CONFIDENTIAL

UNIVERSITI KUALA LUMPUR
Malaysian Institute of Marine Engineering Technology

FINAL EXAMINATION
SEPTEMBER 2016 SESSION

SUBJECT CODE	:	LNB 30203
SUBJECT TITLE	:	SHIP RESISTANCE AND PROPULSION
LEVEL	:	DEGREE
TIME / DURATION	:	9.00 am – 12.00 pm / (3 HOURS)
DATE	:	26 JANUARY 2017

INSTRUCTIONS TO CANDIDATES

a) Please read the instructions given in the question paper CAREFULLY.

b) This question paper is printed on both sides of the paper.

c) Please write your answers on the answer booklet provided. Answer all questions in English only. Graph paper will be provided.

d) Answer should be written in blue or black ink except for sketching, graphic and illustration.

e) This question paper consists of TWO (2) sections; Section A and B. Answer all questions in Section A; For Section B, answer THREE (3) questions only.

f) Attached the Objective Answer Sheet for Section A and attached it together with your answer script.

THERE ARE 15 PAGES OF QUESTIONS, INCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL questions. Please use the objective answer sheet provided. Filled in the correct square with pencil. Make heavy black marks that filled the square completely. Erase clearly any answer you wish to change. Attached the Objective Answer Sheet with your answer script.

1. The fluid flow characteristics for geometrically similar model and full scale are not the same. Select one:
   A. Correct. Both flow in the model scale and the full scale will be not be similar as both of the Reynolds number are not the same.
   B. Incorrect. Both flow in the model scale and the full scale will be similar as both of the Reynolds number are the same.

(2 marks)

2. What will be the physical correction used in model testing in a towing tank to correct the flow to be identical to the full scale flow?
   A. Using studs as turbulence stimulator
   B. Using sand-strip as turbulence stimulator
   C. Using trip wire as turbulence stimulator
   D. All of the above

(2 marks)

3. Describe a way of minimizing a wave resistance of a ship?
   A. Adding a bulbous bow to the ship
   B. Adding a stabiliser to a ship
   C. Adding a bow thruster at the forward end of the ship
   D. Adding a bilge keel to the ship side
   E. Adding a skeg at the baseline of the ship

(2 marks)

4. Describe how bulbous bow plays in minimising the wave resistance of a ship.
   A. The bulbous bow cuts the water pretty well compared with without having one.
   B. The bulbous bow creates its own wave system and cancelling or interfere with the wave system from the stem hence reducing the merged wave system.

C. The bulbous bow reduces the frictional resistance by increasing the wetted surface area

D. The bulbous bow reduces the pressure resistance at the bow region hence reducing the wave resistance.

(2 marks)

5. Why a residuary resistance curve in some cases do exhibits a series of 'hump' and 'hollows'?

   A. The wave resistance oscillate about the mean curve as the frictional resistance become lower and higher with increasing of Froude number
   B. The wave resistance would oscillate about a mean curve depending upon whether the interference effect arising from the bow system and the stern system yields a maximum or minimum resistance.
   C. The wave resistance exhibits 'humps' and 'hollows' as the vessel heave and pitch on the free surface of the water

(2 marks)

6. The resistance of a vessel is greater in shallow water than in deep water conditions. Is this statement correct or incorrect?

   A. Incorrect. The resistance of a vessel is lower in shallow water.
   B. Correct. The resistance of a vessel is greater in shallow water.

(2 marks)

7. The dimensional analysis of the resistance of a ship can be expressed in its final form of three dimensionless numbers as shown below. One of the number is called Euler number. Name the other two non-dimensional numbers.

$$\frac{R}{1/2\rho S V^2} = f\left[-,-,\frac{p}{\rho V^2}\right]$$

   A. Reynolds and Mach number
   B. Cauchy and Mach number
   C. Froude and Reynolds number
   D. Cavitation and Reynolds number
   E. Froude and Weber number

(2 marks)

8. The frictional resistance formula was originally derived from:

A. Experiments of a series of wood planks towed in a towing tank with varying length and roughness

B. Experiments of series of round bilge hull towed in a towing tank with varying length and roughness

C. Sea Trial data

D. Experiments of series of double chine hull towed in a towing tank with varying length and roughness

(2 marks)

9. The Froude's law of comparison states that "The residuary resistance of geometrically similar (geosim) ships are in the ratio of the cubes of their linear dimensions when their speeds are in the ratio of the squares of their lengths" Is this statement correct or incorrect?

