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CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2011 SESSION

SUBJECT CODE	:	FKB 24302
SUBJECT TITLE	:	ENGINEERING MATHEMATICS 3
LEVEL	:	BACHELOR
TIME / DURATION	:	9.00am – 11.00am (2 HOURS)
DATE	:	04 MAY 2011

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers in the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of FIVE (5) questions. You are required to answer FOUR (4) questions only. Answer all questions in English.
6. Fomula is appended.

THERE ARE 12 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

(Total : 75 marks)

INSTRUCTION : Answer FOUR (4) questions only.

Please use the answer booklet provided.

Question 1

(a) Given $\begin{pmatrix} 2x & 3 \end{pmatrix}\begin{pmatrix} 11 \\ -6x \end{pmatrix} = \begin{pmatrix} 100 \end{pmatrix}$. Calculate the value of x.

(1.5 marks)

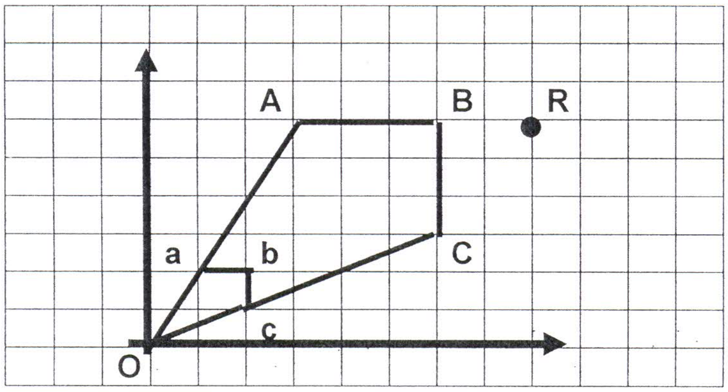
(b) Determine $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$.

(1.5 marks)

(c) In the transformation of two dimensional space, find the matrix operator for a reflection on the line $2y = -3x$.

(3 marks)

(d)



(i) Given the diagram above, figure **OABC** is the image of figure **Oabc**. Describe what has happened to figure **Oabc**.

(ii) What type of transformation involved in this question and find the matrix operator.

(iii) Find the image of point R under the same transformation.

(3 marks)

(e) Given that matrix $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. By using the inverse matrix method, determine the image of the line $x - 2y + z = 1$

(6 marks)

Question 2

(a) Obtain the Cartesian equation of the curve by eliminating the parameter from the parametric equations:

$$x = 5t - 10,\ y = 2t$$

(2 marks)

(b) Convert $x^2 + y^2 + x = \sqrt{x^2 + y^2}$ into Polar Form.

(4 marks)

(c) The parametric equations of an asteroid are $x = \cos^3 t,\ y = \sin^3 t$.

Calculate the arc length of the asteroid on the interval $0 \le t \le \frac{\pi}{2}$.

(5 marks)

(d) Calculate the surface area obtained by revolving the graph of $y = \sqrt{x}$ between $x = 0$ and $x = 1$ about the x-axis.

(4 marks)

Choose the correct formulas:

$A = 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$	$S = \int_A^B \left\{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right\}^{1/2} dt$
$A = 2\pi \int_a^b x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$	$S = \int_A^B \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{1/2} dx$
$A = 2\pi \int_a^b y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$	$A = 2\pi \int_a^b x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Question 3

(a) Given the curve of the polar equation $r = 4\sin 3\theta$. Complete the tables for the interval $0 \le \theta \le 2\pi$ and sketch the graph on the interval on the polar graph in **APPENDIX 5**

(5 marks)

(b) Obtain the polar equation for $y = 3x + 5$.

(5 marks)

(c) A cycloid is the curve traced out by a point on the circumference of a circle when the circle rolls along a straight line in its own plane. The equations of a cycloid is given by

$$x = t - \sin t, \quad y = 1 - \cos t.$$

Calculate the arc length, S of the cycloid over the interval $0 \le t \le \frac{\pi}{2}$.

