UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF MARINE ENGINEERING TECHNOLOGY

FINAL EXAMINATION
JANUARY 2017 SEMESTER

COURSE CODE	: LGB12303
COURSE NAME	: MATHEMATICS 2
PROGRAMME NAME (FOR MPU: PROGRAMME LEVEL)	: BACHELOR OF ENGINEERING TECHNOLOGY (HONS) IN MARINE ELECTRICAL AND ELECTRONIC
DATE	: 13/07/2017 THU
TIME	: 9.00 AM - 12.00 PM
DURATION	: 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please read CAREFULLY the instructions given in the question paper.
2. This question paper has information printed on both sides.
3. This question paper consists of TWO (2) sections; Section A and Section B. Answer ALL questions in Section A and THREE (3) questions from Section B.
4. Please write yours answers on the answer booklet provided.
5. Write your answers only in BLACK or BLUE ink.
6. Answer all questions in English.

THERE ARE 8 PAGES OF QUESTIONS, INCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL FIVE questions.
Please use the answer booklet provided.

Question 1

A box contains 24 resistors, 4 of which are defective. If 4 are sold at random, determine the following probabilities.

(a) Exactly 2 are defective.

(2 marks)

(b) None is defective.

(2 marks)

(c) All are defective.

(2 marks)

(d) At least 1 is defective.

(2 marks)

Question 2

(a) Determine the minimum number of terms that must be taken from the sequences 8, 16, 24, 32, ... so that the sum is more than 120.

(4 marks)

(b) Given that 15, s, b, c, d, e, 30 form an arithmetic progression, find the values of s, b, c, d and e.

(4 marks)

Question 3

Given the vectors $v_1 = 22$ units at $140°$, $v_2 = 40$ units at $190°$ and $v_1 = 15$ units at $290°$.

(a) Sketch the vectors

(2 marks)

(b) Calculate the resultant of $v_1 - v_2 + v_3$.

(6 marks)

Question 4

(a) State the definition of the following terms.

i. Linear first order differential equation

(1 marks)

ii. Homogeneous first order differential equation

(1 marks)

iii. General solution

(1 marks)

iv. Particular solution

(1 marks)

(b) Determine the general solution and particular solution by solving the differential equation $2t\left(t - \frac{d\theta}{dt}\right) = 5$ and given the boundary condition $\theta = 1$ when $t = 1$.

(4 marks)

Question 5

(a) Determine Laplace transform for $f(t) = 4t^2$ using definition of Laplace transform.

(4 marks)

(b) Solve the following Laplace transform of $f(t)$ and $g(t)$.

i. $f(t) = 3t^3 \cos 5t$.

(2 marks)

ii. $g(t) = 2\sin 2te^{3t}$.

(2 marks)

SECTION B (Total: 60 marks)

INSTRUCTION: Total questions are FIVE but answer only THREE questions. Please use the answer booklet provided.

Question 6

(a) A bank manager classifies customers into various categories; **A**: easy to deal with; **B**: a longstanding customer and **C**: has over RM10000 in the bank. The Venn diagram in Figure 1 below gives a visual display data for 55 customers together with the number of customers in each categories. Solve:

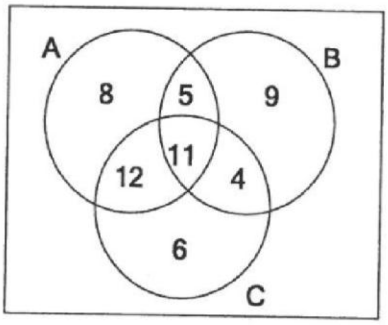

Figure 1

i. Number customers who are easy to deal with or are longstanding customer.

(2 marks)

ii. Number customers who are easy to deal with and have over RM10000 in the bank but are not longstanding customer.

(2 marks)

iii. Number customers who are easy to deal with or are longstanding customer and have over RM10000 in the bank.

(2 marks)

iv. Number of customers who are easy to deal with and have over RM10000 in the bank.

(2 marks)

(b) A clothing store has just imported a new range of jackets that it has advertised at a bargain price on a rack inside the store. The probability that a customer tries on a jacket is 0.40. If a customer tries on a jacket, the probability that they will buy it is 0.70. If a customer does not try on a jacket, the probability that they will buy is 0.15. Calculate the probability that:

i. A customer tries on a jacket and buy it.

(3 marks)

ii. A customer tries on a jacket and does not buy it.

(3 marks)

iii. A customer does not try on a jacket and buy it.

(3 marks)

iv. A customer does not try on a jacket and does not buy it.

