CONFIDENTIAL

UNIVERSITI KUALA LUMPUR
Malaysian Institute of Marine Engineering Technology

FINAL EXAMINATION
MARCH 2025 SEMESTER SESSION

SUBJECT CODE	:	**LEB30703**
SUBJECT TITLE	:	**SIGNALS AND SYSTEMS**
PROGRAMME NAME (FOR MPU: PROGRAMME LEVEL)	:	**BACHELOR OF ELECTRICAL AND ELECTRONICS ENGINEERING TECHNOLOGY (MARINE) WITH HONOURS**
TIME / DURATION	:	**09.00 AM - 12.00 PM (3 HOURS)**
DATE	:	**23 JUNE 2025**

INSTRUCTIONS TO CANDIDATES

1. Please read **CAREFULLY** the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of **TWO (2)** sections; Section A and Section B.
4. Answer **ALL** question in Section A, and **THREE (3)** questions **ONLY** in Section B.
5. Please write your answers on this answer booklet provided.
6. Answer **ALL** questions in English language **ONLY**.
7. Formula is appended as for your reference.

THERE ARE 6 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question 1 (CLO 1, C2, SK2, SP1)

(a) Explain the characteristics of **THREE (3)** types of signals.

(15 marks)

(b) Identify **THREE (3)** main classifications of system.

(3 marks)

(c) Explain random signal.

(2 marks)

Question 2 (CLO 1, C2, SK2, SP1)

(a) Identify **THREE (3)** steps to generate a transpose of any realization.

(6 marks)

(b) In the context of RLC circuits, outline **THREE (3)** key steps involved based on systematic procedure to determine state equations of linear time-invariant systems.

(6 marks)

(c) Explain the criterion for a system with a diagonalized to be considered completely controllable in terms of the matric characteristics.

(4 marks)

(d) Identify **FOUR (4)** types of realization of continuous-time linear time-invariant.

(4 marks)

SECTION B (Total: 60 marks)

INSTRUCTION: Answer THREE (3) questions ONLY.
Please use the answer booklet provided.

Question 3 (CLO 2, C4, SK3, SP3, SP4)

(a) Employing the time-domain method, answer the following questions for a system defined by the following equation:

$$(D^2 + 8D + 5)y(t) = 6Dx(t)$$

Given the initial conditions as $y_n(0) = 0$ and $\dot{y}(0) = 1$.

i. Identify the characteristic polynomial.

(1 mark)

ii. Determine the characteristic roots.

(2 marks)

iii. Determine the zero-input response equations.

(4 marks)

iv. Find the unit impulse response, $h(t)$.

(3 marks)

(b) Develop a passive bandstop filter circuit with the following specifications:

- Resonant Angular Frequency: $\omega_o = 10\ rad/s$
- Quality Factor: $Q = 20$

Your design should include the following items:

i. Passive bandstop filter circuit: Provide a detailed schematic of your designed circuit.

(2 marks)

ii. Bandwidth, B: Determine and present the bandwidth of the filter.

(2 marks)

iii. Component values:

- Resistor(s): Specify the resistance values used in your circuit.

(2 marks)

- Inductor(s): Specify the inductance values used in your circuit.

(2 marks)

- Capacitor(s): Specify the capacitance values used in your circuit.

(2 marks)

Question 4 (CLO 2, C4, SK3, SP3, SP4)

(a) The equation describes an LTI system:

$$y[n] - y[n-1] - 2y[n-2] = x[n-1] + 2x[n-2]$$

i. Determine the z-transform on both sides and neglect initial conditions.

(5 marks)

ii. Calculate the transfer function of the system, $H[z]$.

(2 marks)

iii. Find the impulse response of the system, $h[n]$.

(3 marks)

(b) Refer to Figure 1, answer the following questions:

i. Derive the transfer function, $H(s)$, for the given circuit. Show all the steps involved in obtaining the transfer function.

