CONFIDENTIAL

UNIVERSITI KUALA LUMPUR
Malaysian Institute of Marine Engineering Technology

FINAL EXAMINATION
FEBRUARY 2025 SEMESTER SESSION

SUBJECT CODE	:	**LMB21903**
SUBJECT TITLE	:	**ENGINEERING MATHEMATICS 2**
PROGRAMME NAME (FOR MPU: PROGRAMME LEVEL)	:	**BACHELOR OF MARINE ENGINEERING TECHNOLOGY WITH HONOURS**
TIME / DURATION	:	**2.00 PM - 5.00 PM (3 HOURS)**
DATE	:	**25 JUNE 2025**

INSTRUCTIONS TO CANDIDATES

1. Please read **CAREFULLY** the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of TWO (2) parts; Part A and Part B.
4. Answer **ALL** questions in Part A, and **THREE (3)** questions **ONLY** in Part B.
5. Please write your answers on this answer booklet provided.
6. Answer **ALL** questions in English language **ONLY**.
7. Answer should be written in blue of black ink except for sketching, graphic and illustration.
8. Formula is appended for your reference.

THERE ARE 4 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

PART A (Total: 40 marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question 1
With reference to Calculations with Statistics;

(a) Define the importance of statistics in real life application.

(2 marks)

(b) Classify which the following variables represent as discrete and which represent as continuous.

i. Lengths of 5000 bolts produced in a factory.

(1 mark)

ii. Yearly incomes of university lecturers.

(1 mark)

iii. Lifetimes of television tubes produced by two companies.

(1 mark)

iv. Numbers of shares sold each day in the stock market.

(1 mark)

(c) Calculate the mean, median and mode below set data:

5, 6, 2, 4, 7, 8, 3, 5, 6, 6

(4 marks)

(d) Table 1 below shows the extra time worked by a group of marine engineers at Company XYZ. Calculate the mean and median of the extra time of workers from group data below.

(10 marks)

Table 1: Extra time (Hours)

Extra time (Hours)	Frequency
25 – 29	5
30 - 34	4
35 - 39	7
40 - 44	11
45 - 49	12
50 - 54	8
55 - 59	1

Question 2

With reference to Calculations with Laplace transform;

(a) Briefly explain one application of Laplace transform.

(2 marks)

(b) Express the following functions in its Laplace transform form using the standard Laplace transform table.

i. $f(t) = 1 - 9t + 4t^4$.

(3 marks)

ii. $f(t) = 8\cos(6t) + 7t^4e^{2t}$.

(3 marks)

iii. $f(t) = e^{2t}\sin(3t) - 4t^6$.

(3 marks)

(c) Use Laplace transform to solve the differential equation $2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 0$,

(9 marks)

PART B (Total: 60 marks)

INSTRUCTION: Answer THREE questions.
Please use the answer booklet provided.

Question 3
With reference to Calculations with Differentiation;

(a) Differentiate the following with respect to x for the function models of a non-linear response in a marine mechanical system.

i. $y = (5x^4 - 6x^3)(7 + x^2)$.

(6 marks)

ii. $y = \frac{lnx+7x}{9-4x^2}$.

(6 marks)

(b) The surface area of a sphere is increasing at a constant rate of $3\pi\ cm^2 s^{-1}$. Find the rate at which the volume of the sphere is increasing when the radius is 4 cm.

(8 marks)

Question 4
With reference to Calculations with Integration;

(a) Find the exact value of the area enclosed between the curve and the x-axis over the interval from x = 1 to x = 5.

$$\int_1^5 \frac{5}{x(x+1)}\,dx$$

(8 marks)

(a) Use trapezoidal rule and Simpson's rule to evaluate the following integration using 6 intervals. Give the answers correct to 4 decimal places.

$$\int_0^{\pi} \frac{x}{(sinx+1)}\,dx$$

(12 marks)

Question 5

With reference to Calculations with Differentiation and Integration;

(a) Determine the slope based on the third derivative of the following function, given that x = 0.

i. $y = 3x^2 - 7x^3 + 9x^8$.

(5 marks)

ii. $y = \cos(6x) - 3sinx$.

(5 marks)

(b) Integrate the following with respect to x.

i. $\int \frac{x+2}{(x^2+4x+7)^3} dx$

(5 marks)

ii. $\int x^4 sinx\, dx$.