A. Correct, the residuary resistance of 'geosim' ships are in the ratio of the cubes of their linear dimensions

B. Incorrect, the residuary resistance of 'geosim' ships are in the ratio of the squares of their linear dimensions

(2 marks)

10. It is impossible to model the vessel complying with both Froude numbers and Reynolds number. Therefore it is not possible to directly convert the total resistance of the model to the total resistance of the ship using directly the scale ratio. It requires some extrapolations. Name three methods to extrapolate the model scale drag to full scale drag?

A. Froude's 2 dimensional extrapolation

B. Hughes's 3 dimensional extrapolation

C. ITTC 1978 extrapolation method

D. All of the above

(2 marks)

11. The dimensional analysis of the thrust of a ship propeller can be expressed in its final form of four dimensionless numbers as shown below. But two of the non-dimensionless numbers are missing in the equation below. Name the two missing non-dimensional numbers.

$$\frac{T}{\frac{1}{2}\rho D^2 V_A{}^2} = f\left[\frac{gD}{V_A{}^2}, -, -, \frac{\upsilon}{V_A D}\right]$$

A. Reynolds and Mach number
B. Cauchy and Cavitation number
C. Froude and Reynolds number
D. Advance Coefficient and Cavitation number
E. Froude and Weber number

(2 marks)

12. Why does the angle of the propeller blade change with increasing distance from the hub even though the pitch is constant throughout the radius of the propeller blade?

A. The angle change as to maintain a constant inflow angle. The geometric angle has to increase as radius is decreased. Therefore the angle of the propeller blade change with increasing distance from the hub.
B. The pitch at each radius of the propeller blade decrease with increasing distance from the hub. Therefore the angle of the propeller blade change with increasing distance from the hub.
C. The pitch at each radius of the propeller blade increase with increasing distance from the hub. Therefore the angle of the propeller blade changes with increasing distance from the hub.

(2 marks)

13. When advance coefficient ratio is maintained model propeller revolutions are smaller than a geometrically similar full scale propeller. Is this statement correct or incorrect?

A. Correct. In maintain advance coefficient, the propeller revolutions in model scale are smaller than a geometrically similar full scale propeller.
B. Incorrect. In maintain advance coefficient, the propeller revolutions in model scale are greater than a geometrically similar full scale propeller.

(2 marks)

14. From a design point of view, what can be done to increase the propeller efficiency? Name two (2) design parameters that do not alter the hull design significantly.

A. Decrease the propeller diameter and increasing propeller turning speed.
B. Decrease the propeller diameter and decreasing propeller turning speed
C. Increase propeller diameter and increasing propeller turning speed
D. Increase propeller diameter and decreasing propeller turning speed.

(2 marks)

15. How can cavitation on a propeller blade be reduced? Name two (2) measures to reduce cavitation on propeller blades.

A. Smaller BAR and shorter chord length
B. Smaller BAR and longer chord length
C. Larger BAR and shorter chord length
D. Larger BAR and longer chord length

(2 marks)

16. What are the assumptions made in the Actuator Disk or the Momentum Theory of a propeller?

A. Propeller imparts a uniform acceleration to all fluid passing through the propeller disk. The thrust generated will be uniformly distributed over the propeller disk.
B. The flow is frictionless. Thus thermal energy will not be generated.
C. There is unlimited inflow of water to the propeller.
D. All of the above

(2 marks)

17. One of the components of wake is frictional wake when the frictional drag causes retardation of the flow inside the ship's boundary layer. Describe the other two components of wake.