(6 marks)

ii. Based on the transfer function derived, identify the type of filter. Provide a brief justification for your categorization.

(4 marks)

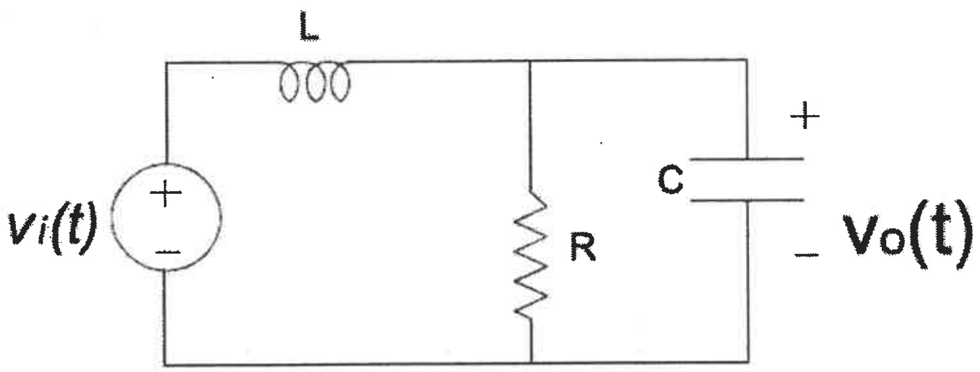

Figure 1

Question 5 (CLO 2, C4, SK3, SP3, SP4)

(a) Refer to the system shown in Figure 2 using the Laplace transform method to answer the following questions:

i. Illustrate the schematic diagram in the s-domain. Use the impedance method to convert the given system into its s-domain equivalent. Provide a clear and labelled schematic diagram.

(5 marks)

ii. Determine the system transfer function, $H(s)$ for the given system. Show all steps involved in the derivation process.

(5 marks)

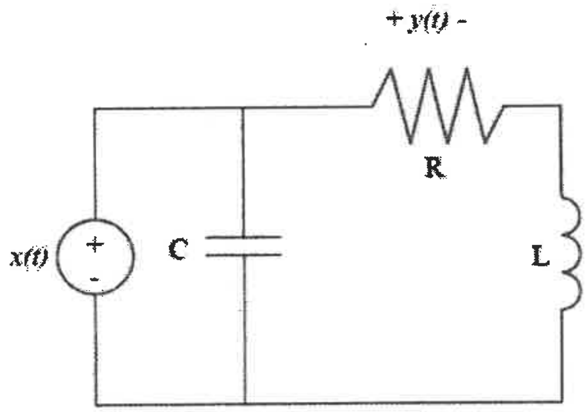

Figure 2

(b) Develop the state space representation for the given circuit shown in Figure 3. Assume that the outputs are the currents flowing through in R_1 and R_2. Given component values:

- $R_1 = 100\Omega$
- $R_2 = 250\Omega$
- $L = 2H$
- $C = 0.5\,F$

Present the state-space equations in matrix form, including the state variables, input, and output matrices.

(10 marks)

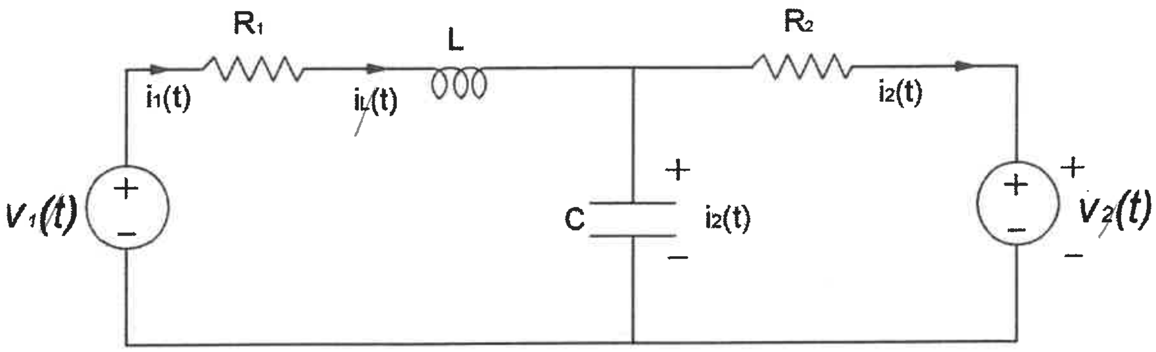

Figure 3

Question 6 (CLO 2, C4, SK3, SP3, SP4)

