UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF MARINE ENGINEERING TECHNOLOGY

FINAL EXAMINATION
JANUARY 2017 SEMESTER

COURSE CODE	:	LGB22003
COURSE NAME	:	MATHEMATICS 3
PROGRAMME NAME (FOR MPU: PROGRAMME LEVEL)	:	BACHELOR OF ENGINEERING TECHNOLOGY (HONS) IN MARINE ELECTRICAL AND ELECTRONIC
DATE	:	04/07/2017 TUE
TIME	:	2.00 PM - 05.00 PM
DURATION	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please read CAREFULLY the instructions given in the question paper.
2. This question paper has information printed on both sides.
3. This question paper consists of TWO (2) sections; Section A and Section B. Answer ALL questions in Section A and THREE (3) questions from Section B.
4. Please write yours answers on the answer booklet provided.
5. Write your answers only in BLACK or BLUE ink.
6. Answer all questions in English.

THERE ARE 6 PAGES OF QUESTIONS, INCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL FIVE questions.
Please use the answer booklet provided.

Question 1 (CLO 1)

(a) Differentiate $z = x^4 \sin(xy^3)$ with respect to x and y respectively for first order partial derivatives.

(4 marks)

(b) Determine $f_x(1,3)$ and $f_y(1,3)$ for the function $f(x,y) = 2x^3y^2 + 2y + 4x.$

(4 marks)

Question 2 (CLO 2)

(a) Describe ONE (1) application of convolution in signal and image processing respectively.

(4 marks)

(b) Let $f(t) = e^{3t}$ and $g(t) = e^{7t}$. Solve $f(t) * g(t)$.

(4 marks)

Question 3 (CLO 3)

Determine the real part and imaginary part for the following complex functions.

(a) $f(z) = z^2 + 6z$

(4 marks)

(b) $f(z) = 4e^{3z}$

(4 marks)

Question 4 (CLO 4)

(a) List TWO (2) properties of z-transform.

(4 marks)

(b) Solve $F(z) = \dfrac{z+1}{z^2 + 0.2z + 0.1}$ using long division method.

(4 marks)

Question 5 (CLO 5)

(a) Distinguish the periodic and non-periodic function in Fourier series.

(4 marks)

(b) Determine the Fourier Transform, $X(\omega)$ if given $x(t) = e^{-3|t|}\sin 2t$.

(4 marks)

SECTION B (Total: 60 marks)

INSTRUCTION: Total questions are FIVE but answer only THREE questions.
Please use the answer booklet provided.

Question 6 (CLO 1)

Determine the solution of the initial-boundary value problem:

$$u_{tt} = 9u_{xx}, \quad 0 < x < \infty, \quad t > 0$$
$$u(x,0) = 0, \quad 0 < x < \infty$$
$$u_t(x,0) = x^3, \quad 0 < x < \infty$$
$$u_x(x,t) = 0, \quad 0 < t < \infty$$

(20 marks)

Question 7 (CLO 2)

Based on Figure 1:

(a) Determine the range for convolution process

(3 marks)

(b) Apply graphical convolution to show process of overlapping signals between the waveform of $x(t)$ and $h(t)$. Use Appendix 1 for the answers. [Hint: Use $h(t)$ as moving signal].

(12 marks)

(c) Sketch and label the result of convolution

(5 marks)

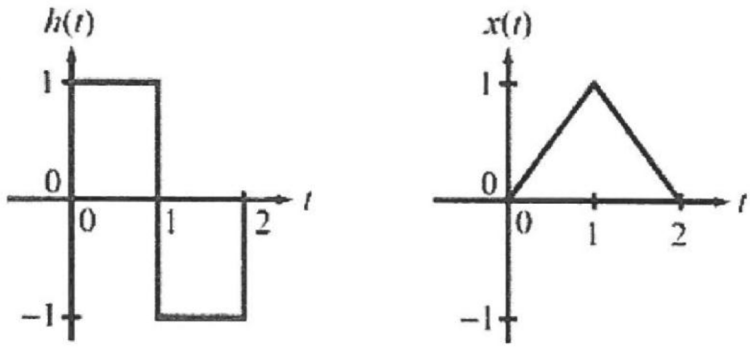


Figure 1

Question 8 (CLO 3)

(a) Given $g(z)=z^2+2$ and $f(z)=z^2+1$. Show that $f(z)+g(z)$ satisfies the Cauchy Riemann equation which is analytic in some region in complex analysis.

