Determine the rectangular equation of the following parametric form:

$$x = 2t - 3 \quad , \quad y = \frac{1}{t^2 + 1}$$

(4 marks)

(c) The line segment of the parametric equation $x = \sin^2 t$ and $y = \cos^2 t$ over the interval $0 \le t \le \frac{\pi}{2}$ is revolved about the x-axis to generate a cone. Calculate the surface area of the cone.

(7 marks)

Formulas:

$$A = \int_a^b \frac{1}{2} r^2 d\theta$$

$$A = 2\pi \int_A^B x \left\{ \left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right\}^{1/2} dt$$

$$A = 2\pi \int_A^B y \left\{ \left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right\}^{1/2} dt$$

Question 3

(a) Sketch, on the same diagram from **APPENDIX 5**, the polar curves

$$\text{Circle}: r = 1$$

$$\text{Cardioid}: r = 1 + \cos\theta$$

(5 marks)

(b) Determine the points of intersection of the circle and the cardioid.

(3 marks)

(c) Hence, show that the area of the region which lies inside the cardioid and outside the circle is $\left(2+\frac{\pi}{4}\right)$ units^2

(7 marks)

Question 4

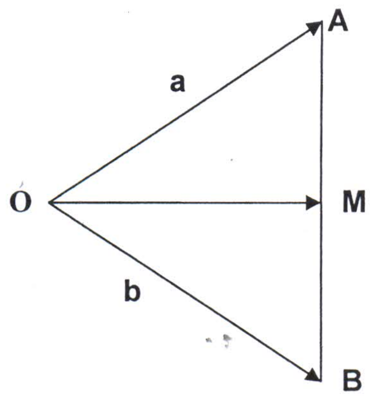

Figure 1

The diagram in **Figure 1** shows a triangle **OAB** with **M** is the midpoint of the line **AB**.

(a) Determine $\overrightarrow{OM}$ in terms of **a** and **b**.

(3 marks)

(b) If $\overrightarrow{OA} = i - j + 2k$ and $\overrightarrow{OB} = -i + 2j + 2k$, determine $\overrightarrow{AM}$ in Cartesian form.

(3 marks)

(c) Hence calculate the length of $\overrightarrow{AM}$.

(2 marks)

(d) Determine the unit vector that has the same direction with $\overrightarrow{AB}$.

(2 marks)

(e) Calculate the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ correct to **two decimal places.**

(5 marks)

Question 5

(a) Given $\overrightarrow{OA} = 3i + 4j$ and $\overrightarrow{OB} = 2hi - hj$.

(i) Determine $\overrightarrow{AB}$ in terms of **h**.

(1 mark)

(ii) If $\overrightarrow{AB} = 9i + kj$, calculate the value of **h** and **k**.

(2 marks)

(b) Given that $\overrightarrow{OA} = 2i + j - 3k$, $\overrightarrow{OB} = i - 2j + k$ and $\overrightarrow{OC} = -i + j - 4k$.
Determine the following:

(i) $\overrightarrow{OA} \cdot \left(\overrightarrow{OB} \times \overrightarrow{OC} \right)$

(2 marks)

(ii) $\overrightarrow{OB} \times \frac{1}{2} \overrightarrow{OA}$

(2 marks)

(iii) $\left| \overrightarrow{OB} \times \frac{1}{2} \overrightarrow{OA} \right|$

(2 marks)

(c) A tetrahedron has vertices at and A (2, 1, 1) , B (1, -1, 2) , C (0, 1, -1) and D (1, -2, 1) . Prove that the volume of the tetrahedron is $\frac{4}{3}$ unit^3 .