(a) Refer to the circuit shown in Figure 4 and answer the following questions for the given input voltage $v_s(t) = 2e^{-3t}u(t)$.

i. Determine the Fourier transform of the input voltage, $v_s(t)$.

(2 marks)

ii. Derive the transfer function, $H(\omega)$, of the circuit in the frequency domain. Show all steps involved in the derivation.

(2 marks)

iii. Using the transfer function and the input voltage, determine the output voltage, $V_o(\omega)$, in the frequency domain.

(2 marks)

iv. Compute the inverse Fourier transform to determine the output voltage, $v_o(t)$, in the time domain.

(4 marks)

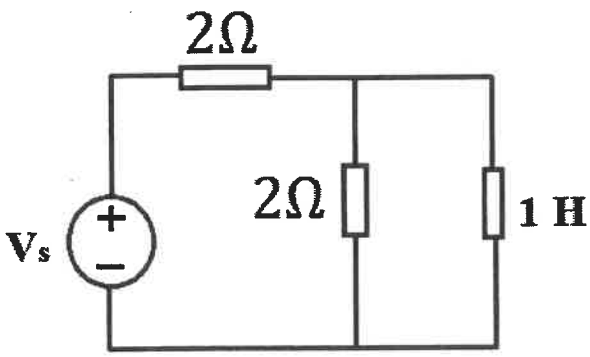

Figure 4

(b) Consider a continuous-time LTI system with the system transfer function as follows:

$$H(s) = \frac{3s + 7}{s^2 + 3s + 2}$$

i. Perform a partial expansion of the transfer function, $H(s)$. Present the expanded form of $H(s)$.

(2 marks)

ii. Identify the parallel realization diagram of the given system. Present the feedback and feedforward coefficient(s).

(5 marks)

iii. Develop the state-space equation of the system using parallel form realization. Clearly provide input and output state-space equation.

(3 marks)

END OF EXAMINATION PAPER

Table of formulae for LEB30703 Signals and Systems

(For use during examination only)

Convolution Table

No.	$x_1(t)$	$x_2(t)$	$x_1(t) * x_2(t) = x_2(t) * x_1(t)$
1	$x(t)$	$\delta(t-T)$	$x(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\dfrac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\dfrac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\dfrac{1}{2}t^2e^{\lambda t}u(t)$
7	$t^N u(t)$	$e^{\lambda t}u(t)$	$\dfrac{N!\,e^{\lambda t}}{\lambda^{N+1}}u(t) - \sum_{k=0}^{N}\dfrac{N!\,t^{N-k}}{\lambda^{k+1}(N-k)!}u(t)$
8	$t^M u(t)$	$t^N u(t)$	$\dfrac{M!N!}{(M+N+1)!}t^{M+N+1}u(t)$
9	$te^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\dfrac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$
10	$t^M e^{\lambda t}u(t)$	$t^N e^{\lambda t}u(t)$	$\dfrac{M!\,N!}{(N+M+1)!}t^{M+N+1}e^{\lambda t}u(t)$
11	$t^M e^{\lambda_1 t}u(t)$	$t^N e^{\lambda_2 t}u(t)$	$\sum_{k=0}^{M}\dfrac{(-1)^k M!(N+k)!\,t^{M-k}e^{\lambda_1 t}}{k!(M-k)!(\lambda_1-\lambda_2)^{N+k+1}}u(t)$
	$\lambda_1 \neq \lambda_2$		$+\sum_{k=0}^{N}\dfrac{(-1)^k N!(M+k)!\,t^{N-k}e^{\lambda_2 t}}{k!(N-k)!(\lambda_2-\lambda_1)^{M+k+1}}u(t)$
12	$e^{-\alpha t}\cos(\beta t+\theta)u(t)$	$e^{\lambda t}u(t)$	$\dfrac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$
			$\phi = \tan^{-1}[-\beta/(\alpha+\lambda)]$
13	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(-t)$	$\dfrac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1} \quad \text{Re}\,\lambda_2 > \text{Re}\,\lambda_1$
14	$e^{\lambda_1 t}u(-t)$	$e^{\lambda_2 t}u(-t)$	$\dfrac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$