A. Potential Wake and Wave Wake
B. Tangential Wake and Wave Wake
C. Turbulence Wake and Separation Wake
D. Nominal Wake and Effective Wake

(2 marks)

18. What are the effects of cavitation to a ship? Name two (2) effects of cavitation.

A. Performance loss and noise

B. Power increase and drag reduction

C. Thrust increase and torque increase

D. Efficiency increase and wave drag increase

(2 marks)

19. The blade element diagram used in the combined blade element-momentum theory is given as in Figure A.1. Describe the item *dD/dr* and *dL/dr* in the Figure A.1.

A. The blade section thrust and the blade section torque

B. The blade section drag and the blade section lift

C. The blade section pressure coefficient and the blade section rate of rotation

(2 marks)

20. The blade element diagram used in the combined blade element-momentum theory is given as in Figure A.1. Describe the angle ϕ and α.

A. The geometric pitch angle and the angle of attack

B. The nose-tail pitch angle and the angle of attack

C. The zero-lift pitch angle and the geometric pitch angle

D. The face pitch angle and the nose-tail pitch angle

E. The inflow angle and the angle of attack

(2 marks)

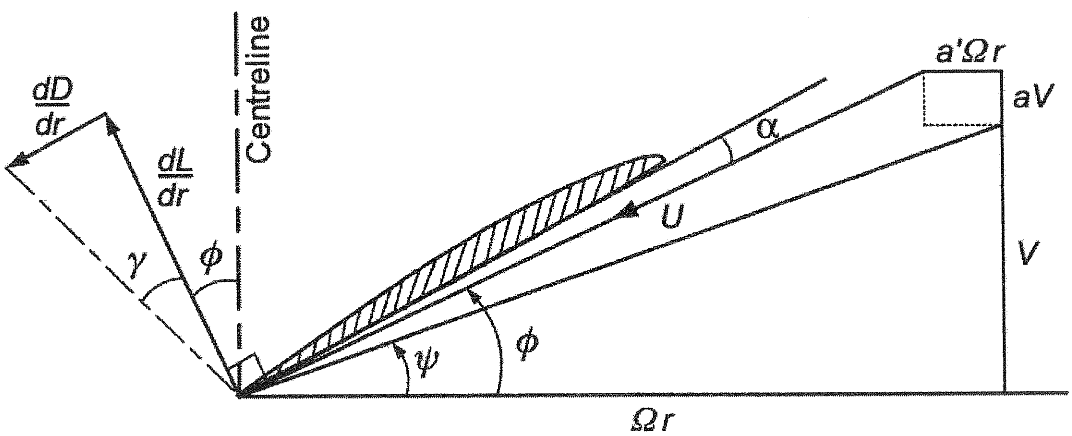


Figure A.1 The blade element diagram`

SECTION B (Total: 60 marks)