(5 marks)

Question 4

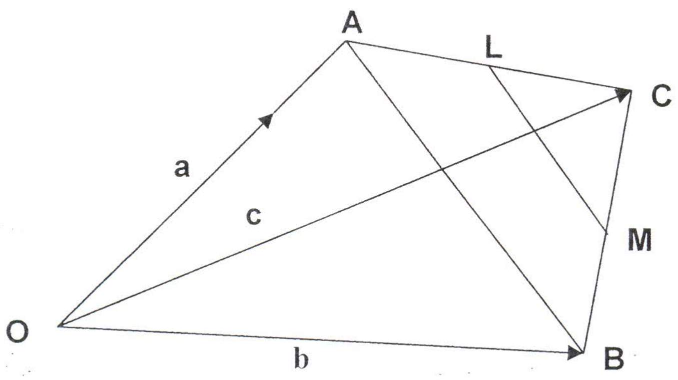
(a) Determine vector **V** if **V** is parallel to the vector $8\mathrm{i}+\mathrm{j}+4\mathrm{k}$ and is equal in magnitude to the vector $\mathrm{i}-2\mathrm{j}+2\mathrm{k}$.

(4 marks)

(b) Determine the coordinates of vector Q if $\left|\overrightarrow{OQ}\right|=1$ and $\overrightarrow{OQ}$ is in the direction of $3\mathrm{i}+2\mathrm{j}+6\mathrm{k}$.

(3 marks)

(c) Refer the following diagram:



A, B and C are the points whose position vectors are $2\mathrm{i}-\mathrm{j}+5\mathrm{k}$, $\mathrm{i}-2\mathrm{j}+\mathrm{k}$ and $3\mathrm{i}+\mathrm{j}-2\mathrm{k}$ respectively. Given that **L** and **M** are the midpoints of **AC** and **CB**.

i) Determine **OL** and **OM**.

(6 marks)

ii) Hence, show that **LM** is parallel to **AB**.

(2 marks)

Question 5

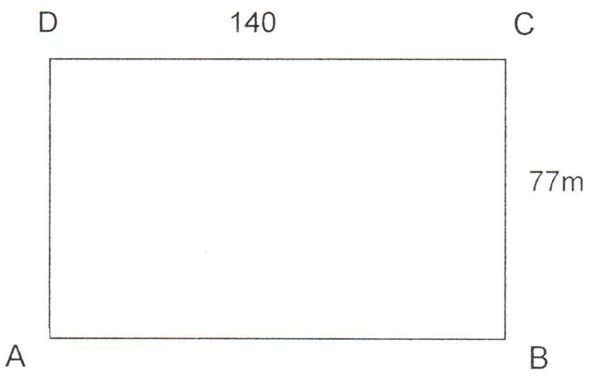
(a) Given $\underset{\sim}{u} = 4i - hj$ and $\underset{\sim}{v} = 6i + 2j$. If $\underset{\sim}{u} = k\underset{\sim}{v}$, find the value of h and k.

(4 marks)

(b) The diagram 1 below shows a rectangular field with dimensions 140m by 77m. A boy pulls a truck directly from A to C with a force of 101.5N. Find the components of this force parallel and perpendicular to $\overrightarrow{AB}$.

(5 marks)

Diagram 1



(c) $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are two vectors such that $\overrightarrow{OA}$ = **a** + 2**b**, $\overrightarrow{OB}$ = 2**a** – **b** and $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$.

Show that **a** • **b** = $\frac{2}{3}(b^2 - a^2)$.

(6 marks)

END OF QUESTION

APPENDIX 1

STANDARD TRANSFORMATIONS

(1)	Rotation through an angle , θ , about the origin	$M = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$
(2)	Rotation through $\frac{\pi}{2}$ ***clockwise*** about the origin	$M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
(3)	Rotation through $\frac{\pi}{2}$ ***anti -clockwise*** about the origin	$M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
(4)	Reflection in the **x-axis**	$M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
(5)	Reflection in the **y-axis**	$M = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
(6)	Reflection in the **line** : $y = x$ or $y - x = 0$	$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
(7)	Reflection in the **line** : $y = -x$ or $y + x = 0$	$M = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
(8)	Reflection in the line : $y = mx$ or $y = x\tan\theta$	$M = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