Table of formulae for LEB30703 Signals and Systems

(For use during examination only)

Laplace Transform Table

No.	$x(t)$	$X(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{\lambda t} u(t)$	$\frac{1}{s-\lambda}$
6	$te^{\lambda t} u(t)$	$\frac{1}{(s-\lambda)^2}$
7	$t^n e^{\lambda t} u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$
8a	$\cos bt\, u(t)$	$\frac{s}{s^2+b^2}$
8b	$\sin bt\, u(t)$	$\frac{b}{s^2+b^2}$
9a	$e^{-at} \cos bt\, u(t)$	$\frac{s+a}{(s+a)^2+b^2}$
9b	$e^{-at} \sin bt\, u(t)$	$\frac{b}{(s+a)^2+b^2}$
10a	$re^{-at} \cos(bt+\theta) u(t)$	$\frac{(r\cos\theta)s+(ar\cos\theta-br\sin\theta)}{s^2+2as+(a^2+b^2)}$
10b	$re^{-at} \cos(bt+\theta) u(t)$	$\frac{0.5re^{j\theta}}{s+a-jb}+\frac{0.5re^{-j\theta}}{s+a+jb}$
10c	$re^{-at} \cos(bt+\theta) u(t)$; $r=\sqrt{\frac{A^2c+B^2-2ABa}{c-a^2}}$; $\theta=\tan^{-1}\left(\frac{Aa-B}{A\sqrt{c-a^2}}\right)$; $b=\sqrt{c-a^2}$	$\frac{As+B}{s^2+2as+c}$
10d	$e^{-at}\left[A\cos bt+\frac{B-Aa}{b}\sin bt\right]u(t)$; $b=\sqrt{c-a^2}$	$\frac{As+B}{s^2+2as+c}$

Table of formulae for LEB30703 Signals and Systems

(For use during examination only)

Summary of Laplace Transform Operation

Operation	$x(t)$	$X(s)$
Addition	$x_1(t) + x_2(t)$	$X_1(s) + X_2(s)$
Scalar multiplication	$kx(t)$	$kX(s)$
Time differentiation	$\frac{dx}{dt}$	$sX(s) - x(0^-)$
	$\frac{d^2x}{dt^2}$	$s^2X(s) - sx(0^-) - \dot{x}(0^-)$
	$\frac{d^3x}{dt^3}$	$s^3X(s) - s^2x(0^-) - s\dot{x}(0^-) - \ddot{x}(0^-)$
	$\frac{d^nx}{dt^n}$	$s^nX(s) - \sum_{k=1}^{n} s^{n-k}x^{(k-1)}(0^-)$
Time integration	$\int_{0^-}^{t} x(\tau)\,d\tau$	$\frac{1}{s}X(s)$
	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{s}X(s) + \frac{1}{s}\int_{-\infty}^{0^-} x(t)\,dt$

