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SET B

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2011 SESSION

SUBJECT CODE	:	FKB 15103
SUBJECT TITLE	:	ENGINEERING MATHEMATICS 1
LEVEL	:	BACHELOR
TIME / DURATION	:	9.00 am – 12.00 noon (3 HOURS)
DATE	:	04 MAY 2011

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of SIX (6) questions. Answer FIVE (5) questions only.
6. Answer all questions in English.

THERE ARE 5 PAGES OF QUESTIONS AND 3 PAGES OF APPENDIX , EXCLUDING THIS PAGE.

(Total: 100 marks)

INSTRUCTION: Answer only FIVE (5) questions.
Please use the answer booklet provided.

Question 1

(a) Given a polynomial with real coefficient $P(z) = z^3 + 2z^2 - 6z + 8$.

(i) Show that $z - (1 + j)$ is a factor of $P(z)$.

(ii) Hence, factorize $P(z)$ completely in **Complex Domain.**

(6 marks)

(b) Given that *a*, *b* and *c* are real numbers in the polynomial

$$P(z) = 2z^4 + az^3 + bz^2 + cz + 3$$

Determine the value of *a*, *b* and *c* such that the numbers 2 and *j* are the roots of $P(z)$.

(6 marks)

(c) The transform of a signal is given by $F(s) = \dfrac{-6s + 2}{s^2 + 2s + 17}$.

Decompose $F(s)$ completely in the **Complex Domain**.

(8 marks)

Question 2

(a) Minor M_{ij} is defined as the determinant of the matrix that results from removing the ith row and j^{th} column of the matrix A. If matrix $A = \begin{bmatrix} a & -6 & b \\ -1 & 2 & 0 \\ -2 & 0 & 3 \end{bmatrix}$ and the minor $M_{31} = -2$ and $M_{22} = 5$, determine the values of *a* and *b*.

(4 marks)

(b) Table (1) below shows the number of boxes of milk A, B and C which were supplied by a dairy to three houses in a village every week.

	First House	Second House	Third House
Milk A	1	2	3
Milk B	2	4	5
Milk C	3	5	6

Table (1)

The payment collected by the dairy owner from the first, second and third houses are RM 130, RM 235 and RM 295 respectively. If x, y and z are the prizes for each box of milk, determine the values of x, y and z by using the **CRAMER'S RULE**.

(11 marks)

(c) Based on the given augmented matrix $\left[\begin{array}{ccc|c} 1 & -2 & -1 & -1 \\ 0 & 5 & 5 & 15 \\ 0 & 0 & k^2-25 & k-5 \end{array}\right]$,

(i) For what values of k does the system has infinitely many solutions?

(2.5 marks)

(ii) For what values of k is the system inconsistent?

(2.5 marks)

Question 3

(a) Solve the following complex equation to find value of w and v.

$$Z^2 - 4 - 2iZ = (Z + iv)^2 - wZi$$

(5 marks)

(b) A system is said to be stable if all the poles of the transfer function lie within the unit circle, $|z| < 1$. The system is said to be critically stable if it has a pole on the unit circle, $|z| = 1$. Poles occur where the denominator of the transfer function is zero. Determine whether the system with the following transfer functions are stable or critically stable:

$$G(z) = \frac{10}{3z^2 + 2z + 1}$$

(7 marks)

(c) Figure 1 shows three coplanar forces F_1, F_2 and F_3 which are acting at a point. Given that F_1, 10 N acting at angle of 45°; F_2, 8 N acting at angle of 120° and F_3, 15 N acting at angle of 210°. Determine using complex number, the magnitude and direction of the resultant of the coplanar forces.