INSTRUCTION: Answer three (3) questions only

Question 1

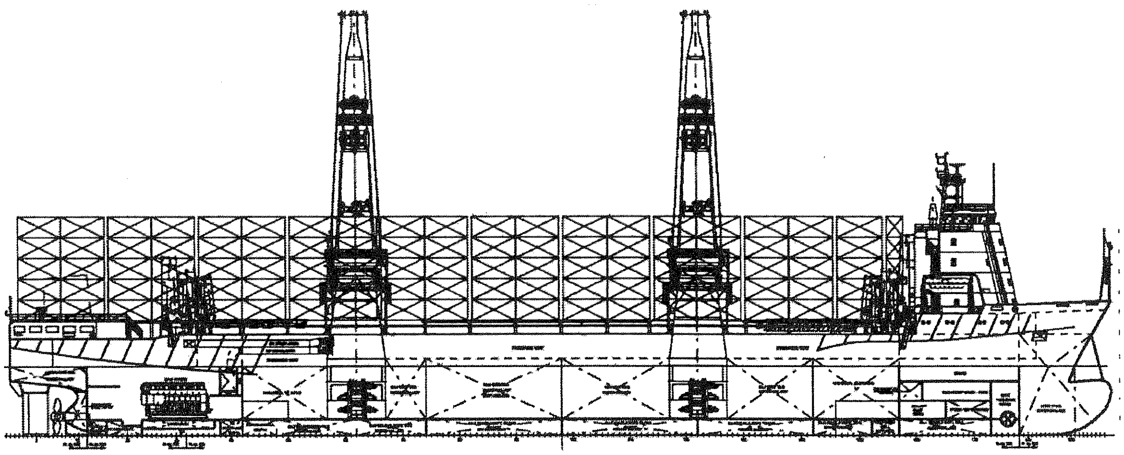


Figure 1.1 The longitudinal view of a proposed 18,680 dwt multi-purpose carrier with L = 142 m. Source: Grubisic et al (2008) – A 18680 dwt Multipurpose / Heavy Lift Cargo Vessel, Part 1, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb.

You are an engineer working in a commercial towing tank facility. For a client Asianta Shipping, you have tested two different designs, design A and design B (see Table 1.1) for a 142 m multi-purpose carrier (shown in Figure 1.1). Both designs have the same length, same displacement and same speed and were tested at 1/71 model scale. Surprisingly, both have the same drag of 0.9 N regarding to your towing tank results.

a) Which design would you suggest to your client, Asianta Shipping? Use ITTC 1978 approach including form factor *(1 + k)* of 1.12 to support your suggestion. Assume the correlation allowance, C_A to be 0.0004. The air resistance coefficient C_{AAS}, was calculated to be 0.0000386.

(15 marks)

b) Based on your experience as a ship designer, which design would you expect to be more beneficial from a resistance point of view at higher speed when wave-making dominates the total resistance? Explain.

(5 marks)

Table 1.1 Main particulars of two proposed designs of a multi-purpose carrier.

Parameter	units	design A	design B
Speed	[kn]	12.0	12.0
Displacement	[m^3]	13,500	13,500
Length	[m]	142	142
Breadth	[m]	23.4	17.8
Draft	[m]	5.00	7.10
wet. surface area	[m^2]	2540	2360
model scale ratio	-	1/71.0	1/71.0
drag at model scale	[N]	0.900	0.900

(Total 20 marks)

Question 2

A towing tank results were recorded for a model having the principal particulars given below as:

L_{WL} = 2.496 m

Displacement = 28.41 kg

WSA = 0.952 m^2

Scale factor = 50

Model Speed (m/sec)	Drag (grams)	Forward Sinkage (mm)	Aft Sinkage (mm)
1.39	498	- 2.2	- 1.7
1.52	655	- 2.2	- 3.2
1.66	720	- 3.0	- 2.9
1.81	824	- 3.8	- 3.4
1.96	1008	- 3.3	- 6.7
2.11	1217	- 1.3	- 10.8
2.25	1407	0.6	- 14.2

a) Determine the full scale total resistance and the effective power using ITTC 57 Frictional Line and 2D extrapolation procedure at full scale speed of 29 knots.

(10 marks)

b) Extrapolate the model forward and aft sinkage to full scale for **all model speed**.

(3 marks)

c) Calculate the dynamic trim of the vessel at each speed in degree **if the distance between the forward sinkage measurement post and the aft sinkage measurement post is 338mm.**

(3 marks)

d) Plot the Forward Sinkage and the Aft Sinkage in full-scale with respect to the ship speed in knots.

(2 marks)

e) Plot the dynamic trim in degree with respect to the ship speed in knots.

(2 marks)

(Total 20 marks)

Question 3

The towing tank results were recorded as in Table 3.1 for a model having the principal particulars given below as:

L_{WL}	=	4.30 m
B	=	0.25 m
Draft	=	0.049 m
C_M	=	0.95
Displacement	=	50 kg
WSA	=	1.18 m^2
Scale factor	=	29

Table 3.1 The towing tank test results for a 1/29 scaled ship model.