Operation	$x(t)$	$X(s)$
Time shifting	$x(t - t_0)u(t - t_0)$	$X(s)e^{-st_0} \quad t_0 \geq 0$
Frequency shifting	$x(t)e^{s_0t}$	$X(s - s_0)$
Frequency differentiation	$-tx(t)$	$\frac{dX(s)}{ds}$
Frequency integration	$\frac{x(t)}{t}$	$\int_{s}^{\infty} X(z)\,dz$
Scaling	$x(at),\ a \geq 0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi j}X_1(s) * X_2(s)$
Initial value	$x(0^+)$	$\lim_{s\to\infty} sX(s) \quad (n > m)$
Final value	$x(\infty)$	$\lim_{s\to 0} sX(s)$ [poles of $sX(s)$ in LHP]

Table of formulae for LEB30703 Signals and Systems

(For use during examination only)

Fourier Transform Table

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$	
13	$\cos\omega_0 t\,u(t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\,u(t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\,u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\,u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$

16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\text{rect}\left(\dfrac{t}{\tau}\right)$	$\tau\,\text{sinc}\left(\dfrac{\omega\tau}{2}\right)$	
18	$\dfrac{W}{\pi}\,\text{sinc}\,(Wt)$	$\text{rect}\left(\dfrac{\omega}{2W}\right)$	
19	$\Delta\left(\dfrac{t}{\tau}\right)$	$\dfrac{\tau}{2}\,\text{sinc}^2\left(\dfrac{\omega\tau}{4}\right)$	
20	$\dfrac{W}{2\pi}\,\text{sinc}^2\left(\dfrac{Wt}{2}\right)$	$\Delta\left(\dfrac{\omega}{2W}\right)$	
21	$\displaystyle\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\displaystyle\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\dfrac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}\,e^{-\sigma^2\omega^2/2}$	

Table of formulae for LEB30703 Signals and Systems
(For use during examination only)
Summary of Fourier Transform Operation

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$

Operation	$x(t)$	$X(\omega)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{\infty}^{t} x(u)\, du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

Table of formulae for LEB30703 Signals and Systems

(For use during examination only)

Z- Transform Table

No.	x[n]	X[z]
1	$\delta[n-k]$	z^{-k}
2	$u[n]$	$\frac{z}{z-1}$
3	$nu[n]$	$\frac{z}{(z-1)^2}$
4	$n^2u[n]$	$\frac{z(z+1)}{(z-1)^3}$
5	$n^3u[n]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^n u[n]$	$\frac{z}{z-\gamma}$
7	$\gamma^{n-1}u[n-1]$	$\frac{1}{z-\gamma}$
8	$n\gamma^n u[n]$	$\frac{\gamma z}{(z-\gamma)^2}$
9	$n^2\gamma^n u[n]$	$\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\frac{n(n-1)(n-2)\ldots(n-m+1)}{\gamma^m m!}\gamma^n u[n]$	$\frac{z}{(z-\gamma)^{m+1}}$
11a	$\lvert\gamma^n\rvert \cos\beta n\, u[n]$	$\frac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
11b	$\lvert\gamma\rvert^n \sin\beta n\ u[n]$	$\frac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12a	$r\lvert\gamma\rvert^n\cos(\beta n+\theta)\,u[n]$	$\frac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12b	$r\lvert\gamma\rvert^n\cos(\gamma n+\theta)\,u[n] \qquad \gamma=\lvert\gamma\rvert e^{j\beta}$	$\frac{(0.5re^{j\theta})z}{z-\gamma}+\frac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r\lvert\gamma\rvert^n\cos(\gamma n+\theta)u[n]$ $r=\sqrt{\frac{A^2\lvert\gamma\rvert^2+B^2-2AaB}{\lvert\gamma\rvert^2-a^2}}$ $\beta=\cos^{-1}\frac{-a}{\lvert\gamma\rvert},$ $\theta=\tan^{-1}\frac{Aa-B}{A\sqrt{\lvert\gamma\rvert^2-a^2}}$	$\frac{z-(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$