(8 marks)

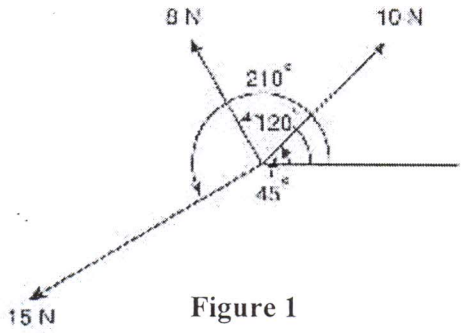

Figure 1

Question 4

(a) Find the angle between the plane $x+y+z=10$

and the line $\dfrac{x-1}{2}=\dfrac{y+3}{3}=2-z$

(6 marks)

(b) Given the following 2 lines,

$$l_1:\ x=1+2\lambda,\ \ y=-1+\lambda \ \text{ and } \ z=2+4\lambda$$

$$l_2:\ \frac{x+2}{4}=\frac{y}{-3}=z+1$$

(i) Determine whether the given 2 lines, l_1 and l_2 are parallel, intersecting or skewed.

(8 marks)

(ii) Find the shortest distance between lines, l_1 and l_2.

(6 marks)

Question 5

(a) By using the **first principle method**, determine the derivative of the function

$$f(x) = \frac{3x}{x^2+1} \quad \text{at } x = -4.$$

(5 marks)

(b) Find the values of ***a, b*** and ***c*** if the derivative of $y = \dfrac{x}{\sqrt{3+\sqrt{x-1}}}$

is given by $\dfrac{dy}{dx} = \dfrac{a\sqrt{x-1}+bx+c}{4\sqrt{x-1}\sqrt{\left(3+\sqrt{x-1}\right)^3}}$.

(8 marks)

(c) If $y.\sinh^{-1}x = x.\sinh^{-1}y$, evaluate $\dfrac{dy}{dx}$ when $x = 2,\ y = 1$.

(7 marks)

Question 6

(a) Solve the following integral

$$\int \frac{x.\sin^{-1}x}{\sqrt{1-x^2}}\,dx$$

(8 marks)

(b) Determine the following indefinite integral by using appropriate substitutions

$$\int \frac{1}{e^x\left(1+e^x\right)}\,dx$$

(12 marks)

END OF QUESTION

APPENDIX 1 - Trigonometric Identities and Formulas

Fundamental Identities

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Formulas For Negatives

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

Addition Formulas

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Subtraction Formulas

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Half-Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\sin\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\ldots\ldots\ldots = 1 - 2\sin^2\theta$$

$$\ldots\ldots\ldots = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Product-To-Sum Formulas

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

Sum-To-Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

APPENDIX 2 – Table of Differentiation

Trigonometric Functions
$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$
$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$
$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec f(x)) = f'(x)\csc f(x)\tan f(x)$
$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$

Inverse Trigonometric Functions
$\frac{d}{dx}(\sin^{-1}U) = \frac{1}{\sqrt{1-U^2}}\frac{dU}{dx}$, $\lvert U\rvert < 1$
$\frac{d}{dx}(\cos^{-1}U) = \frac{-1}{\sqrt{1-U^2}}\frac{dU}{dx}$, $\lvert U\rvert < 1$
$\frac{d}{dx}(\tan^{-1}U) = \frac{1}{1+U^2}\frac{dU}{dx}$
$\frac{d}{dx}(\csc^{-1}U) = \frac{-1}{\lvert U\rvert\sqrt{U^2-1}}\frac{dU}{dx}$, $\lvert U\rvert > 1$
$\frac{d}{dx}(\sec^{-1}U) = \frac{1}{\lvert U\rvert\sqrt{U^2-1}}\frac{dU}{dx}$, $\lvert U\rvert > 1$
$\frac{d}{dx}(\cot^{-1}U) = \frac{-1}{1+U^2}\frac{dU}{dx}$

Hyperbolic Functions
$\frac{d}{dx}(\sinh U) = \cosh U\frac{dU}{dx}$
$\frac{d}{dx}(\cosh U) = \sinh U\frac{dU}{dx}$
$\frac{d}{dx}(\tanh U) = \operatorname{sech}^2 U\frac{dU}{dx}$
$\frac{d}{dx}(\operatorname{csch} U) = -\operatorname{csch} U\coth U\frac{dU}{dx}$
$\frac{d}{dx}(\operatorname{sech} U) = -\operatorname{sech} U\tanh U\frac{dU}{dx}$
$\frac{d}{dx}(\coth U) = -\operatorname{csch}^2 U\frac{dU}{dx}$