(8 marks)

(b) Calculate the contour integral $\oint_C \frac{3}{z(z^2+4)}dz$ if C is:

i. the circle $|z|=0.5$

ii. the circle $|z|=1$

(12 marks)

Question 9 (CLO 4)

(a) Obtain the inverse z-transform of the function $F(z)=\frac{1}{z^2(z-0.5)}$.

(10 marks)

(b) Solve the linear difference equation $x(k+2)-\left(\frac{3}{2}\right)x(k+1)+\left(\frac{1}{2}\right)x(k)=1(k)$ with the initial conditions $x(0)=1, x(1)=\frac{5}{2}$.

(10 marks)

Question 10 (CLO 5)

(a) Determine the Fourier Transform of $f(t) = 3te^{-6t}$ by using definition method for the range between $0 < t < \infty$.

(10 marks)

(b) Obtain a Fourier series for the periodic square wave function $f(x)$ defined as shown as:

$$f(x) = \begin{cases} -A, & when \quad -\pi < x < 0 \\ +A, & when \quad 0 < x < \pi \end{cases}$$

The function is periodic outside of this range with period 2π.

(10 marks)

END OF QUESTIONS PAPER

TRIGONOMETRY IDENTITIES

FUNDAMENTAL IDENTITIES	FORMULAS FOR NEGATIVES
$\csc\theta = \dfrac{1}{\sin\theta}$	$\sin(-\theta) = -\sin\theta$
$\sec\theta = \dfrac{1}{\cos\theta}$	$\cos(-\theta) = \cos\theta$
$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$	$\tan(-\theta) = -\tan\theta$
$\cos^2\theta = \dfrac{1}{2}(1+\cos 2\theta)$	$\csc(-\theta) = -\csc\theta$
$\sin^2\theta + \cos^2\theta = 1$	$\sec(-\theta) = \sec\theta$
$1+\tan^2\theta = \sec^2\theta$	$\cot(-\theta) = -\cot\theta$
$1+\cot^2\theta = \csc^2\theta$	$\sin^2\theta = \dfrac{1}{2}(1-\cos 2\theta)$

ADDITION FORMULAS	SUBTRACTION FORMULAS
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A\tan B}$	$\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\tan B}$

HALF-ANGLE FORMULAS	DOUBLE-ANGLE FORMULAS
$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$	$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ .... $= 1-2\sin^2\theta$ .... $= 2\cos^2\theta - 1$
$\tan\dfrac{\theta}{2} = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$	$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

PRODUCT-TO-SUM FORMULAS	SUM-TO-PRODUCT FORMULAS
$\sin\alpha\cos\beta = \dfrac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$	$\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$
$\cos\alpha\sin\beta = \dfrac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$	$\sin\alpha - \sin\beta = 2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$
$\cos\alpha\cos\beta = \dfrac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$	$\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

DIFFERENTIATION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$

INTEGRATION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \cos x\,dx = \sin x + c$	$\int \cos f(x)dx = \frac{\sin f(x)}{f'(x)} + c$
$\int \sin x\,dx = -\cos x + c$	$\int \sin f(x)dx = \frac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x\,dx = \tan x + c$	$\int \sec^2 f(x)dx = \frac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x\,dx = \sec x + c$	$\int \sec f(x) \tan f(x)dx = \frac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x\,dx = -\csc x + c$	$\int \csc f(x) \cot f(x)dx = \frac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x\,dx = -\cot x + c$	$\int \csc^2 f(x)dx = \frac{-\cot f(x)}{f'(x)} + c$
$\int \tan x\,dx = \ln\lvert\sec x\rvert + c$	$\int \tan x\,dx = \frac{\ln\lvert\sec f(x)\rvert}{f'(x)} + c$
$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + c$	$\int \sec x\ dx = \frac{\ln\lvert\sec f(x) + \tan f(x)\rvert}{f'(x)} + c$
$\int \cot x\,dx = \ln\lvert\sin x\rvert + c$	$\int \cot x\,dx = \frac{\ln\lvert\sin f(x)\rvert}{f'(x)} + c$
$\int \csc x\,dx = -\ln\lvert\csc x + \cot x\rvert + c$	$\int \csc x dx = \frac{-\ln\lvert\csc f(x) + \cot f(x)\rvert}{f'(x)} + c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int e^x dx = e^x + c$	$\int e^{f(x)}\,dx = \frac{e^{f(x)}}{f'(x)} + c$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \frac{1}{x}dx = \ln\lvert x\rvert + c$	$\int \frac{1}{f(x)}dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + c$