Summary of Z-Transform Operation

Operation	x[n]	X[z]
Addition	$x_1[n] + x_2[n]$	$X_1[z] + X_2[z]$
Scalar multiplication	$ax[n]$	$aX[z]$
Right-shifting	$x[n-m]u[n-m]$	$\frac{1}{z^m}X[z]$
	$x[n-m]u[n]$	$\frac{1}{z^m}X[z] + \frac{1}{z^m}\sum_{n=1}^{m} x[-n]z^n$
	$x[n-1]u[n]$	$\frac{1}{z}X[z] + x[-1]$
	$x[n-2]u[n]$	$\frac{1}{z^2}X[z] + \frac{1}{z}x[-1] + x[-2]$
	$x[n-3]u[n]$	$\frac{1}{z^3}X[z] + \frac{1}{z^2}x[-1] + \frac{1}{z}x[-2] + x[-3]$
Left-shiting	$x[n+m]u[n]$	$z^m X[z] - z^m \sum_{n=0}^{m-1} x[n]z^{-n}$
	$x[n+1]u[n]$	$zX[z] - zx[0]$
	$x[n+2]u[n]$	$z^2X[z] - z^2x[0] - zx[1]$
	$x[n+3]u[n]$	$z^3X[z] - z^3x[0] - z^2x[1] - zx[2]$
Multiplication by γ^n	$\gamma^n x[n]u[n]$	$X\left[\frac{z}{\gamma}\right]$
Multiplication by n1	$nx[n]u[n]$	$-z\frac{d}{dz}X[z]$
Time convolution	$x_1[n] * x_2[n]$	$X_1[z]X_2[z]$
Frequency convolution	$x_1[n]x_2[n]$	$\frac{1}{2\pi j}\oint X_1[u]X_2\left[\frac{z}{u}\right]u^{-1}du$
Time reversal	$x[-n]$	$X[1/z]$
Initial value	$x[0]$	$\lim_{z\to}(z-1)X[z]$ poles of $z(z-1)X[z]$ inside the unit circle

Table of formulae for LEB30703 Signals and Systems

(For use during examination only)

Formulae

Series Resonance

Resonance frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}}\, rad/s$$

Half-power frequency,

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Quality factor,

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

Bandwidth,

$$B = \frac{R}{L} = \frac{\omega_O}{Q}$$

For $Q \geq 10$,

$$\omega_1 \cong \omega_0 - \frac{B}{2}, \qquad \omega_2 \cong \omega_0 + \frac{B}{2},$$

Parallel Resonance

$$\omega_0 = \frac{1}{\sqrt{LC}}\, rad/s$$

Half-power frequency,

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Bandwidth,

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

Quality factor,

$$Q = \frac{\omega_o}{B} = \omega_0 RC = \frac{R}{\omega_O L}$$

For $Q \geq 10$,

$$\omega_1 \cong \omega_0 - \frac{B}{2}, \qquad \omega_2 \cong \omega_0 + \frac{B}{2},$$

Table of formulae for LEB30703 Signals and Systems

(For use during examination only)

PASSIVE FILTER

Highpass filter

$$H(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

$$\omega_c = \frac{1}{RC}$$

Lowpass filter

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

$$\omega_c = \frac{1}{RC}$$

Bandpass filter

$$H(\omega) = \frac{R}{R + j(\omega L - \frac{1}{\omega C})}$$

$$\omega_c = \frac{1}{\sqrt{RC}}$$

Bandstop filter

$$H(\omega) = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$$

$$\omega_c = \frac{1}{\sqrt{LC}}$$

ACTIVE FILTERS

First-order Lowpass filter

$$H(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_f R_f}$$

$$\omega_c = \frac{1}{R_f C_f}$$

First-order Highpass Filter

$$H(\omega) = -\frac{j\omega C_i R_f}{1 + j\omega C_i R_i}$$

$$\omega_c = \frac{1}{R_i C_i}$$

Bandpass filter

$$H(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_1 R}\frac{j\omega C_2 R}{1 + j\omega C_2 R}$$

$$\omega_2 = \frac{1}{RC_1}$$

$$\omega_1 = \frac{1}{RC_2}$$

$$K = \frac{R_f}{R_i}\frac{\omega_2}{\omega_1 + \omega_2}$$

Table of formulae for LEB30703 Signals and Systems

(For use during examination only)