Inverse Hyperbolic Functions
$\frac{d}{dx}(\sinh^{-1} U) = \frac{1}{\sqrt{1+U^2}}\frac{dU}{dx}$
$\frac{d}{dx}(\cosh^{-1} U) = \frac{1}{\sqrt{U^2-1}}\frac{dU}{dx}$, $U > 1$
$\frac{d}{dx}(\tanh^{-1} U) = \frac{1}{1-U^2}\frac{dU}{dx}$, $\lvert U\rvert < 1$
$\frac{d}{dx}(\operatorname{csch}^{-1} U) = \frac{-1}{\lvert U\rvert\sqrt{1+U^2}}\frac{dU}{dx}$, $U \neq 0$
$\frac{d}{dx}(\operatorname{sech}^{-1} U) = \frac{-1}{U\sqrt{1-U^2}}\frac{dU}{dx}$, $0 < U < 1$
$\frac{d}{dx}(\coth^{-1} U) = \frac{1}{1-U^2}\frac{dU}{dx}$, $\lvert U\rvert > 1$

Exponential Function
$\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$

Natural Logarithmic Function
$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)}f'(x)$

APPENDIX 3 – Table of Integration

Trigonometric Functions Where $f(x) = ax + b$
$\int \cos f(x)dx = \frac{\sin f(x)}{f'(x)} + C$
$\int \sin f(x)dx = -\frac{\cos f(x)}{f'(x)} + C$
$\int \sec^2 f(x)dx = \frac{\tan f(x)}{f'(x)} + C$
$\int \csc^2 f(x)dx = -\frac{\cot f(x)}{f'(x)} + C$
$\int \sec f(x)\tan f(x)dx = \frac{\sec f(x)}{f'(x)} + C$
$\int \csc f(x)\cot f(x)dx = -\frac{\csc f(x)}{f'(x)} + C$

Inverse Trigonometric Functions
$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{-1}{\lvert x \rvert \sqrt{x^2 - a^2}} dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \frac{1}{\lvert x \rvert \sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \frac{-1}{a^2 + x^2} dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$

Hyperbolic Functions Where $f(x) = ax + b$
$\int \cosh f(x)dx = \frac{\sinh f(x)}{f'(x)} + C$
$\int \sinh f(x)dx = \frac{\cosh f(x)}{f'(x)} + C$
$\int \text{sech}^2 f(x)dx = \frac{\tanh f(x)}{f'(x)} + C$
$\int \text{csch}^2 f(x)dx = -\frac{\coth f(x)}{f'(x)} + C$
$\int \text{sech}\, f(x)\tanh f(x)dx = \frac{-\text{sech}\, f(x)}{f'(x)} + C$
$\int \text{csch}\, f(x)\coth f(x)dx = -\frac{\text{csch}\, f(x)}{f'(x)} + C$

Inverse Hyperbolic Functions
$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \sinh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad a > 0$
$\int \frac{-1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad x > a$
$\int \frac{1}{a^2 - x^2} dx = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \frac{1}{a^2 - x^2} dx = \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \frac{1}{x\sqrt{a^2 + x^2}} dx = -\frac{1}{a}\text{csch}^{-1}\left(\frac{x}{a}\right) + C \quad , \quad 0 < x < a$
$\int \frac{1}{x\sqrt{a^2 - x^2}} dx = -\frac{1}{a}\text{sech}^{-1}\left(\frac{x}{a}\right) + C \quad , \quad 0 < x < a$

Exponential Function Where $f(x) = ax + b$
$\int e^{f(x)} dx = \frac{e^{f(x)}}{f'(x)} + C$

Form $\frac{1}{f(x)}$, where $f(x) = ax + b$
$\int \frac{1}{f(x)} dx = \frac{\ln\lvert f(x) \rvert}{f'(x)} + C$