(6 marks)

END OF QUESTION

APPENDIX 1

STANDARD TRANSFORMATIONS

(1)	Rotation through an angle , θ , about the origin	$M = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$
(2)	Rotation through $\frac{\pi}{2}$ ***clockwise*** about the origin	$M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
(3)	Rotation through $\frac{\pi}{2}$ ***anti -clockwise*** about the origin	$M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
(4)	Reflection in the **x-axis**	$M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
(5)	Reflection in the **y-axis**	$M = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
(6)	Reflection in the **line** : $y = x$ or $y - x = 0$	$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
(7)	Reflection in the **line** : $y = -x$ or $y + x = 0$	$M = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
(8)	Reflection in the line : $y = mx$ or $y = x\tan\theta$ NOTE : $m = \tan\theta$	$M = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ where :

		$\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-m^2}{1+m^2}$ $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta} = \dfrac{2m}{1+m^2}$
(9)	Shear of θ in the direction O_x	$M = \begin{pmatrix} 1 & \tan\theta \\ 0 & 1 \end{pmatrix}$
(10)	Shear of θ in the direction Oy	$M = \begin{pmatrix} 1 & 0 \\ \tan\theta & 1 \end{pmatrix}$

APPENDIX 2

TRIGONOMETRIC IDENTITIES AND FORMULAS

FUNDAMENTAL IDENTITIES	FORMULAS FOR NEGATIVES
$\csc\theta = \frac{1}{\sin\theta}$	$\sin(-\theta) = -\sin\theta$
$\sec\theta = \frac{1}{\cos\theta}$	$\cos(-\theta) = \cos\theta$
$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$	$\tan(-\theta) = -\tan\theta$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\csc(-\theta) = -\csc\theta$
$\sin^2\theta + \cos^2\theta = 1$	$\sec(-\theta) = \sec\theta$
$1 + \tan^2\theta = \sec^2\theta$	$\cot(-\theta) = -\cot\theta$
$1 + \cot^2\theta = \csc^2\theta$	

ADDITION FORMULAS	SUBTRACTION FORMULAS
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$	$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

HALF-ANGLE FORMULAS	DOUBLE-ANGLE FORMULAS
$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$	$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\ldots\ldots\ldots\ldots = 1 - 2\sin^2\theta$ $\ldots\ldots\ldots\ldots = 2\cos^2\theta - 1$
$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$	$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

PRODUCT-TO-SUM FORMULAS	SUM-TO-PRODUCT FORMULAS
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$	$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$	$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$	$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$	$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

APPENDIX 3

DIFFERENTIATION

TRIGONOMETRIC FUNCTIONS

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin f(x)) = \cos f(x) f'(x)$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos f(x)) = -\sin f(x) f'(x)$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan f(x)) = \sec^2 f(x). f'(x)$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\csc f(x)) = -\csc f(x) \cot f(x). f'(x)$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\sec f(x)) = \sec f(x) \tan f(x). f'(x)$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot f(x)) = -\csc^2 f(x). f'(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx} e^x = e^x$	$\frac{d}{dx} e^{f(x)} = e^{f(x)}. f'(x)$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx} \ln x = \frac{1}{x}$	$\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}$

APPENDIX 4

INTEGRATION

TRIGONOMETRIC FUNCTIONS

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int \cos x dx = \sin x + c$	$\int \cos f(x) dx = \frac{\sin f(x)}{f'(x)} + c$
$\int \sin x dx = -\cos x + c$	$\int \sin f(x) dx = \frac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x dx = \tan x + c$	$\int \sec^2 f(x) dx = \frac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x dx = \sec x + c$	$\int \sec f(x) \tan f(x) dx = \frac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x dx = -\csc x + c$	$\int \csc f(x) \cot f(x) dx = \frac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x dx = -\cot x + c$	$\int \csc^2 f(x) dx = \frac{-\cot f(x)}{f'(x)} + c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int e^x dx = e^x + c$	$\int e^{f(x)} dx = \frac{e^{f(x)}}{f'(x)} + c$

FORM OF 1/x

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int \frac{1}{x} dx = \ln\|x\| + c$	$\int \frac{1}{f(x)} dx = \frac{\ln\|f(x)\|}{f'(x)} + c$

APPENDIX 5

NAME : ....

ID NUMBER : ....