LAPLACE TRANSFORM

	$f(t)$	$F(s) = \mathfrak{L}[f(t)](s)$
1.	1	$\frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	$t^n (n = 1, 2, 3, \cdots)$	$\frac{n!}{s^{n+1}}$
4.	$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
5.	e^{at}	$\frac{1}{s-a}$
6.	te^{at}	$\frac{1}{(s-a)^2}$
7.	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
8.	$\frac{1}{a-b}(e^{at} - e^{bt})$	$\frac{1}{(s-a)(s-b)}$
9.	$\frac{1}{a-b}(ae^{at} - be^{bt})$	$\frac{s}{(s-a)(s-b)}$
10.	$\frac{(c-b)e^{at} + (a-c)e^{bt} + (b-a)e^{ct}}{(a-b)(b-c)(c-a)}$	$\frac{1}{(s-a)(s-b)(s-c)}$
11.	$\sin(at)$	$\frac{a}{s^2+a^2}$
12.	$\cos(at)$	$\frac{s}{s^2+a^2}$
13.	$1 - \cos(at)$	$\frac{a^2}{s(s^2+a^2)}$
14.	$at - \sin(at)$	$\frac{a^3}{s^2(s^2+a^2)}$
15.	$\sin(at) - at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
16	$\sin(at) + at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
17.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$
18.	$t\cos(at)$	$\frac{(s^2-a^2)}{(s^2+a^2)^2}$
19.	$\frac{\cos(at) - \cos(bt)}{(b-a)(b+a)}$	$\frac{s}{(s^2+a^2)(s^2+b^2)}$
20.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
21.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
22.	$\sinh(at)$	$\frac{a}{s^2-a^2}$
23.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
24.	$\sin(at)\cosh(at) - \cos(at)\sinh(at)$	$\frac{4a^3}{s^4+4a^4}$
25.	$\sin(at)\sinh(at)$	$\frac{2a^2 s}{s^4+4a^4}$

	$f(t)$	$F(s) = \mathcal{L}[f(t)](s)$
26.	$\sinh(at) - \sin(at)$	$\frac{2a^3}{s^4 - a^4}$
27.	$\cosh(at) - \cos(at)$	$\frac{2a^2 s}{s^4 - a^4}$
28.	$\frac{1}{\sqrt{\pi t}} e^{at}(1 + 2at)$	$\frac{s}{(s-a)^{3/2}}$
29.	$J_0(at)$	$\frac{1}{\sqrt{s^2 + a^2}}$
30.	$J_n(at)$	$\frac{1}{a^n} \frac{\left(\sqrt{s^2 + a^2} - s\right)^n}{\sqrt{s^2 + a^2}}$
31.	$J_0(2\sqrt{at})$	$\frac{1}{s} e^{-a/s}$
32.	$\frac{1}{t} \sin(at)$	$\tan^{-1}\left(\frac{a}{s}\right)$
33.	$\frac{2}{t}[1 - \cos(at)]$	$\ln\left(\frac{s^2 + a^2}{s^2}\right)$
34.	$\frac{2}{t}[1 - \cosh(at)]$	$\ln\left(\frac{s^2 - a^2}{s^2}\right)$
35.	$\frac{1}{\sqrt{\pi t}} - a e^{a^2 t} \operatorname{erfc}\left(\frac{a}{\sqrt{t}}\right)$	$\frac{1}{\sqrt{s} + a}$
36.	$\frac{1}{\sqrt{\pi t}} + a e^{a^2 t} \operatorname{erf}\left(\frac{a}{\sqrt{t}}\right)$	$\frac{\sqrt{s}}{s - a^2}$
37.	$e^{a^2 t} \operatorname{erf}(a\sqrt{t})$	$\frac{a}{\sqrt{s}(s - a^2)}$
38.	$e^{a^2 t} \operatorname{erfc}(a\sqrt{t})$	$\frac{1}{\sqrt{s}(\sqrt{s} + a)}$
39.	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{1}{s} e^{-a\sqrt{s}}$
40.	$\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\frac{1}{\sqrt{s}} e^{-a\sqrt{s}}$
41.	$\frac{1}{\sqrt{\pi(t + a)}}$	$\frac{1}{\sqrt{s}} e^{as} \operatorname{erfc}(\sqrt{as})$
42.	$\frac{1}{\pi t} \sin(2a\sqrt{t})$	$\operatorname{erf}\left(\frac{a}{\sqrt{s}}\right)$
43.	$f\left(\frac{t}{a}\right)$	$aF(as)$
44.	$e^{bt/a} f\left(\frac{t}{a}\right)$	$aF(as - b)$
45.	$\delta_\epsilon(t)$	$\frac{e^{-\epsilon s}(1 - e^{-\epsilon s})}{\epsilon s}$
46.	$\delta(t - a)$	e^{-as}
47.	$L_n(t)$ (Laguerre polynomial)	$\frac{1}{s}\left(\frac{s-1}{s}\right)^n$