(3 marks)

Question 7

(a) According to the new remuneration scheme, the starting pay of a technician is RM1250 per month and the annual increment is RM108. Kim who is 21 years old become technician in factory.

i. What will his monthly salary be when he is 63 years old?

(6 marks)

ii. What will his estimate age be when he gets a monthly payment of RM3500?

(6 marks)

(b) The seventh term of geometric sequence is 32 and the last term is 512. If the first term is $\frac{1}{2}$, find the number of terms and the sum of the progression.

(8 marks)

Question 8

(a) If $p = 4i + j - 2k$, $q = 3i - 2j + k$ and $r = i - 2k$, determine:

i. $(p - 2q) x r$

(5 marks)

ii. $p x (2r x 3q)$

(5 marks)

(b) Determine the moment and the magnitude of the moment of a force of $(i + 2j - 3k)$ newtons about the point **B** having coordinates $(0, 1, 1)$, when force acts on a line through **A** whose coordinates are $(1, 3, 4)$.

(10 marks)

Question 9

(a) Determine the general solution and particular solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 4$ when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 0$.

(10 marks)

(b) The oscillations of heavily dumped pendulum satisfy the differential equation $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 0$ where x cm is the displacement at time t seconds. The initial displacement $(t = 0)$ is equal to 4 cm and initial velocity, $\frac{dx}{dt} = 8$ cm/s. Solve the equation for x.

(10 marks)

Question 10

(a) Determine the Laplace transform of the causal function f(t) defined by:

$$f(t)=\begin{cases}3t^2 & (0\le t<4)\\ 2t-3 & (4\le t<6)\\ 5 & (t\ge 6)\end{cases}$$

(10 marks)

(b) Determine the inverse of Laplace transform $Y(s)=\dfrac{3s-137}{s^2+2s+401}$

(10 marks)

END OF EXAMINATION PAPERS

TRIGONOMETRY IDENTITIES

FUNDAMENTAL IDENTITIES	FORMULAS FOR NEGATIVES
$\csc\theta = \dfrac{1}{\sin\theta}$	$\sin(-\theta) = -\sin\theta$
$\sec\theta = \dfrac{1}{\cos\theta}$	$\cos(-\theta) = \cos\theta$
$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$	$\tan(-\theta) = -\tan\theta$
$\cos^2\theta = \dfrac{1}{2}(1+\cos 2\theta)$	$\csc(-\theta) = -\csc\theta$
$\sin^2\theta + \cos^2\theta = 1$	$\sec(-\theta) = \sec\theta$
$1+\tan^2\theta = \sec^2\theta$	$\cot(-\theta) = -\cot\theta$
$1+\cot^2\theta = \csc^2\theta$	$\sin^2\theta = \dfrac{1}{2}(1-\cos 2\theta)$

ADDITION FORMULAS	SUBTRACTION FORMULAS
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A\tan B}$	$\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\tan B}$

HALF-ANGLE FORMULAS	DOUBLE-ANGLE FORMULAS
$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$	$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\ldots\ldots\ldots = 1-2\sin^2\theta$ $\ldots\ldots\ldots = 2\cos^2\theta - 1$
$\tan\dfrac{\theta}{2} = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$	$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

PRODUCT-TO-SUM FORMULAS	SUM-TO-PRODUCT FORMULAS
$\sin\alpha\cos\beta = \dfrac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$	$\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$
$\cos\alpha\sin\beta = \dfrac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$	$\sin\alpha - \sin\beta = 2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$
$\cos\alpha\cos\beta = \dfrac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$	$\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

DIFFERENTIATION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x)=\cos x$	$\frac{d}{dx}(\sin f(x))=f'(x)\cos f(x)$
$\frac{d}{dx}(\cos x)=-\sin x$	$\frac{d}{dx}(\cos f(x))=-f'(x)\sin f(x)$
$\frac{d}{dx}(\tan x)=\sec^2 x$	$\frac{d}{dx}(\tan f(x))=f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc x)=-\csc x\cot x$	$\frac{d}{dx}(\csc f(x))=-f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec x)=\sec x\tan x$	$\frac{d}{dx}(\sec f(x))=f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot x)=-\csc^2 x$	$\frac{d}{dx}(\cot f(x))=-f'(x)\csc^2 f(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}e^x=e^x$	$\frac{d}{dx}e^{f(x)}=f'(x)e^{f(x)}$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}\ln x=\frac{1}{x}$	$\frac{d}{dx}\ln f(x)=\frac{f'(x)}{f(x)}$