Speed	**Drag**	$C_{FM,\ 24^oC}*10^3$	$C_{TM,\ 24^oC}*10^3$
(m/sec)	(grams)	(-)	(-)
0.65	117	3.73	4.60
0.78	167	3.60	4.56
0.92	227	3.49	4.46
1.05	295	3.40	4.45
1.11	330	3.37	4.45

The test above was conducted in a towing tank with a width of 4m and a depth of 2.5m and the water temperature was measured to be 24°C. The form factor (1+k) was determined earlier using Prohaska's plot at 1.17. At **Froude depth number of 0.212**:

a) Correct the model speed using Shuster's blockage corrector.

(14 marks)

b) Correct the coefficient of the total resistance, $C_{TM,\ 24^oC}$ to a standard 15°C,

(6 marks)

(Total 20 marks)

Question 4

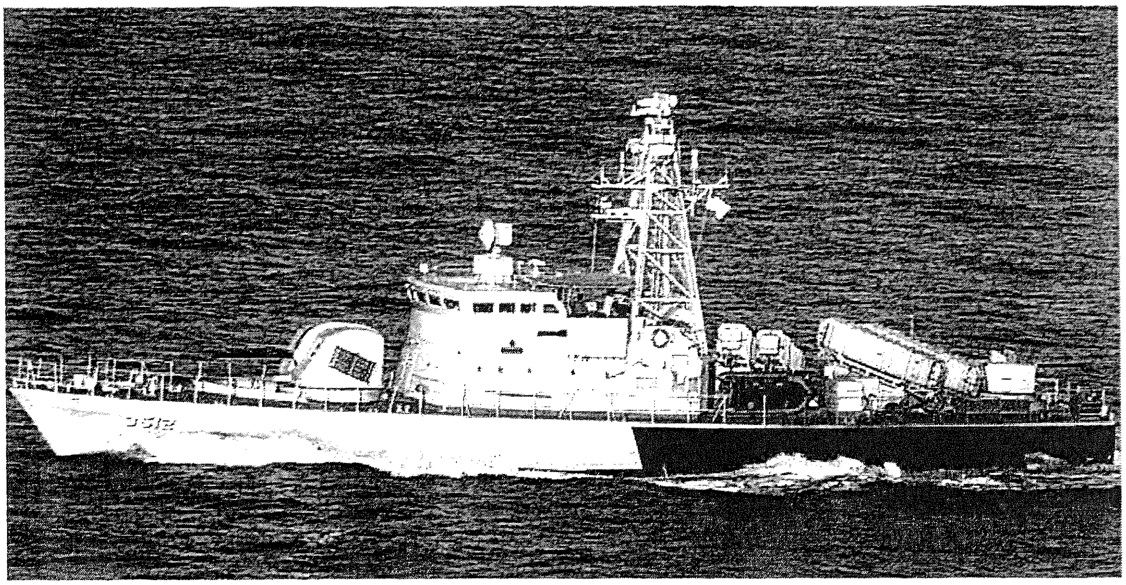


Figure 4.1 KD Perkasa, a 'Handalan' Class fast attack craft which was based on the design of Swedish SPICA class torpedo boat, were built by Karlskrona Varvet AB, Sweden in 1978. This craft are powered by three MTU 16V 538 TB91 diesel engines.

A triple screw fast attack craft owned by the Royal Malaysian Navy, 'KD Perkasa' as shown in Figure 4.1, had undergone a re-engine program and were planned to be fitted with three new Gawn series propellers. The Gawn propellers selected will be with a diameter of 1.2 meter and a blade area ratio of 0.95, for minimising cavitation. The total delivered power **at the three shafts** are estimated at 6.750 MW. The wake fraction is estimated to be 0.21. At the ship cruising speed of 25 knots;

a) Determine the optimum P/D that attained the maximum propeller efficiency.

(17 marks)

b) Calculate the shaft speed in rev/min at this cruising speed.

(3 marks)

Plot the K_Q-J^3 curve in the Gawn K_T-K_Q-J chart given in **Annex 1**. Attached it to the answer script.