FOURIER TRANSFORM

Entry	$x(t)$	$X(f)$	$X(\omega)$
1	$\delta(t)$	1	1
2	$\text{rect}(t)$	$\text{sinc}(f)$	$\text{sinc}\left(\frac{\omega}{2\pi}\right)$
3	$\text{tri}(t)$	$\text{sinc}^2(f)$	$\text{sinc}^2\left(\frac{\omega}{2\pi}\right)$
4	$\text{sinc}(t)$	$\text{rect}(f)$	$\text{rect}\left(\frac{\omega}{2\pi}\right)$
5	$\cos(2\pi\alpha t)$	$0.5[\delta(f+\alpha)+\delta(f-\alpha)]$	$\pi[\delta(\omega+2\pi\alpha)+\delta(\omega-2\pi\alpha)]$
6	$\sin(2\pi\alpha t)$	$j0.5[\delta(f+\alpha)-\delta(f-\alpha)]$	$j\pi[\delta(\omega+2\pi\alpha)-\delta(\omega-2\pi\alpha)]$
7	$e^{-\alpha t}u(t)$	$\frac{1}{\alpha+j2\pi f}$	$\frac{1}{\alpha+j\omega}$
8	$te^{-\alpha t}u(t)$	$\frac{1}{(\alpha+j2\pi f)^2}$	$\frac{1}{(\alpha+j\omega)^2}$
9	$e^{-\alpha\|t\|}$	$\frac{2\alpha}{\alpha^2+4\pi^2 f^2}$	$\frac{2\alpha}{\alpha^2+\omega^2}$
10	$e^{-\pi t^2}$	$e^{-\pi f^2}$	$e^{-\omega^2/4\pi}$
11	$\text{sgn}(t)$	$\frac{1}{j\pi f}$	$\frac{2}{j\omega}$
12	$u(t)$	$0.5\delta(f)+\frac{1}{j2\pi f}$	$\pi\delta(\omega)+\frac{1}{j\omega}$
13	$e^{-\alpha t}\cos(2\pi\beta t)u(t)$	$\frac{\alpha+j2\pi f}{(\alpha+j2\pi f)^2+(2\pi\beta)^2}$	$\frac{\alpha+j\omega}{(\alpha+j\omega)^2+(2\pi\beta)^2}$
14	$e^{-\alpha t}\sin(2\pi\beta t)u(t)$	$\frac{2\pi\beta}{(\alpha+j2\pi f)^2+(2\pi\beta)^2}$	$\frac{2\pi\beta}{(\alpha+j\omega)^2+(2\pi\beta)^2}$
15	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$
16	$x_p(t)=\sum_{k=-\infty}^{\infty}X[k]e^{j2\pi k f_0 t}$	$\sum_{k=-\infty}^{\infty}X[k]\delta(f-kf_0)$	$\sum_{k=-\infty}^{\infty}2\pi X[k]\delta(\omega-k\omega_0)$

