UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF MARINE ENGINEERING TECHNOLOGY

FINAL EXAMINATION
JANUARY 2017 SEMESTER

COURSE CODE	: LGB10403
COURSE NAME	: ENGINEERING MATHEMATICS 2
PROGRAMME NAME (FOR MPU: PROGRAMME LEVEL)	: BACHELOR OF ENGINEERING TECHNOLOGY (HONS) IN MARINE ENGINEERING BACHELOR OF ENGINEERING TECHNOLOGY (HONS) IN NAVAL ARCHITECTURE & SHIPBUILDING
DATE	: 03/07/2017 MON
TIME	: 9.00 AM - 12.00 PM
DURATION	: 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please read CAREFULLY the instructions given in the question paper.
2. This question paper has information printed on both sides.
3. This question paper consists of TWO (2) sections; Section A and Section B. Answer ALL questions in Section A and THREE (3) questions from Section B.
4. Please write yours answers on the answer booklet provided.
5. Write your answers only in BLACK or BLUE ink.
6. Answer all questions in English.

THERE ARE 6 PAGES OF QUESTIONS, INCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALLFIVE questions.
Please use the answer booklet provided.

Question 1

(a) Differentiate $5y-6xy=y^2+6$ with respect to x and y respectively using implicit differentiation.

(4 marks)

(b) If $y=\tan^{-1}\frac{3}{t^2}$, determine $\frac{dy}{dt}$.

(4 marks)

Question 2

(a) Solve $\int \sin^4\theta d\theta$.

(3 marks)

(b) Let $f(x)=\begin{cases} x-2, & x\geq 2 \\ 2-x, & x<2 \end{cases}$ and $g(x)=x(3-x)$. Determine the integration $\int_0^5 (f(x)+g(x))dx.$

(5 marks)

Question 3

(a) Determine the real part and imaginary part for $\frac{MP}{N}$ if given $M=4-5i, N=2+6i$ and $P=3i$

(4 marks)

(b) Express $z=-3-4i$ in exponential form.

(4 marks)

Question 4

(a) Distinguish homogeneous second order differential equation and nonhomogeneous second order differential equation.

(2 marks)

(b) Solve the differential equation $\frac{dy}{d\theta} = \sec\theta + y\tan\theta$ using linear first order differential equation, given the boundary condition $y = 1$ when $\theta = 0$.

(6 marks)

Question 5

(a) Show that by using definition of Laplace transform for $f(t) = 5e^{4t}$ is $\frac{5}{s-4}$.

(4 marks)

(b) Solve the following Laplace transform using of first shifting properties.

i. $f(t) = 5e^{-3t}\sin 2t$

(2 marks)

ii. $g(t) = 2t^4e^{3t}$

(2 marks)

SECTION B (Total: 60 marks)

INSTRUCTION: Total questions are FIVE but answer only THREE questions.
Please use the answer booklet provided.

Question 6

(a) A conical water tank with vertex down has a radius of 10 feet at the top and is 24 feet high. If water flows out of the tank at a rate of $20 ft^3/\text{min}$, how fast is the depth of the water decreasing when the water is 16 feet dep? Hint: $v = \frac{1}{3}\pi r^2 h$.

(10 marks)

(b) Given a closed triangle box with volume of $72m^3$. The length of the box is twice its width. Determine the minimum area of the box.

(10 marks)

Question 7

(a) Evaluate the definite integrals $\int_1^4 \sqrt{1+x^3}\,dx$ using

i. Simpson's Rule.

ii. Trapezoidal Theorem.

In each of the approximate methods use 6 intervals and give the answer correct to 3 decimal places

(10 marks)

(b) Determine the area bounded the given curves, $f(x) = x^2 + 3$ and $g(x) = 7 - 3x$.

(10 marks)

Question 8

(a) A delta-connected impedance Z_A in ohm unit (electronic unit) is given by $Z_A = Z_1Z_2 + Z_1Z_2Z_3 + \frac{Z_1}{Z_3}$. Determine Z_A in both trigonometry and polar form if given $Z_1 = (1-3j), Z_2 = (-2-5j)$ and $Z_3 = (-3-j4)$.

