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CONFIDENTIAL

SET B

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2011 SESSION

SUBJECT CODE	:	FKB 23302
SUBJECT TITLE	:	ENGINEERING MATHEMATICS 3
LEVEL	:	BACHELOR
TIME / DURATION	:	12.30 pm – 2.30 pm (2 HOURS)
DATE	:	04 MAY 2011

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of FIVE (5) questions. You are required to answer FOUR (4) questions only. Answer all questions in English.
6. Fomula is appended.

THERE ARE 2 PAGES OF QUESTIONS, AND 5 PAGES OF APPENDICES EXCLUDING THIS PAGE.

(Total : 60 marks)

INSTRUCTION : Answer FOUR (4) questions only.
Please use the answer booklet provided.

Question 1

(a) Determine $\sum_{n=1}^{60}(3r-4)^3$ by using the standard result for natural number series.

(7 marks)

(b) Express the recurring decimal $0.\dot{0}\dot{7}\dot{2}$ as a fraction in its lowest term.

(5 marks)

(c) Use the table of Maclaurin series to determine the expansion of $\sin 2x$ up to and including the term in x^5.

(3 marks)

Question 2

Determine the particular solution to the differential equation

$$(x^2+1)y''+(x^2-1)y=0 \qquad \textit{given that } y(0)=1\ ,\ \ y'(0)=0$$

(15 marks)

Question 3

Determine the centre, radius and interval of convergence for

$$\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{2}{3}\right)^{2n}(x-1)^n$$

Also, determine the behavior of the series at the endpoints of the interval.

(15 marks)

Question 4

Use the table of Z-transform to determine

(a) $Z\left\{4^n\left(2+\sin\dfrac{n\pi}{2}\right)^2\right\}$.

(7 marks)

(b) $Z\{\sin 3t\cos 5t+\cos^2 3t\}$.

(6 marks)

(c) $Z\{u(n-5)+2n^2\}$

(2 marks)

Question 5

Solve the given difference equation using Z-Transform

$$y_{n+2}-4y_{n+1}+4y_n=3 \quad \text{if} \quad y_0=1\,,\ \ y_1=0$$

(15 marks)

END OF QUESTION

APPENDIX 1

MACLAURIN SERIES FOR COMMON FUNCTIONS

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots \text{ for } -1 < x < 1$$

$$\text{cn}(x, k) = 1 - \frac{1}{2}x^2 + \frac{1}{24}(1 + 4k^2)x^4 + \ldots$$

$$\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \ldots \text{ for } -\infty < x < \infty$$

$$\cos^{-1} x = \frac{1}{2}\pi - x - \frac{1}{6}x^3 - \frac{3}{40}x^5 - \frac{5}{112}x^7 - \ldots \text{ for } -1 < x < 1$$

$$\cosh x = 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{720}x^6 + \frac{1}{40{,}320}x^8 + \ldots$$

$$\cot^{-1} x = \frac{1}{2}\pi - x + \frac{1}{3}x^3 - \frac{1}{5}x^5 + \frac{1}{7}x^7 - \frac{1}{9}x^9 + \ldots$$

$$\text{dn}(x, k) = 1 - \frac{1}{2}k^2x^2 + \frac{1}{24}k^2(4 + k^2)x^4 + \ldots$$

$$\text{erf}(x) = \frac{1}{\sqrt{\pi}}\left(2x - \frac{2}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{21}x^7 + \ldots\right)$$

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \ldots \text{ for } -\infty < x < \infty$$

$${}_2F_1(\alpha, \beta; \gamma; x) = 1 + \frac{\alpha\beta}{1!\,\gamma}x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{2!\,\gamma(\gamma+1)}x^2 + \ldots$$

$$\ln(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots \text{ for } -1 < x < 1$$

$$\ln\left(\frac{1+x}{1-x}\right) = 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + \frac{2}{7}x^7 + \ldots \text{ for } -1 < x < 1$$

$$\sec x = 1 + \frac{1}{2}x^2 + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \ldots$$

$$\text{sech}\, x = 1 - \frac{1}{2}x^2 + \frac{5}{24}x^4 - \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \ldots$$

$$\sin x = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \ldots \text{ for } -\infty < x < \infty$$

$$\sin^{-1} x = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \frac{35}{1152}x^9 + \ldots$$

$$\sinh x = x + \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{5040}x^7 + \frac{1}{362880}x^9 + \ldots$$

$$\sinh^{-1} x = x - \frac{1}{6}x^3 + \frac{3}{40}x^5 - \frac{5}{112}x^7 + \frac{35}{1152}x^9 - \ldots$$

$$\text{sn}(x, k) = x - \frac{1}{6}(1 + k^2)x^3 + \frac{1}{120}(1 + 14k^2 + k^4)x^5 + \ldots$$

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \ldots$$

$$\tan^{-1} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \ldots \text{ for } -1 < x < 1$$

$$\tanh x = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \ldots$$

$$\tanh^{-1} x = x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \frac{1}{9}x^9 + \ldots$$