Property	$x(t)$	$X(f)$	$X(\omega)$
Similarity	$X(t)$	$x(-f)$	$2\pi x(-\omega)$
Time Scaling	$x(\alpha t)$	$\frac{1}{\lvert\alpha\rvert}X\left(\frac{f}{\alpha}\right)$	$\frac{1}{\lvert\alpha\rvert}X\left(\frac{\omega}{\alpha}\right)$
Folding	$x(-t)$	$X(-f)$	$X(-\omega)$
Time Shift	$x(t-\alpha)$	$e^{-j2\pi f\alpha}X(f)$	$e^{-j\omega\alpha}X(\omega)$
Frequency Shift	$e^{j2\pi\alpha t}x(t)$	$X(f-\alpha)$	$X(\omega-2\pi\alpha)$
Convolution	$x(t)\star h(t)$	$X(f)H(f)$	$X(\omega)H(\omega)$
Multiplication	$x(t)h(t)$	$X(f)\star H(f)$	$\frac{1}{2\pi}X(\omega)\star H(\omega)$
Modulation	$x(t)\cos(2\pi\alpha t)$	$0.5[X(f+\alpha)+X(f-\alpha)]$	$0.5[X(\omega+2\pi\alpha)+X(\omega-2\pi\alpha)]$
Derivative	$x'(t)$	$j2\pi fX(f)$	$j\omega X(\omega)$
Times-t	$-j2\pi tx(t)$	$X'(f)$	$2\pi X'(\omega)$
Integration	$\int_{-\infty}^{t} x(t)\,dt$	$\frac{1}{j2\pi f}X(f)+0.5X(0)\delta(f)$	$\frac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$
Conjugation	$x^*(t)$	$X^*(-f)$	$X^*(-\omega)$
Correlation	$x(t)\star\star y(t)$	$X(f)Y^*(f)$	$X(\omega)Y^*(\omega)$
Autocorrelation	$x(t)\star\star x(t)$	$X(f)X^*(f)=\lvert X(f)\rvert^2$	$X(\omega)X^*(\omega)=\lvert X(\omega)\rvert^2$

Fourier Transform Theorems	
Central ordinates	$x(0)=\int_{-\infty}^{\infty}X(f)\,df=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)\,d\omega \qquad X(0)=\int_{-\infty}^{\infty}x(t)\,dt$
Parseval's theorem	$E=\int_{-\infty}^{\infty}x^2(t)\,dt=\int_{-\infty}^{\infty}\lvert X(f)\rvert^2\,df=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2\,d\omega$
Plancherel's theorem	$\int_{-\infty}^{\infty}x(t)y^*(t)\,dt=\int_{-\infty}^{\infty}X(f)Y^*(f)\,df=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)Y^*(\omega)\,d\omega$

COMPLEX ANALYSIS

$$\int_{\Gamma} f(z)dz = \int_{a}^{b} f(\Gamma(t))\Gamma'(t)dt$$

$$\oint_{c} \frac{f(z)}{z - z_0} dz = 2\pi i f(z_0)$$

Residue at pole z $= \lim_{z \to z_0} [(z - z_0)f(z)]$

Residue at pole z $= \frac{1}{(m-1)!} \lim_{z \to z_0} \left\{ \frac{d^{m-1}}{dz^{m-1}} \left[(z - z_0)^m f(z) \right] \right\}$

$$\oint_{c} f(z)dz = 2\pi j\left[\text{sum of residue of } f(z) \text{ at poles inside } C\right]$$

EULER'S FORMULAS

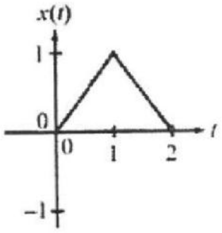
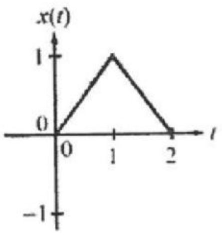
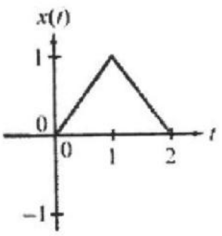
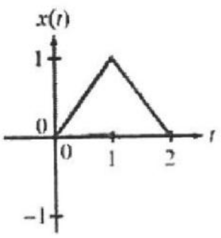
$$\cos x = \operatorname{Re}(e^{ix}) = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin x = \operatorname{Im}(e^{ix}) = \frac{e^{ix} - e^{-ix}}{2i}$$

