



UNIVERSITI KUALA LUMPUR
KAMPUS CAWANGAN MALAYSIAN SPANISH INSTITUTE

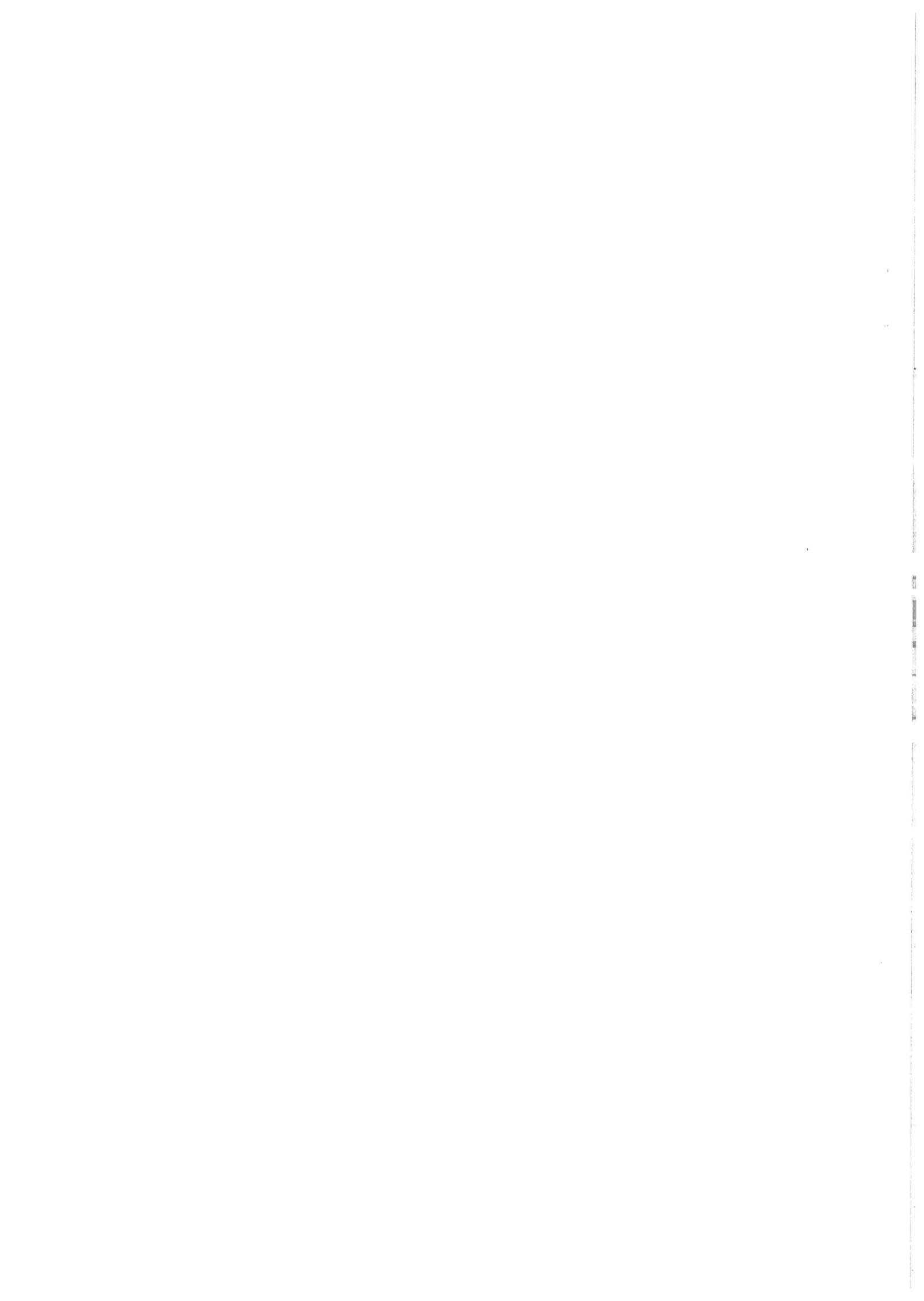
FINAL EXAMINATION
JULY 2023 SEMESTER

COURSE CODE : SSP 01204
COURSE NAME : MATHEMATICS
PROGRAMME NAME : FOUNDATION IN SCIENCE AND ARTS
DATE : 26 DECEMBER 2023
TIME : 09.00 AM – 11.00 AM
DURATION : 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please CAREFULLY read the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of TWO (2) sections; Section A and Section B.
4. Answer ALL questions in Section A. For Section B, answer TWO (2) questions.
5. Please write your answers on the answer booklet provided.
6. Answer all questions in English language ONLY.
7. Formula sheet has been appended for your reference.