(5 marks)

Question 6

With reference to Calculations with Differentiation and Integration;

(a) Given the implicit equation below, use implicit differentiation to find $\frac{dy}{dx}$ and the value of gradient if given y = 1 when x = 1

$$2y^2 + 4x^3 = 6y$$

(10 marks)

(b) Find the area between the curves $y = x^2$ and $y = \sqrt{x}$ with respect to x axis.

(10 marks)

END OF EXAMINATION PAPER

TRIGONOMETRY IDENTITIES

FUNDAMENTAL IDENTITIES	FORMULAS FOR NEGATIVES
$\csc\theta = \frac{1}{\sin\theta}$	$\sin(-\theta) = -\sin\theta$
$\sec\theta = \frac{1}{\cos\theta}$	$\cos(-\theta) = \cos\theta$
$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$	$\tan(-\theta) = -\tan\theta$
$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$	$\csc(-\theta) = -\csc\theta$
$\sin^2\theta + \cos^2\theta = 1$	$\sec(-\theta) = \sec\theta$
$1+\tan^2\theta = \sec^2\theta$	$\cot(-\theta) = -\cot\theta$
$1+\cot^2\theta = \csc^2\theta$	$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

ADDITION FORMULAS	SUBTRACTION FORMULAS
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$	$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$

HALF-ANGLE FORMULAS	DOUBLE-ANGLE FORMULAS
$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$	$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ ....$= 1-2\sin^2\theta$ ....$= 2\cos^2\theta - 1$
$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$	$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

PRODUCT-TO-SUM FORMULAS	SUM-TO-PRODUCT FORMULAS
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$	$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$	$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$	$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

DIFFERENTIATION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x)=\cos x$	$\frac{d}{dx}(\sin f(x))=f'(x)\cos f(x)$
$\frac{d}{dx}(\cos x)=-\sin x$	$\frac{d}{dx}(\cos f(x))=-f'(x)\sin f(x)$
$\frac{d}{dx}(\tan x)=\sec^2 x$	$\frac{d}{dx}(\tan f(x))=f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc x)=-\csc x\cot x$	$\frac{d}{dx}(\csc f(x))=-f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec x)=\sec x\tan x$	$\frac{d}{dx}(\sec f(x))=f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot x)=-\csc^2 x$	$\frac{d}{dx}(\cot f(x))=-f'(x)\csc^2 f(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}e^x=e^x$	$\frac{d}{dx}e^{f(x)}=f'(x)e^{f(x)}$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}\ln x=\frac{1}{x}$	$\frac{d}{dx}\ln f(x)=\frac{f'(x)}{f(x)}$

INTEGRATION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int \cos x\,dx = \sin x + c$	$\int \cos f(x)dx = \dfrac{\sin f(x)}{f'(x)} + c$
$\int \sin x\,dx = -\cos x + c$	$\int \sin f(x)dx = \dfrac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x\,dx = \tan x + c$	$\int \sec^2 f(x)dx = \dfrac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x\,dx = \sec x + c$	$\int \sec f(x) \tan f(x)dx = \dfrac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x\,dx = -\csc x + c$	$\int \csc f(x) \cot f(x)dx = \dfrac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x\,dx = -\cot x + c$	$\int \csc^2 f(x)dx = \dfrac{-\cot f(x)}{f'(x)} + c$
$\int \tan x\,dx = \ln\lvert\sec x\rvert + c$	$\int \tan x\,dx = \dfrac{\ln\lvert\sec f(x)\rvert}{f'(x)} + c$
$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + c$	$\int \sec x\;dx = \dfrac{\ln\lvert\sec f(x) + \tan f(x)\rvert}{f'(x)} + c$
$\int \cot x\,dx = \ln\lvert\sin x\rvert + c$	$\int \cot x\,dx = \dfrac{\ln\lvert\sin f(x)\rvert}{f'(x)} + c$
$\int \csc x\,dx = -\ln\lvert\csc x + \cot x\rvert + c$	$\int \csc x dx = \dfrac{-\ln\lvert\csc f(x) + \cot f(x)\rvert}{f'(x)} + c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int e^x dx = e^x + c$	$\int e^{f(x)}\,dx = \dfrac{e^{f(x)}}{f'(x)} + c$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int \dfrac{1}{x} dx = \ln\lvert x\rvert + c$	$\int \dfrac{1}{f(x)} dx = \dfrac{\ln\lvert f(x)\rvert}{f'(x)} + c$