TABLE OF Z-TRANSFORM

$x(t)$	$X(s)$	$X(z)$
1. $\delta(t) = \begin{cases} 1 & t = 0, \\ 0 & t = kT, k \neq 0 \end{cases}$	1	1
2. $\delta(t - kT) = \begin{cases} 1 & t = kT, \\ 0 & t \neq kT \end{cases}$	e^{-kTs}	z^{-k}
3. $u(t)$, unit step	$1/s$	$\frac{z}{z-1}$
4. t	$1/s^2$	$\frac{Tz}{(z-1)^2}$
5. t^2	$2/s^3$	$\frac{T^2 z(z+1)}{(z-1)^3}$
6. e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z - e^{-aT}}$
7. $1 - e^{-at}$	$\frac{a}{s(s+a)}$	$\frac{(1 - e^{-aT})z}{(z-1)(z - e^{-aT})}$
8. te^{-at}	$\frac{1}{(s+a)^2}$	$\frac{Tze^{-aT}}{(z - e^{-aT})^2}$
9. $t^2 e^{-at}$	$\frac{2}{(s+a)^3}$	$\frac{T^2 e^{-aT} z(z + e^{-aT})}{(z - e^{-aT})^3}$
10. $be^{-bt} - ae^{-at}$	$\frac{(b-a)s}{(s+a)(s+b)}$	$\frac{z[z(b-a) - (be^{-aT} - ae^{-bT})]}{(z - e^{-aT})(z - e^{-bT})}$
11. $\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$\frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$
12. $\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	$\frac{z(z - \cos \omega T)}{z^2 - 2z \cos \omega T + 1}$
13. $e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$	$\frac{(ze^{-aT} \sin \omega T)}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$

APPENDIX 1

Range and signal overlapping	Process Convolution (Calculation)	End Value
Range: 		
Range: 		
Range: 		
Range: 		
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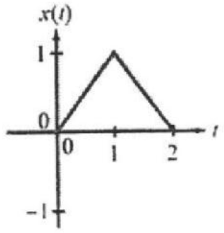
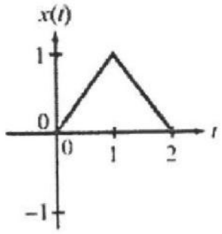
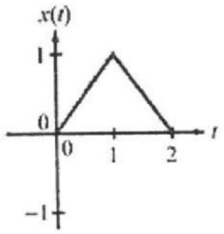
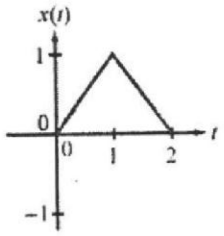
		
Range: 		
Range: 		
Range: 		

TABLE OF Z-TRANSFORM 2

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
$\delta[n-m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
$\begin{cases} a^n & 0 \le n \le N-1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\|z\| > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\|z\| > 1$
$r^n \cos(\omega_0 n)u[n]$	$\frac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2 z^{-2}}$	$\|z\| > r$

FOURIER SERIES

$$f(\theta) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\theta + \sum_{n=1}^{\infty} b_n \sin n\theta$$

Where for the range of $-\pi \le \theta \le \pi$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)d\theta$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\cos n\theta d\theta \quad n = 1, 2, \cdots$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\sin n\theta d\theta \quad n = 1, 2, \cdots$$

$$\cos(a)\cos(b) = \frac{1}{2}\big(\cos(a+b) + \cos(a-b)\big)$$

$$\sin(a)\sin(b) = \frac{1}{2}\big(\cos(a-b) - \cos(a+b)\big)$$

$$\sin(a)\cos(b) = \frac{1}{2}\big(\sin(a+b) + \sin(a-b)\big)$$

$$\cos(a)\sin(b) = \frac{1}{2}\big(\sin(a+b) - \sin(a-b)\big)$$