(Total 20 marks)

Question 5

Your company is assigned to design a propeller for a 260m cruise ship for STX France for their newly designed cruise ship *Elena*, as shown in Figure 5.1. The ship is designed to be a single screw ship with a design speed of 25 knots. Early estimation reveals that the vessel will required a thrust of 1,915 kN with a 5.33m diameter four bladed propeller. The wake fraction is estimated to be 0.25.

a) Using Keller's equation (Keller, 1966), determine the required expanded blade area ratio if the propeller shaft axis immersion is 3.5 m. The ship has a transom stern, therefore the k in the Keller's equation is assumed to be 0.1.

(4 marks)

b) Early powering prediction reveals that the vessel will be installed with a 21,740 kW internal combustion engine. Using the B_P-δ chart for propeller Wageningen series, select the appropriate B_P-δ chart **based on your estimation in (a)**, see Annex 2, 3, 4 and 5, and assuming the shafting efficiency to be 0.98, calculate

   i. the open water efficiency,
   ii. pitch-diameter ratio
   iii. and the propeller shaft speed in rev/min.

(16 marks)

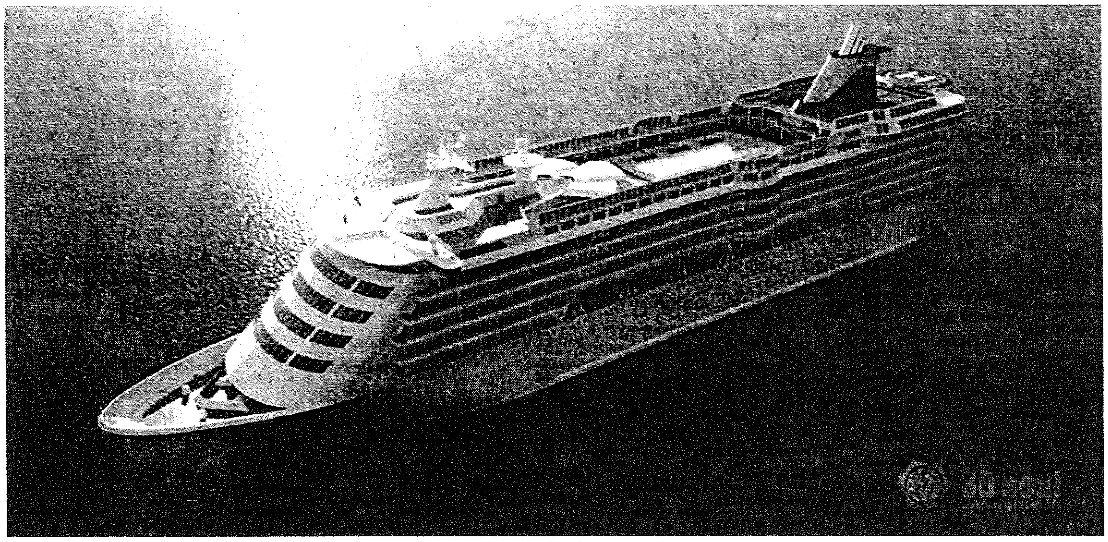


Figure 5.1 The new 260m long luxury cruise line, *Elena*, designed by STX France, capable of accommodating 550 passengers with spacious living areas, restaurants and salons. (Image taken from Rhinoceros.com, modeled in Rhino, rendered in V-Ray)

(Total 20 marks)

Question 6

A wake survey was done using Laser Doppler Velocimetry (LDV) for a model with a twin screw configuration in a towing tank. The local wake measurement was taken at the propeller plane. The results are presented in a contour plot as shown in Figure 6.1. The measurement were taken at radius 1.0R, 0.75R and 0.50R. The propeller diameter is at 7 m with the hub diameter at 1 m. Integrate the radial wake fraction to obtain the nominal mean wake fraction for this twin screw ship.