INVERSE TRIGONOMETRIC FUNCTION FUNCTION

$$\frac{d}{dx}\left[\sin^{-1}\left(\frac{x}{a}\right)\right] = \frac{1}{\sqrt{a^2 - x^2}}$$

$$\frac{d}{dx}\left[\cos^{-1}\left(\frac{x}{a}\right)\right] = -\frac{1}{\sqrt{a^2 - x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1}\left(\frac{x}{a}\right)\right] = \frac{a}{a^2 + x^2}$$

$$\frac{d}{dx}\left[\csc^{-1}\left(\frac{x}{a}\right)\right] = -\frac{a}{x\sqrt{x^2 - a^2}}$$

$$\frac{d}{dx}\left[\sec^{-1}\left(\frac{x}{a}\right)\right] = \frac{a}{x\sqrt{x^2 - a^2}}$$

$$\frac{d}{dx}\left[\cot^{-1}\left(\frac{x}{a}\right)\right] = -\frac{a}{a^2 + x^2}$$

FIRST AND SECOND ORDER DIFFERENTIAL EQUATION

If the roots of the auxiliary equation are:

(i) **real and different**, say $m = \alpha$ and $m = \beta$, then the general solution is

$$y = Ae^{\alpha x} + Be^{\beta x}$$

(ii) **real and equal, say** $m = \alpha$ twice, then the general solution is

$$y = (Ax + B)e^{\alpha x}$$

(iii) **complex**, say $m = \alpha \pm j\beta$, then the general solution is

$$y = e^{\alpha x}\{A\cos\beta x + B\sin\beta x\}$$

Table 51.1 Form of particular integral for different functions

Type	*Straightforward cases* Try as particular integral:	*'Snag' cases* Try as particular integral:	*See problem*
(a) $f(x)$ = a constant	$v=k$	$v=kx$ (used when C.F. contains a constant)	1, 2
(b) $f(x)$ = polynomial (i.e. $f(x)=L+Mx+Nx^2+\cdots$ where any of the coefficients may be zero)	$v=a+bx+cx^2+\cdots$		3
(c) $f(x)$ = an exponential function (i.e. $f(x)=Ae^{ax}$)	$v=ke^{ax}$	(i) $v=kxe^{ax}$ (used when e^{ax} appears in the C.F.)	4, 5
		(ii) $v=kx^2e^{ax}$ (used when e^{ax} and xe^{ax} both appear in the C.F.)	6
(d) $f(x)$ = a sine or cosine function (i.e. $f(x)=a\sin px+b\cos px$, where a or b may be zero)	$v=A\sin px+B\cos px$	$v=x(A\sin px+B\cos px)$ (used when $\sin px$ and/or $\cos px$ appears in the C.F.)	7, 8
(e) $f(x)$ = a sum e.g.			9
(i) $f(x)=4x^2-3\sin 2x$	(i) $v=ax^2+bx+c+d\sin 2x+e\cos 2x$		
(ii) $f(x)=2-x+e^{3x}$	(ii) $v=ax+b+ce^{3x}$		
(f) $f(x)$ = a product e.g. $f(x)=2e^x\cos 2x$	$v=e^x(A\sin 2x+B\cos 2x)$		10

Table of Laplace Transforms

	$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$		$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\ n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at},\ n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	e^{-cs}
27.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	28.	$u_c(t)g(t)$	$e^{-cs}\mathcal{L}\{g(t+c)\}$
29.	$e^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^{\infty}F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

STATISTICS

Means:

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

Modes:

$$= L + \left[\frac{a}{a+b}\right]c$$

Median:

$$= L + \left[\frac{\frac{N}{2} - f_L}{f_m}\right]c$$

Ungroup data:

Variance:

$$s^2 = \frac{\sum_i (x_i - \bar{x})^2}{n-1}$$

or

$$s^2 = \frac{\sum_i x_i^2 - \frac{\left(\sum_i x_i\right)^2}{n}}{n-1}$$

Standard deviation

$$s = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}$$

Grouped data:

Variance:

$$s^2 = \frac{\left(\sum_{i=1}^{n} x_i^2 f\right) - \frac{\left(\sum_{i=1}^{n} x_i f\right)^2}{n}}{n-1}$$

Standard Deviation:

$$s = \sqrt{\frac{\left(\sum_{i=1}^{n} x_i^2 f\right) - \frac{\left(\sum_{i=1}^{n} x_i f\right)^2}{n}}{n-1}}$$