THERE ARE 5 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.



SECTION A (Total: 60 marks)

INSTRUCTION: Answer ALL questions.

Please use the answer booklet provided.

Question 1

- (a) Solve the equation $8^{x+1} = 3^{x+2}$, giving your answer to 4 decimal places. (5 marks)
- (b) Determine the slope of the curve $x^2y^4 - 2x^3 + y^2 = 4$ using implicit differentiation. (5 marks)

Question 2

- (a) Determine $(f \circ g)(x)$ if $f(x) = 3x^2 - 2$ and $g(x) = 4x - 5$. (5 marks)
- (b) Calculate $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$. (5 marks)

Question 3

- (a) Let $A = \begin{pmatrix} 3 & -2 & 2 \\ 1 & 2 & -1 \\ 3 & 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 & -1 \\ 2 & 5 & 1 \\ 0 & -3 & 3 \end{pmatrix}$. Determine BA . (5 marks)
- (b) Evaluate $\int_2^5 2x^2 + 3 \, dx$. (5 marks)

Question 4

- (a) An arithmetic series has first term 49 and 15th term 7. Determine the value of the
- i. common difference. (3 marks)
 - ii. sum of the first 15 terms of the series. (2 marks)
- (b) Determine the number of terms in the geometric sequence 1,3,9,27,.....,531441. (5 marks)

Question 5

- (a) Eight consecutive heart beats of a patient per minute are 75,71,74,(q + 3),72,73,76,q. If the sample mean is 75, determine
- i. the value of q. (3 marks)
 - ii. Hence, determine the sample median. (2 marks)
- (b) The following data shows the ages of a group of people at company A.

Stem	Leaf
2	1 2 3 7 8
3	3 3 4 5 7 9
4	0 1 2 3 4 4 5 7 8 9
5	2 3 5 7 8

Key: 3|4 means 34 years old

Determine the:

- i. first quartile, (2 marks)
- ii. third quartile, (2 marks)
- iii. interquartile range. (1 mark)

Question 6

- (a) An ordinary dice is thrown. Determine the probability that the number obtained is
- i. an odd number
 - ii. greater than 2
 - iii. divisible by 2

(5 marks)

- (b) The probability that a student in a college is a male is 0.4 and that a student rides a bike is 0.32. If these two events are independent, determine the probability that a student selected at random is
- i. a male and rides a bike
 - ii. a female or rides a bike

(5 marks)

SECTION B (Total: 40 marks)

INSTRUCTION: Answer ONLY TWO (2) questions.

Please use the answer booklet provided.

Question 1

(a) Given $z = -7 - 5i$.i. Express z in polar form.

(7 marks)

ii. Determine the value of z^3 by using De Moivre's theorem.

(3 marks)

(b) A spherical balloon is inflated so that its volume is increasing at the rate of $2 \text{ cm}^3 / \text{s}$. How fast is the diameter of the balloon increasing when the radius is 3 cm ? Given

$$\text{volume of sphere, } V = \frac{4}{3} \pi r^3.$$

(10 marks)

Question 2

(a) Determine the expansion of $\sqrt{1+3x}$ up to and including the term in x^3 . By substituting in $x = 0.01$ in the expansion, determine an approximation to $\sqrt{103}$.

(10 marks)

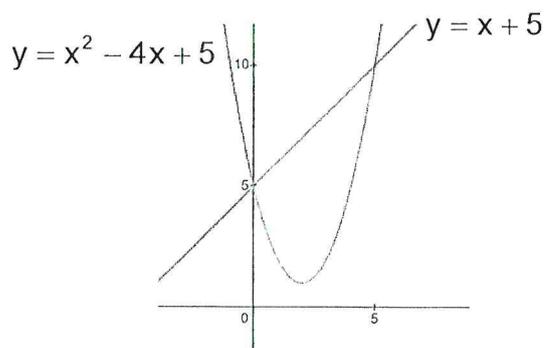
(b) Calculate the area of the region enclosed by the curve $y = x^2 - 4x + 5$ and the line $y = x + 5$ as shown in Figure 1.

