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SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JULY 2010 SESSION

SUBJECT CODE	:	FKB 16103
SUBJECT TITLE	:	ENGINEERING MATHEMATICS 2
LEVEL	:	BACHELOR
TIME / DURATION	:	12.30pm – 2.30pm (2 HOURS)
DATE	:	10 NOVEMBER 2010

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of FIVE (5) questions. Answer FOUR (4) questions only.
6. Answer all questions in English.

THERE ARE 5 PAGES OF QUESTIONS AND 5 PAGES OF APPENDIX , EXCLUDING THIS PAGE.

(Total: 80 marks)

INSTRUCTION: Answer only FOUR questions.
Please use the answer booklet provided.

Question 1

(a) Given $f(x,y) = xy.\ln(x^2 + xy)$. Determine the **second order partial derivative** $\dfrac{\partial^2 f}{\partial x \partial y}$ at the point (1,1).

(10 marks)

(b) A planned community with a sports complex at the center is created from a square tract of land measuring 6,000 feet on a side. Land values are modeled by the formula : $P(x,y) = 5 - (0.0000002)(1.1x^2 + 1.2y^2)$ where $P(x,y)$ is the price of land (in Ringgit Malaysia /sq. ft) at the point (x,y), measured from the center of the community. The closer land is to the sport complex, the more expensive a plot is. Given the average price of land is given by :

$$\frac{1}{36{,}000{,}000}\int_{-3000}^{3000}\int_{-3000}^{3000} P(x,y)\;dydx$$

Determine the **average price** of the land in this community.

(10 marks)

Question 2

(a) The variation of resistance ***R*** ohm of a copper conductor with temperature, $\theta^\circ C$, is given by

$$\frac{dR}{d\theta} = \alpha R$$

where α is the temperature coefficient of resistance of copper.

(i) If $R = R_0$ at $\theta = 0^\circ C$, solve the equation for R ***by using method of differential equation.***

(6 marks)

(ii) Taking α as 3.9×10^{-4} per $^\circ C$, find the resistance of a copper conductor at $20^\circ C$, correct to four significant digits, when its resistance at $80^\circ C$ is 57.4Ω.

(4 marks)

(b) A body moves in a straight line so that its distance ***s*** meters from the origin after time ***t*** seconds is given by :

$$\frac{d^2 s}{dt^2} + a^2 s = 0$$

where ***a*** is a constant. ***By using method of differential equation,*** solve the equation for ***s*** given that $s = k$ and $\frac{ds}{dt} = 0$ when $t = \frac{2\pi}{a}$.

(10 marks)

Question 3

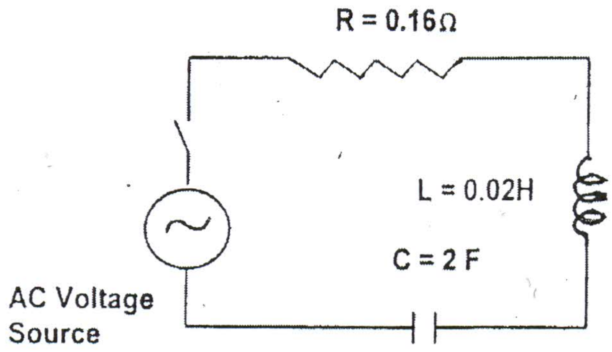


Figure 1

Refer to **Figure 1** above, consider the circuit with a resistor *R* ohms, an inductor *L* henrys and a capacitor *C* farads in series with an AC voltage source of $E_{rms} = 12V$. By using **Laplace transform method**, find the charge *Q* as function of time *t* in an RCL circuit measured in coulombs which satisfies

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E_{rms}$$

if $R = 0.16\Omega$, $L = 0.02\,Henry$, $C = 2\,Farads$, $E_{rms} = 12\ Volts$ with the initial condition $Q(t) = 0$ and $Q'(t) = 0$ at $t = 0$ second (when the switch is closed).