	NOTE : $m = \tan\theta$	where : $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-m^2}{1+m^2}$ $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta} = \dfrac{2m}{1+m^2}$
(9)	Shear of θ in the direction O_x	$M = \begin{pmatrix} 1 & \tan\theta \\ 0 & 1 \end{pmatrix}$
(10)	Shear of θ in the direction Oy	$M = \begin{pmatrix} 1 & 0 \\ \tan\theta & 1 \end{pmatrix}$

APPENDIX 2

TRIGONOMETRIC IDENTITIES AND FORMULAS

FUNDAMENTAL IDENTITIES		FORMULAS FOR NEGATIVES
$\csc\theta = \dfrac{1}{\sin\theta}$		$\sin(-\theta) = -\sin\theta$
$\sec\theta = \dfrac{1}{\cos\theta}$		$\cos(-\theta) = \cos\theta$
$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$		$\tan(-\theta) = -\tan\theta$
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$		$\csc(-\theta) = -\csc\theta$
$\sin^2\theta + \cos^2\theta = 1$		$\sec(-\theta) = \sec\theta$
$1 + \tan^2\theta = \sec^2\theta$		$\cot(-\theta) = -\cot\theta$
$1 + \cot^2\theta = \csc^2\theta$		
ADDITION FORMULAS		**SUBTRACTION FORMULAS**
$\sin(A+B) = \sin A\cos B + \cos A\sin B$		$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$		$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$		$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$
HALF-ANGLE FORMULAS		**DOUBLE-ANGLE FORMULAS**
$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$		$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$		$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $.........= 1 - 2\sin^2\theta$ $.........= 2\cos^2\theta - 1$
$\tan\dfrac{\theta}{2} = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$		$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
PRODUCT-TO-SUM FORMULAS		**SUM-TO-PRODUCT FORMULAS**
$\sin\alpha\cos\beta = \dfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$		$\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$
$\cos\alpha\sin\beta = \dfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$		$\sin\alpha - \sin\beta = 2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$		$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$		$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

APPENDIX 3

DIFFERENTIATION

TRIGONOMETRIC FUNCTIONS

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin f(x)) = \cos f(x) f'(x)$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos f(x)) = -\sin f(x) f'(x)$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan f(x)) = \sec^2 f(x). f'(x)$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\csc f(x)) = -\csc f(x) \cot f(x). f'(x)$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\sec f(x)) = \sec f(x) \tan f(x). f'(x)$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot f(x)) = -\csc^2 f(x). f'(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}e^{x} = e^{x}$	$\frac{d}{dx}e^{f(x)} = e^{f(x)}.f'(x)$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM

$\dfrac{d}{dx}\ln x = \dfrac{1}{x}$	$\dfrac{d}{dx}\ln f(x) = \dfrac{f'(x)}{f(x)}$

APPENDIX 4

INTEGRATION

TRIGONOMETRIC FUNCTIONS

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \cos x\,dx = \sin x + c$	$\int \cos f(x)dx = \dfrac{\sin f(x)}{f'(x)} + c$
$\int \sin x\,dx = -\cos x + c$	$\int \sin f(x)dx = \dfrac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x\,dx = \tan x + c$	$\int \sec^2 f(x)dx = \dfrac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x\,dx = \sec x + c$	$\int \sec f(x) \tan f(x)dx = \dfrac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x\,dx = -\csc x + c$	$\int \csc f(x) \cot f(x)dx = \dfrac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x\,dx = -\cot x + c$	$\int \csc^2 f(x)dx = \dfrac{-\cot f(x)}{f'(x)} + c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int e^x dx = e^x + c$	$\int e^{f(x)}dx = \dfrac{e^{f(x)}}{f'(x)} + c$

FORM OF 1/x

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$	$\int \frac{1}{f(x)}\,dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + c$