Figure 1

(10 marks)

Question 3

- (a) Determine the value of y to the given system of equations using Cramer's rule.

$$\begin{aligned}2x + y + z &= 18 \\x - 2y - z &= -6 \\-x + 3z - 2y &= -6\end{aligned}$$

(10 marks)

- (b) A random sample of 80 college graduates was asked how long it took them to be promoted. The time was recorded and summarized in the table below.

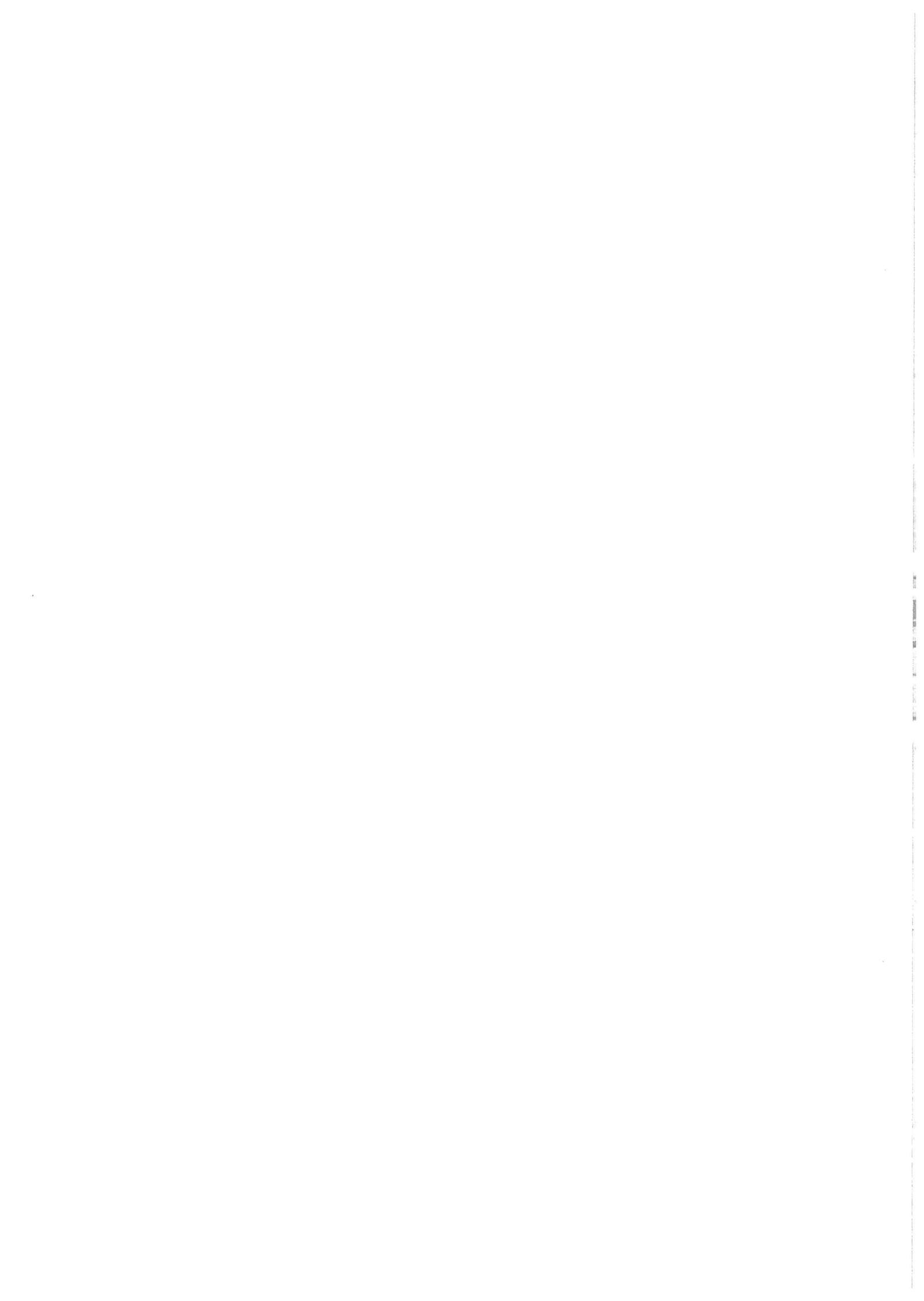
Table 1

Time (years)	0 – 3	3 – 6	6 – 9	9 – 12	12 – 15
Number of college graduates	5	15	20	30	10