Notch filter

$$H(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_1 R} - \frac{j\omega C_2 R}{1 + j\omega C_2 R}$$

$$\omega_2 = \frac{1}{RC_1}$$

$$\omega_1 = \frac{1}{RC_2}$$

$$K = \frac{R_f}{R_i}\frac{2\omega_1}{\omega_1 + \omega_2}$$

OTHER TYPES OF FILTER

Butterworth Filters (BF):

Roots of the Butterworth polynomial,

$$s_m = -\sin[(2m-1)(\pi/2n)] + j\cos[(2m-1)(\pi/2n)] = \sigma_m + j\omega_m; \qquad m = 1,2,\ldots,2n$$

The magnitude function of the nth order Butterworth filter:

$$|H_{Bn}(j\omega)| = \frac{1}{\sqrt{[1+(\omega/\omega_c)^{2n}}}; \qquad n = positive\ integer;\ \omega_c = \text{cut-off frequency}$$

Normalizing magnitude function of the nth order Butterworth filter is

$$|H_{Bn}(j\omega)| = \frac{1}{\sqrt{1 + (\omega)^{2n}}}$$

List of polynomials Butterworth Filters (BF) up to n=7:

n	*Polynomial*
1	$(s + 1)$
2	$(s^2 + 1.414s + 1)$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)$
5	$(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)$
6	$(s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.939s + 1)$

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<u>*Chebyshev Filters (CF):*</u>

Minimum value of ripple (dB),

$$dB(\gamma) = 10\log_{10}(1+\varepsilon^2)$$

Roots of the Chebyshev polynomial,

$$s_m = -\left[\sin\left((2m-1)\left(\frac{\pi}{2n}\right)\right)\right]\sinh\left[\left(\frac{1}{n}\right)\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right] + j\left[\cos\left((2m-1)\left(\frac{\pi}{2n}\right)\right)\right]\cosh\left[\left(\frac{1}{n}\right)\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right]$$

Normalizing magnitude function of the nth order Chebyshev filter is

$$|H_{Cn}(j\omega)| = \frac{1}{\sqrt{[1+\xi^2 C_n^2(\omega)]}}$$

Transfer function of CF for normalized frequency,

$$H_{c_n}(s) = \frac{K}{(-1)^n \prod_{m=1}^{n}\left(\frac{s}{s_m}-1\right)}$$, K = specified gain.

Transfer function of CF for denormalized frequency,

$$H_{c_n}(s) = \frac{K}{(-1)^n \prod_{m=1}^{n}\left(\frac{s}{s_m \omega_c}-1\right)}$$

<u>*List of polynomials Chebyshev Filters (CF) up to n=8:*</u>

n	$C_n(\omega)$
0	1
1	ω
2	$2\omega^2 - 1$
3	$4\omega^3 - 3\ \omega$
4	$8\omega^4 - 8\omega^2 + 1$
5	$16\omega^5 - 20\omega^3 + 5\omega$
6	$32\omega^7 - 48\omega^4 + 18\omega^2 - 1$
7	$74\omega^7 - 112\omega^5 + 57\omega^3 - 7\omega$
8	$128\omega^8 - 256\omega^7 + 170\omega^4 - 32\omega^2 + 1$

Table of formulae for LEB30703 Signals and Systems

(For use during examination only)

FIR FILTER:

$$H_{lc}(z) = H_l(z)\ z^{-(N-1)/2}$$

$$H_l(z) = h(0) + \sum_{\ell=1}^{\frac{N-1}{2}} h(\ell T)\left(z^{\ell} + z^{-\ell}\right)$$