INTEGRATION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int \cos x\,dx = \sin x + c$	$\int \cos f(x)dx = \frac{\sin f(x)}{f'(x)} + c$
$\int \sin x\,dx = -\cos x + c$	$\int \sin f(x)dx = \frac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x\,dx = \tan x + c$	$\int \sec^2 f(x)dx = \frac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x\,dx = \sec x + c$	$\int \sec f(x) \tan f(x)dx = \frac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x\,dx = -\csc x + c$	$\int \csc f(x) \cot f(x)dx = \frac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x\,dx = -\cot x + c$	$\int \csc^2 f(x)dx = \frac{-\cot f(x)}{f'(x)} + c$
$\int \tan x\,dx = \ln\lvert\sec x\rvert + c$	$\int \tan x\,dx = \frac{\ln\lvert\sec f(x)\rvert}{f'(x)} + c$
$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + c$	$\int \sec x\ dx = \frac{\ln\lvert\sec f(x) + \tan f(x)\rvert}{f'(x)} + c$
$\int \cot x\,dx = \ln\lvert\sin x\rvert + c$	$\int \cot x\,dx = \frac{\ln\lvert\sin f(x)\rvert}{f'(x)} + c$
$\int \csc x\,dx = -\ln\lvert\csc x + \cot x\rvert + c$	$\int \csc x dx = \frac{-\ln\lvert\csc f(x) + \cot f(x)\rvert}{f'(x)} + c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int e^x dx = e^x + c$	$\int e^{f(x)}\,dx = \frac{e^{f(x)}}{f'(x)} + c$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$	$\int \frac{1}{f(x)} dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + c$

LAPLACE TRANSFORM

$Laplace\ transforms - Table$			
$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
e^{-at}	$\frac{1}{s+a}$	$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2 + \omega^2}$
$\frac{1}{2}t^2 e^{-at}$	$\frac{1}{(s+a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$
$\frac{1}{(n-1)!} t^{n-1} e^{-at}$	$\frac{1}{(s+a)^n}$	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2} \quad s > \|\omega\|$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2} \quad s > \|\omega\|$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1 + at)]$	$\frac{1}{s(s+a)^2}$	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n = 1,2,3..$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s} F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$	$\delta(t)$ *unit impulse*	$1 \quad$ *all s*
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2 f}{df^2}$	$s^2 F(s) - sf(0) - f'(0)$
$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) - \cdots - f^{n-1}(0)$		

FIRST AND SECOND ORDER DIFFERENTIAL EQUATION

If the roots of the auxiliary equation are:

(i) **real and different**, say $m = \alpha$ and $m = \beta$, then the general solution is

$$y = Ae^{\alpha x} + Be^{\beta x}$$

(ii) **real and equal, say** $m = \alpha$ twice, then the general solution is

$$y = (Ax + B)e^{\alpha x}$$

(iii) **complex**, say $m = \alpha \pm j\beta$, then the general solution is

$$y = e^{\alpha x}\{A \cos \beta x + B \sin \beta x\}$$

Table 51.1 Form of particular integral for different functions

Type	*Straightforward cases* Try as particular integral:	*'Snag' cases* Try as particular integral:	*See problem*
(a) $f(x)=$ a constant	$v=k$	$v=kx$ (used when C.F. contains a constant)	1, 2
(b) $f(x)=$ polynomial (i.e. $f(x)=L+Mx+Nx^2+\cdots$ where any of the coefficients may be zero)	$v=a+bx+cx^2+\cdots$		3
(c) $f(x)=$ an exponential function (i.e. $f(x)=Ae^{ax}$)	$v=ke^{ax}$	(i) $v=kxe^{ax}$ (used when e^{ax} appears in the C.F.)	4, 5
		(ii) $v=kx^2e^{ax}$ (used when e^{ax} and xe^{ax} both appear in the C.F.)	6
(d) $f(x)=$ a sine or cosine function (i.e. $f(x)=a\sin px+b\cos px$, where a or b may be zero)	$v=A\sin px+B\cos px$	$v=x(A\sin px+B\cos px)$ (used when $\sin px$ and/or $\cos px$ appears in the C.F.)	7, 8
(e) $f(x)=$ a sum e.g.			9
(i) $f(x)=4x^2-3\sin 2x$	(i) $v=ax^2+bx+c+d\sin 2x+e\cos 2x$		
(ii) $f(x)=2-x+e^{3x}$	(ii) $v=ax+b+ce^{3x}$		
(f) $f(x)=$ a product e.g. $f(x)=2e^x\cos 2x$	$v=e^x(A\sin 2x+B\cos 2x)$		10