- i. Determine the mean and median.
- ii. Determine the variance.
- iii. Calculate the Pearson's coefficient of skewness.

(10 marks)

END OF EXAMINATION PAPER



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NUMBERS

1. Law of Exponents

- a. $a^m \cdot a^n = a^{m+n}$
- b. $\frac{a^m}{a^n} = a^{m-n}$
- c. $(a^m)^n = a^{mn}$
- d. $(ab)^n = a^n b^n$
- e. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- f. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

2. Law of Logarithms

- a. $\log_a xy = \log_a x + \log_a y$
- b. $\log_a \frac{x}{y} = \log_a x - \log_a y$
- c. $\log_a 1 = 0$
- d. $\log_a a = 1$
- e. $\log_a x^n = n \log_a x$

3. Change of base

$$\log_a N = \frac{\log_b N}{\log_b a}$$

4. Complex Number

Polar form

$$z = r(\cos \theta + i \sin \theta)$$

De Moivre's Theorem

$$z^n = r^n (\cos(n\theta) + i \sin(n\theta))$$

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SEQUENCES AND SERIES

1. Arithmetic Sequences and Series

a. $T_n = a + (n-1)d$

b. $S_n = \frac{n}{2}[2a + (n-1)d]$

c. $S_n = \frac{n}{2}[a + l]$

2. Geometric Sequences and Series

a. $T_n = ar^{n-1}$

b. $S_n = \frac{a(1-r^n)}{1-r}$

3. Sum to Infinity

$$S_\infty = \frac{a}{1-r}, |r| < 1$$

4. Binomial expansion

a. $(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + \binom{n}{n}b^n$

Where $T_{r+1} = {}^nC_r a^{n-r} b^r$ and ${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$.