(10 marks)

(b) Use De Moivre's theorem to determine:

i. $[-2+j3]^6$

ii. $\sqrt{5+j12}$

Leave the answer in polar form.

(10 marks)

Question 9

(a) Determine the general solution and particular solution of $9\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 3x - 1$ when $x = 0,\ y = 0$ and $\frac{dy}{dx} = -\frac{4}{3}$.

(12 marks)

(b) $L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$ is equation representing in electric circuit. If inductance L is 0.25, capacitance C is 29.76×10^{-6} farads and R is 250 ohms, solve the equation for i given the boundary condition that when $t = 0,\ i = 0$ and $\frac{di}{dt} = 34$.

(8 marks)

Question 10

(a) Use Laplace transform to solve the differential equation $2\frac{d^2y}{dx^2}+5\frac{dy}{dx}-3y=0$, given that when $y(0)=4$ and $\frac{dy}{dx}=9$.

(10 marks)

(c) Determine the inverse of Laplace transform $F(s)=\frac{s-3}{s^2-4s-140}$.

(10 marks)

END OF EXAMINATION PAPER

TRIGONOMETRY IDENTITIES

FUNDAMENTAL IDENTITIES	FORMULAS FOR NEGATIVES
$\csc\theta = \dfrac{1}{\sin\theta}$	$\sin(-\theta) = -\sin\theta$
$\sec\theta = \dfrac{1}{\cos\theta}$	$\cos(-\theta) = \cos\theta$
$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$	$\tan(-\theta) = -\tan\theta$
$\cos^2\theta = \dfrac{1}{2}(1+\cos 2\theta)$	$\csc(-\theta) = -\csc\theta$
$\sin^2\theta + \cos^2\theta = 1$	$\sec(-\theta) = \sec\theta$
$1+\tan^2\theta = \sec^2\theta$	$\cot(-\theta) = -\cot\theta$
$1+\cot^2\theta = \csc^2\theta$	$\sin^2\theta = \dfrac{1}{2}(1-\cos 2\theta)$

ADDITION FORMULAS	SUBTRACTION FORMULAS
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A\tan B}$	$\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\tan B}$

HALF-ANGLE FORMULAS	DOUBLE-ANGLE FORMULAS
$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$	$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ .... $= 1-2\sin^2\theta$ .... $= 2\cos^2\theta - 1$
$\tan\dfrac{\theta}{2} = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$	$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

PRODUCT-TO-SUM FORMULAS	SUM-TO-PRODUCT FORMULAS
$\sin\alpha\cos\beta = \dfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$	$\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$
$\cos\alpha\sin\beta = \dfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$	$\sin\alpha - \sin\beta = 2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$
$\cos\alpha\cos\beta = \dfrac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$	$\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

DIFFERENTIATION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$

INTEGRATION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \cos x\, dx = \sin x + c$	$\int \cos f(x) dx = \dfrac{\sin f(x)}{f'(x)} + c$
$\int \sin x\, dx = -\cos x + c$	$\int \sin f(x) dx = \dfrac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x\, dx = \tan x + c$	$\int \sec^2 f(x) dx = \dfrac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x\, dx = \sec x + c$	$\int \sec f(x) \tan f(x) dx = \dfrac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x\, dx = -\csc x + c$	$\int \csc f(x) \cot f(x) dx = \dfrac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x\, dx = -\cot x + c$	$\int \csc^2 f(x) dx = \dfrac{-\cot f(x)}{f'(x)} + c$
$\int \tan x\, dx = \ln\lvert\sec x\rvert + c$	$\int \tan x\, dx = \dfrac{\ln\lvert\sec f(x)\rvert}{f'(x)} + c$
$\int \sec x\, dx = \ln\lvert\sec x + \tan x\rvert + c$	$\int \sec x\ dx = \dfrac{\ln\lvert\sec f(x) + \tan f(x)\rvert}{f'(x)} + c$
$\int \cot x\, dx = \ln\lvert\sin x\rvert + c$	$\int \cot x\, dx = \dfrac{\ln\lvert\sin f(x)\rvert}{f'(x)} + c$
$\int \csc x\, dx = -\ln\lvert\csc x + \cot x\rvert + c$	$\int \csc x dx = \dfrac{-\ln\lvert\csc f(x) + \cot f(x)\rvert}{f'(x)} + c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int e^x dx = e^x + c$	$\int e^{f(x)}\, dx = \dfrac{e^{f(x)}}{f'(x)} + c$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \dfrac{1}{x} dx = \ln\lvert x\rvert + c$	$\int \dfrac{1}{f(x)} dx = \dfrac{\ln\lvert f(x)\rvert}{f'(x)} + c$