IIR FILTER :

Bilinear z-transform characteristic:

$$s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$$

$$\omega_{ac} = \frac{2}{T}tan\left(\frac{\omega_{dc}T}{2}\right)$$

$n = \frac{log(A^2-1)}{2log\left(\frac{\omega_a}{\omega_c}\right)}$ or since usually A>>1, $n \approx \frac{log\ A}{log\left(\frac{\omega_a}{\omega_c}\right)}$

STATE-SPACE ANALYSIS – Continuous-Time Systems

$$Q(s) = \phi(s)q(0) + \phi(s)BX(s)$$

$$\phi(s) = (sI - A)^{-1}$$

$$Y(s) = CQ(s) + DX(s)$$

$$H(s) = C\phi(s)B + D$$

$$e^{At} = \beta_0 I + \beta_1 A + \beta_2 A^2 + \cdots + \beta_{N-1}A^{N-1}$$

where $\begin{bmatrix}\beta_0\\ \beta_1\\ \vdots\\ \beta_{N-1}\end{bmatrix} = \begin{bmatrix}1 & \lambda_1 & \lambda_1^2\cdots & \lambda_1^{N-1}\\ 1 & \lambda_2 & \lambda_2^2\cdots & \lambda_2^{N-1}\\ \vdots & \vdots & \vdots\cdots & \vdots\\ 1 & \lambda_N & \lambda_N^2\cdots & \lambda_N^{N-1}\end{bmatrix}^{-1}\begin{bmatrix}e^{\lambda_1 t}\\ e^{\lambda_2 t}\\ \vdots\\ e^{\lambda_N t}\end{bmatrix}$

$$q(t) = e^{At}q(0) + e^{At} * Bx(t)$$

$$y(t) = Ce^{At}q(0) + [Ce^{At}B + D\delta(t)] * x(t) = e^{At}q(0) + h(t) * x(t)$$

$$h(t) = C\phi B + D\delta(t)$$

$$\dot{w} = \hat{A}w + \hat{B}x, \quad y = \hat{C}w + Dx$$

$$\hat{A} = PAP^{-1},\ \hat{B} = PB,\ \hat{C} = CP^{-1},\ w = Pq$$

$\dot{z} = \Lambda z + \hat{B}x, \quad Y = \hat{C}z + Dx$

$\Lambda = PAP^{-1}$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0\cdots & 0 \\ 0 & \lambda_2 & 0\cdots & 0 \\ \vdots & \vdots & \vdots\cdots & \vdots \\ 0 & 0 & \ldots & \lambda_N \end{bmatrix}$$

STATE-SPACE ANALYSIS – Discrete-Time Systems

$q[n] = A^n q[0] + A^{n-1}u[n-1] * Bx[n]$

$y[n] = Cq + Dx = CA^n q[0] + CA^{n-1}u[n-1] * Bx[n] + Dx$

$A^n = \beta_0 I + \beta_1 A + \beta_2 A^2 + \cdots + \beta_{N-1}A^{N-1}$

where $\begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{N-1} \end{bmatrix} = \begin{bmatrix} 1 & \lambda_1 & \lambda_1^2\cdots & \lambda_1^{N-1} \\ 1 & \lambda_2 & \lambda_2^2\cdots & \lambda_2^{N-1} \\ \vdots & \vdots & \vdots\cdots & \vdots \\ 1 & \lambda_N & \lambda_N^2\cdots & \lambda_N^{N-1} \end{bmatrix}^{-1} \begin{bmatrix} \lambda_1^n \\ \lambda_2^n \\ \vdots \\ \lambda_N^n \end{bmatrix}$

$Q[z] = (I - z^{-1}A)^{-1}q[0] + (zI - A)^{-1}BX[z]$

$Y[z] = C(I - z^{-1}A)^{-1}q[0] + H[z]X[z]$

$H[z] = C(zI - A)^{-1}B + D$