APPENDIX 2

TRIGONOMETRIC IDENTITIES AND FORMULAS

FUNDAMENTAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

FORMULAS FOR NEGATIVES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

ADDITION FORMULAS

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

SUBTRACTION FORMULAS

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A - B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

HALF-ANGLE FORMULAS

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1 - \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 + \cos\theta}$$

DOUBLE-ANGLE FORMULAS

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$.......... = 1 - 2\sin^2\theta$$

$$.......... = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

PRODUCT-TO-SUM FORMULAS

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

SUM-TO-PRODUCT FORMULAS

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha + \beta}{2}\sin\frac{\alpha - \beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha + \beta}{2}\sin\frac{\alpha - \beta}{2}$$

APPENDIX 3

TABLE OF Z-TRANSORM

	$X(z)$	**Region of existence**
1	$\frac{z}{z-1}$	All z
$(-1)^n$	$\frac{z}{z+1}$	$\lvert z\rvert > 1$
$a^n u(n)$	$\frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
$u(n-m)$	$z^{-m}.\frac{z}{z-1}$	$\lvert z\rvert > 1$
n	$\frac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
n^2	$\frac{z^2+z}{(z-1)^3}$	$\lvert z\rvert > 1$
$n(n-1)$	$\frac{2z}{(z-1)^2}$	$\lvert z\rvert > 1$
n^k	$\frac{k!z}{(z-1)^{k+1}}$	$\lvert z\rvert > 1$
$u(n-1)$	$\frac{1}{z-1}$	$\lvert z\rvert > 1$
na^n	$\frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
$u(n)\cos n\theta$	$\frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$	$\lvert z\rvert > 1$
$u(n)\sin n\theta$	$\frac{z\sin\theta}{z^2-2z\cos\theta+1}$	$\lvert z\rvert > 1$
$r^n\cos n\theta$	$\frac{z(z-r\cos\theta)}{z^2-2zr\cos\theta+r^2}$	$\lvert z\rvert > \lvert r\rvert$

$r^n \sin n\theta$	$\dfrac{zr\sin\theta}{z^2 - 2zr\cos\theta + r^2}$	$\lvert z\rvert > \lvert r\rvert$
$a^n x(n)$	$X\left(\dfrac{z}{a}\right)$	$\lvert z\rvert > \lvert a\rvert$
$nx(n)$	$\dfrac{1}{z}\dfrac{dX(z)}{dz^{-1}}$	
$x(n)$ or $f(t)$	$X(z)$ or $F(z)$	
$\dfrac{1}{n}$	$\log\left(\dfrac{z}{z-1}\right)$	$\lvert z\rvert > 1$
$\delta(n)$	1	
$\delta(n-k)$	$\dfrac{1}{z^k}$	
$a^n u(n)$	$\dfrac{z}{z-a}$	
$a^n \cos n\theta \,.\, u(n)$	$\dfrac{z(z - a\cos\theta)}{z^2 - 2az\cos\theta + a^2}$	
$a^n \sin n\theta \,.\, u(n)$	$\dfrac{az\sin\theta}{z^2 - 2az\cos\theta + a^2}$	
$na^n \,.\, u(n)$	$\dfrac{az}{(z-a)^2}$	
$(n+1)a^n \,.\, u(n)$	$\dfrac{z^2}{(z-a)^2}$	
$n(n-1)a^n \,.\, u(n)$	$\dfrac{2a^2 z}{(z-a)^3}$	
$u(n)$	$\dfrac{z}{z-1}$	$\lvert z\rvert > 1$
$Z(t)$	$\dfrac{Tz}{(z-1)^2}$	
$Z(t^2)$	$\dfrac{T^2 z(z+1)}{(z-1)^3}$	

$Z(t^3)$	$\frac{T^3 z(1+4z-z^2)}{(z-1)^4}$	
$Z(t^k)$	$-Tz\frac{d}{dz}\left[Z(t^{k-1})\right]$	
$a^n \cos\frac{n\pi}{2}$	$\frac{z^2}{z^2+a^2}$	
$a^n \sin\frac{n\pi}{2}$	$\frac{az}{z^2+a^2}$	
$a^n f(t)$	$F\left(\frac{z}{a}\right)$	
$nf(nT)=nf(t)$	$-z\frac{d}{dz}[F(z)]$	
k	$\frac{kz}{z-1}$	$\|z\| > 1$
e^{-at}	$\frac{z}{z-e^{-aT}}$	$\|z\| > \|e^{-aT}\|$
e^{at}	$\frac{z}{z-e^{aT}}$	$\|z\| > \|e^{aT}\|$
$\cos\omega t$	$\frac{z(z-\cos\varpi T)}{z^2-2z\cos\omega T+1}$	$\|z\| > 1$
$\sin\omega t$	$\frac{z\sin\omega T}{z^2-2z\cos\omega T+1}$	$\|z\| > 1$