INVERSE TRIGONOMETRIC FUNCTION FUNCTION

$$\frac{d}{dx}\left[\sin^{-1}\left(\frac{x}{a}\right)\right] = \frac{1}{\sqrt{a^2 - x^2}}$$
$$\frac{d}{dx}\left[\cos^{-1}\left(\frac{x}{a}\right)\right] = -\frac{1}{\sqrt{a^2 - x^2}}$$
$$\frac{d}{dx}\left[\tan^{-1}\left(\frac{x}{a}\right)\right] = \frac{a}{a^2 + x^2}$$
$$\frac{d}{dx}\left[\csc^{-1}\left(\frac{x}{a}\right)\right] = -\frac{a}{x\sqrt{x^2 - a^2}}$$
$$\frac{d}{dx}\left[\sec^{-1}\left(\frac{x}{a}\right)\right] = \frac{a}{x\sqrt{x^2 - a^2}}$$
$$\frac{d}{dx}\left[\cot^{-1}\left(\frac{x}{a}\right)\right] = -\frac{a}{a^2 + x^2}$$

FIRST AND SECOND ORDER DIFFERENTIAL EQUATION

If the roots of the auxiliary equation are:

(i) **real and different**, say $m=\alpha$ and $m=\beta$, then the general solution is

$$y = Ae^{\alpha x} + Be^{\beta x}$$

(ii) **real and equal, say** $m=\alpha$ twice, then the general solution is

$$y = (Ax + B)e^{\alpha x}$$

(iii) **complex**, say $m=\alpha \pm j\beta$, then the general solution is

$$y = e^{\alpha x}\{A\cos\beta x + B\sin\beta x\}$$

Table 51.1 Form of particular integral for different functions

Type	*Straightforward cases* Try as particular integral:	*'Snag' cases* Try as particular integral:	*See problem*
(a) $f(x)$ = a constant	$v=k$	$v=kx$ (used when C.F. contains a constant)	1, 2
(b) $f(x)$ = polynomial (i.e. $f(x)=L+Mx+Nx^2+\cdots$ where any of the coefficients may be zero)	$v=a+bx+cx^2+\cdots$		3
(c) $f(x)$ = an exponential function (i.e. $f(x)=Ae^{ax}$)	$v=ke^{ax}$	(i) $v=kxe^{ax}$ (used when e^{ax} appears in the C.F.)	4, 5
		(ii) $v=kx^2e^{ax}$ (used when e^{ax} and xe^{ax} both appear in the C.F.)	6
(d) $f(x)$ = a sine or cosine function (i.e. $f(x)=a\sin px+b\cos px$, where a or b may be zero)	$v=A\sin px+B\cos px$	$v=x(A\sin px+B\cos px)$ (used when $\sin px$ and/or $\cos px$ appears in the C.F.)	7, 8
(e) $f(x)$ = a sum e.g. (i) $f(x)=4x^2-3\sin 2x$	(i) $v=ax^2+bx+c+d\sin 2x+e\cos 2x$		9
(ii) $f(x)=2-x+e^{3x}$	(ii) $v=ax+b+ce^{3x}$		
(f) $f(x)$ = a product e.g. $f(x)=2e^x\cos 2x$	$v=e^x(A\sin 2x+B\cos 2x)$		10