(20 marks)

Question 4

The periodic function $f(t)$ is defined as

$$f(t)=\begin{cases} 24, & -\pi < t < -\dfrac{\pi}{2} \\ -24, & -\dfrac{\pi}{2} < t < 0 \\ 24, & 0 < t < \dfrac{\pi}{2} \\ -24, & \dfrac{\pi}{2} < t < \pi \end{cases}$$

(a) Sketch $f(t)$ in the range $-\dfrac{3\pi}{2} \le t \le \dfrac{3\pi}{2}$.

(2 marks)

(b) Determine whether the function $f(t)$ is even function, odd function or neither odd nor even function. State your reasoning.

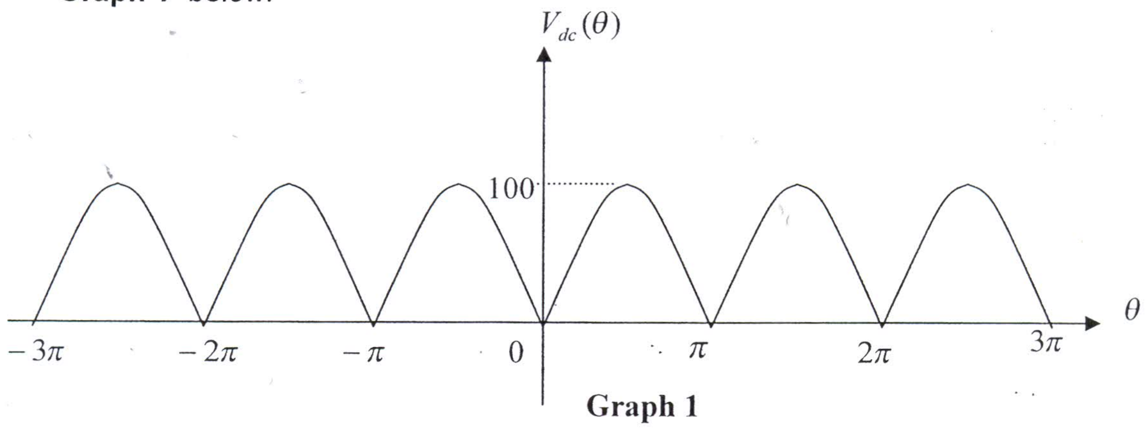
(2 marks)

(c) Obtain the Fourier series of the function $f(t)$.

(16 marks)

Question 5

(a) The output voltage given by a single-phase, full wave bridge rectifier is shown in **Graph 1** below.



Graph 1

Given that : $V_{dc}(\theta) = \begin{cases} -100\sin\theta, & when \ -\pi < \theta < 0 \\ 100\sin\theta, & when \ 0 < \theta < \pi \end{cases}$

(i) Determine the period of the function $V_{dc}(\theta)$.

(1 marks)

(ii) Determine the Fourier coefficients of a_n of $V_{dc}(\theta)$.

(16 marks)

Given formula :

$$\int(\sin a\theta \sin b\theta)\, d\theta = \frac{\sin(a-b)\theta}{2(a-b)} - \frac{\sin(a+b)\theta}{2(a+b)} + C, \qquad a \neq b$$

$$\int(\cos a\theta \cos b\theta)\, d\theta = \frac{\sin(a-b)\theta}{2(a-b)} + \frac{\sin(a+b)\theta}{2(a+b)} + C, \qquad a \neq b$$

$$\int(\sin a\theta \cos b\theta)\, d\theta = \frac{-\cos(a-b)\theta}{2(a-b)} - \frac{\cos(a+b)\theta}{2(a+b)} + C, \qquad a \neq b$$

(iii) Given that the Fourier series of $V_{dc}(\theta)$ is given by

$$V_{dc}(\theta) = \frac{200}{\pi} - \frac{400}{\pi}\left[\frac{\cos 2\theta}{3} + \frac{\cos 4\theta}{15} + \frac{\cos 6\theta}{35} + \ldots\ldots\right]$$

Deduce a series for $\frac{1}{2}$ when $\theta = 2\pi$.