b. $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$

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MATRICES

1. Cofactor, $C_{ij} = (-1)^{i+j} M_{ij}$

2. Determinant:

a. 2×2 matrix

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

b. 3×3 matrix

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

3. Inverse matrix

a. 2×2 matrix

$$A^{-1} = \frac{1}{|A|} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

b. 3×3 matrix

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

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DERIVATIVES

1. Differentiations of Exponential & Logarithmic Functions

a. $\frac{d}{dx}(e^{f(x)}) = e^{f(x)} \cdot f'(x)$

b. $\frac{d}{dx}(\ln|f(x)|) = \frac{f'(x)}{f(x)}$

2. Differentiations of Trigonometric Functions

a. $\frac{d}{dx}(\sin f(x)) = \cos f(x) \cdot f'(x)$

b. $\frac{d}{dx}(\cos f(x)) = -\sin f(x) \cdot f'(x)$

c. $\frac{d}{dx}(\tan f(x)) = \sec^2 f(x) \cdot f'(x)$

d. $\frac{d}{dx}(\cot f(x)) = -\csc^2 f(x) \cdot f'(x)$

e. $\frac{d}{dx}(\sec f(x)) = \sec f(x) \tan f(x) \cdot f'(x)$

f. $\frac{d}{dx}(\csc f(x)) = -\csc f(x) \cot f(x) \cdot f'(x)$

3. Related Rates

a. **Circumference of Circle**

$$C = 2\pi r$$

b. **Area of Circle**

$$A = \pi r^2$$

c. **Volume of sphere**

$$V = \frac{4}{3}\pi r^3$$

d. **Volume of Conical Tank**

$$V = \frac{1}{3}\pi r^2 h$$

e. **Pythagorean Theorem**

$$c^2 = a^2 + b^2$$

INTEGRALS

1. Differentiations of Exponential & Logarithmic Functions

a. $\int e^{f(x)} dx = \frac{e^{f(x)}}{f'(x)} + C$

b. $\int \frac{1}{f(x)} dx = \frac{\ln|f(x)|}{f'(x)} + C$

2. Differentiations of Trigonometric Functions

a. $\int \sin f(x) dx = -\frac{\cos f(x)}{f'(x)} + C$

b. $\int \cos f(x) dx = \frac{\sin f(x)}{f'(x)} + C$

c. $\int \sec^2 f(x) dx = \frac{\tan f(x)}{f'(x)} + C$

3. Area and Volume

a. $A = \int_a^b f(x) dx$

b. $A = \int_a^b f(x) - g(x) dx$

c. $V = \pi \int_a^b [f(x)]^2 dx$

d. $V = \pi \int_a^b [f(x)]^2 - [g(x)]^2 dx$

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INTRODUCTION TO STATISTICS

1. Ungrouped Data

a. **Mean,** $\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{\sum x}{n}$

b. **Median** = $\left(\frac{n+1}{2}\right)^{\text{th}}$ (odd number)

c. **Median** = $\frac{\left(\frac{n}{2}\right)^{\text{th}} + \left(\frac{n}{2} + 1\right)^{\text{th}}}{2}$ (even number)

d. **Percentiles**

$$P_k = \begin{cases} x_{[s]} & \text{when } s \text{ is not an integer} \\ \frac{x_s + x_{s+1}}{2} & \text{when } s \text{ is an integer} \end{cases}$$

Where $s = \frac{kn}{100}$ and $[s]$ is the least integer greater than s .

e. **Variance,** $s^2 = \frac{1}{n-1} \left[\sum x^2 - \frac{(\sum x)^2}{n} \right]$

f. **Standard deviation,** $s = \sqrt{\frac{1}{n-1} \left[\sum x^2 - \frac{(\sum x)^2}{n} \right]}$

2. Grouped Data

a. **Mean,** $\bar{x} = \frac{\sum fx}{\sum f}$

b. **Median** = $L_B + \left(\frac{\frac{N}{2} - F_{k-1}}{f_k} \right) c$

where

L_B = lower boundary of the median class

N = total frequency

F_{k-1} = cumulative frequency before median class

f_k = frequency of the median class

c = size of the median class

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c. **Mode** $= L_B + \left(\frac{d_1}{d_1 + d_2} \right) c$

where

L_B = lower boundary of the modal class

d_1 = the difference between the frequency of the modal class and the previous class

d_2 = the difference between the frequency of the modal class and the next class

c = size of the modal class

d. **Quartiles** $Q_k = L_k + \left(\frac{\frac{k}{4}N - F_{k-1}}{f_k} \right) c_k$

where

L_k = lower boundary of the class where Q_k lies

N = total frequency

F_{k-1} = cumulative frequency before the quartile class

f_k = frequency of the class where Q_k lies

c_k = size of the class where Q_k lies

e. **Percentiles** $P_k = L_k + \left(\frac{\frac{k}{100}N - F_{k-1}}{f_k} \right) c_k$

where

L_k = lower boundary of the class where P_k lies

N = total frequency

F_{k-1} = cumulative frequency before the percentile class

f_k = frequency of the class where P_k lies

c_k = size of the class where P_k lies

f. **Interquartile range**, $IQR = Q_3 - Q_1$

g. **Variance**, $s^2 = \frac{1}{n-1} \left[\sum fx^2 - \frac{(\sum fx)^2}{n} \right]$

h. **Standard deviation**, $s = \sqrt{\frac{1}{n-1} \left[\sum fx^2 - \frac{(\sum fx)^2}{n} \right]}$

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3. Pearson's Coefficient of Skewness

a. $s_k = \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}$

b. $s_k = \frac{3(\text{Mean} - \text{Median})}{\text{Standard deviation}}$

INTRODUCTION TO PROBABILITY

1. Probability of an Event

$$P(A) = \frac{n(A)}{n(S)}$$

2. Additional rule

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

b. $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

3. De Morgan's rule

a. $P(A' \cup B') = P(A \cap B)'$

b. $P(A' \cap B') = P(A \cup B)'$

4. Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