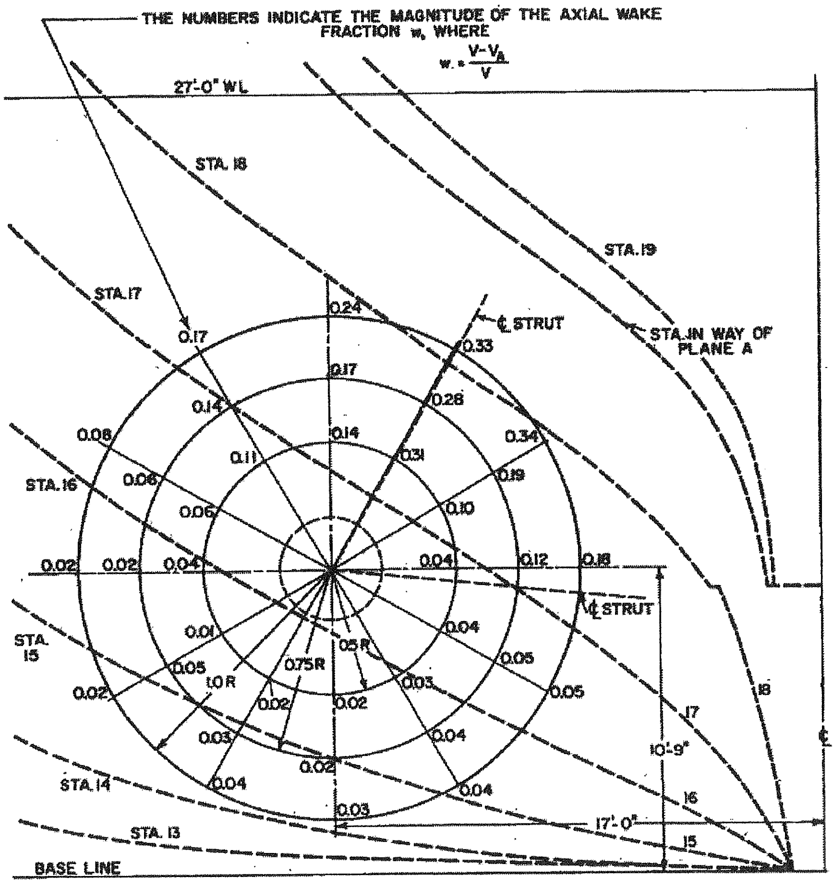


Fig. 98(a) Wake diagram for TS ship fitted with shaft-struts

LOAD DISPLACEMENT, SHIP SPEED 20 KNOTS.
TRANSVERSE SECTION, LOOKING FORWARD. THE WAKE SURVEY WAS MADE IN A PLANE PERPENDICULAR TO THE LONGITUDINAL AXIS AT A DISTANCE OF 5.77 FT. AFT OF STATION 18½.

Figure 6.1 The local radial wake measurements for a twin screw ship

(Total 20 marks)

Question 7

You are a towing tank engineer and are required to make a scale effect correction to the propeller thrust and torque using the method proposed by Lindgren et al (1978) which is currently used in the ITTC 1978 Extrapolation procedure no 7.5-02-03-01.4 to an open water propeller test results tabulated in Table 7.1 below.

Table 7.1 The open water test results for a 1/29 scaled propeller running at 21 rev/secs.

V_A **(metre /sec)**	Q (Newton.metre)	T (Newton)
0.29	0.46	24.46
0.8	0.40	21.16
1.31	0.32	17.64
1.81	0.26	12.48

The open water test was done in a towing tank, with the advance velocity, V_A varied at each run as listed in the first column in Table 7.1. The propeller was set at a constant shaft speed of 21 rev/sec. The model propeller is 1:29 of the full scale propeller.

a) Calculate the corrected full scale propeller thrust and torque coefficient at $J = 0.52$. The propeller drawing is shown **in Annex 6 (all units in meter).** You may find all the necessary data in the drawing. (Some interpolations are required).