(3 marks)

END OF QUESTION

APPENDIX 1

Table of Differentiation

Trigonometric Functions - GENERAL FORM
$\frac{d}{dx}(\sin f(x)) = f'(x) \cos f(x)$
$\frac{d}{dx}(\cos f(x)) = -f'(x) \sin f(x)$
$\frac{d}{dx}(\tan f(x)) = f'(x) \sec^2 f(x)$
$\frac{d}{dx}(\csc f(x)) = -f'(x) \csc f(x) \cot f(x)$
$\frac{d}{dx}(\sec f(x)) = f'(x) \sec f(x) \tan f(x)$
$\frac{d}{dx}(\cot f(x)) = -f'(x) \csc^2 f(x)$

Exponential Function - GENERAL FORM
$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)\, e^{f(x)}$

Logarithmic Function - GENERAL FORM
$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)} f'(x)$

APPENDIX 2

Table of Integration

Trigonometric Functions - GENERALFORM Where : $f(x)=ax+b$
$\int \cos f(x)dx = \frac{\sin f(x)}{f'(x)} + C$
$\int \sin f(x)dx = \frac{-\cos f(x)}{f'(x)} + C$
$\int \sec^2 f(x)dx = \frac{\tan f(x)}{f'(x)} + C$
$\int \sec f(x)\tan f(x)dx = \frac{\sec f(x)}{f'(x)} + C$
$\int \csc f(x)\cot f(x)dx = \frac{-\csc f(x)}{f'(x)} + C$
$\int \csc^2 f(x)dx = \frac{-\cot f(x)}{f'(x)} + C$

Exponential Function - GENERALFORM Where : $f(x)=ax+b$
$\int e^{f(x)} dx = \frac{e^{f(x)}}{f'(x)} + C$

Logarithmic Function - GENERALFORM Where : $f(x)=ax+b$
$\int \frac{1}{f(x)} dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + C$

APPENDIX 3

Trigonometric Identities and Formulas

FUNDAMENTAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

FORMULAS FOR NEGATIVES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

ADDITION FORMULAS

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

SUBTRACTION FORMULAS

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

HALF-ANGLE FORMULAS

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

DOUBLE-ANGLE FORMULAS

$$\sin 2\theta = 2\sin\theta\sin\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\ldots\ldots\ldots = 1 - 2\sin^2\theta$$

$$\ldots\ldots\ldots = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

PRODUCT-TO-SUM FORMULAS

$$\sin\alpha\text{in}\alpha\text{c} = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos\alpha\text{os}\alpha\text{s} = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

$$\cos\alpha\text{os}\alpha\text{c} = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$\sin\alpha\text{in}\alpha\text{s} = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

SUM-TO-PRODUCT FORMULAS

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

APPENDIX 4

Fundamental Hyperbolic Identities	
$\cosh x + \sinh x = e^{x}$ $\cosh x - \sinh x = e^{-x}$	$\sinh(-x) = -\sinh x$ $\cosh(-x) = \cosh x$
$\text{csch}\, x = \dfrac{1}{\sinh x}$ $\text{sech}\, x = \dfrac{1}{\cosh x}$	$\tanh x = \dfrac{\sinh x}{\cosh x}$ $\coth x = \dfrac{1}{\tanh x} = \dfrac{\cosh x}{\sinh x}$
$\cosh^2 x - \sinh^2 x = 1$ $1 - \tanh^2 x = \text{sech}^2 x$ $\coth^2 x - \text{csch}^2 x = 1$	$\sinh 2x = 2\sinh x \cosh x$ $\cosh 2x = \cosh^2 x + \sinh^2 x$ $\cosh 2x = 2\sinh^2 x + 1$ $\cosh 2x = 2\cosh^2 x - 1$ $\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$

$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$
$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$
$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$
$\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$
$\tanh(x + y) = \dfrac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$

APPENDIX 5

TABLE OF LAPLACE TRANSORM

$f(t)$	$F(s)$
1	$\frac{1}{s}$
k	$\frac{k}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
t	$\frac{1}{s^2}$
t^2	$\frac{2!}{s^3}$
t^k	$\frac{k!}{s^{k+1}}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}t^k$	$\frac{k!}{(s-a)^{k+1}}$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
$e^{at}\sinh bt$	$\frac{b}{(s-a)^2-b^2}$
$e^{at}\cosh bt$	$\frac{s-a}{(s-a)^2-b^2}$
$t\sin at$	$\frac{2as}{(s^2+a^2)^2}$
$t\cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$