APPENDIX 5

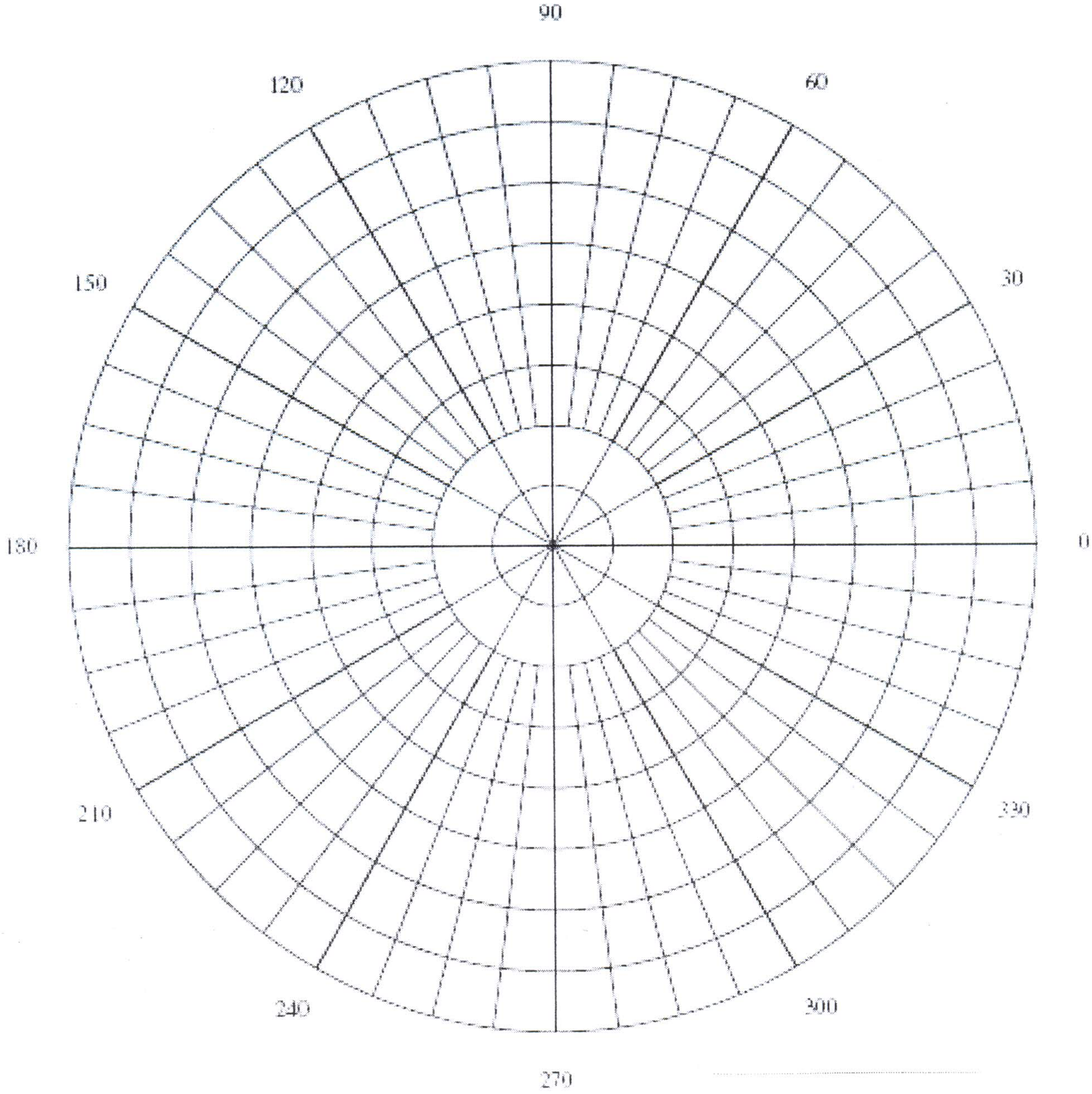
NAME : ....

ID NUMBER : ....

Question 3(a)

$0 \le r \le 2\pi$

θ								
$r = 4\sin 3\theta$								
θ								
$r = 4\sin 3\theta$								


90
120
60
150
30
180
0
210
330
240
300
270