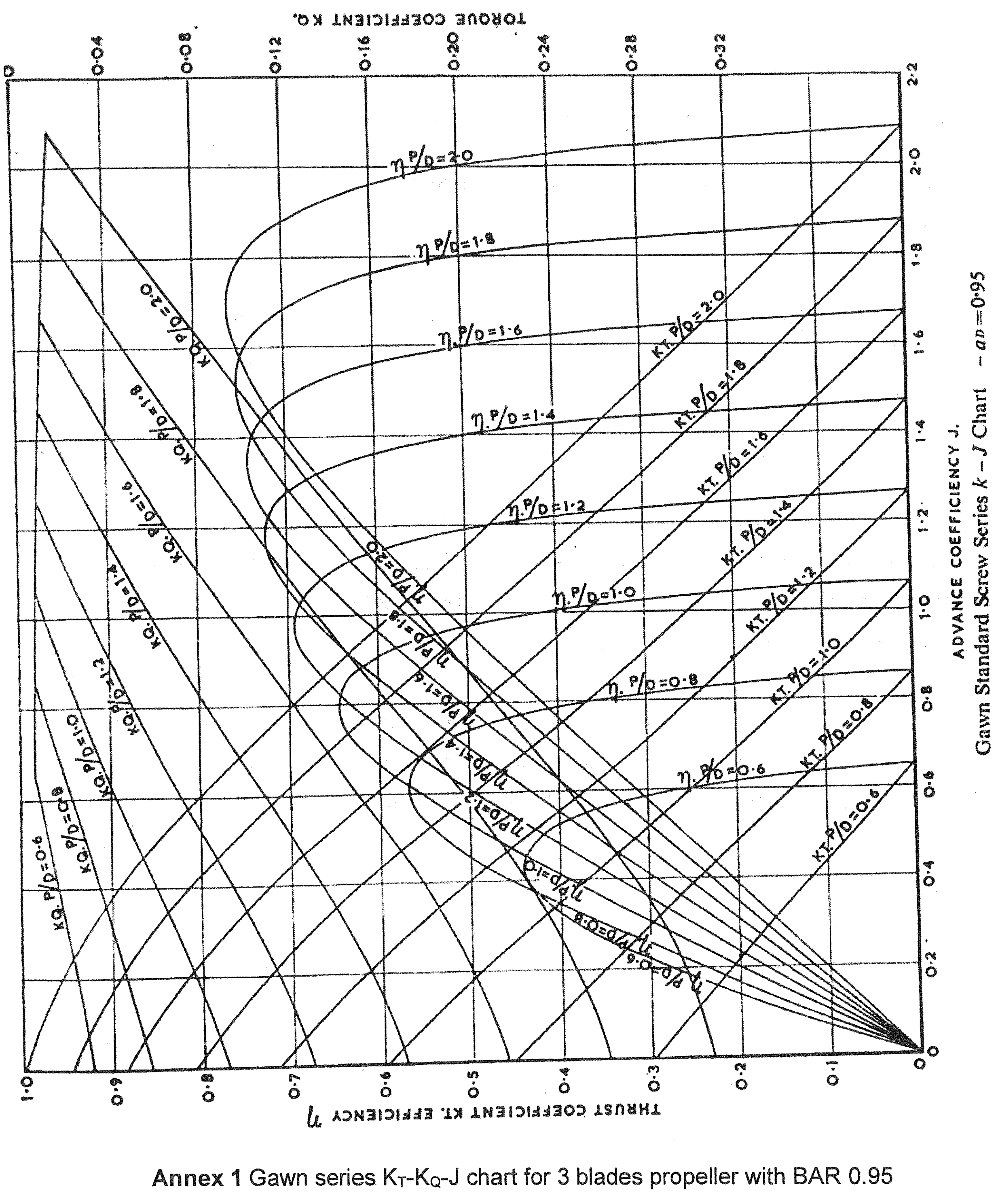
(15 marks)

b) If this open water test was conducted in a cavitation tunnel with a circulation speed of 5.0 m/s, **maintaining J at 0.52**, determine the new rate of rotation for the same model propeller and the new local Reynolds number at 0.75 radius fraction of the propeller blade.

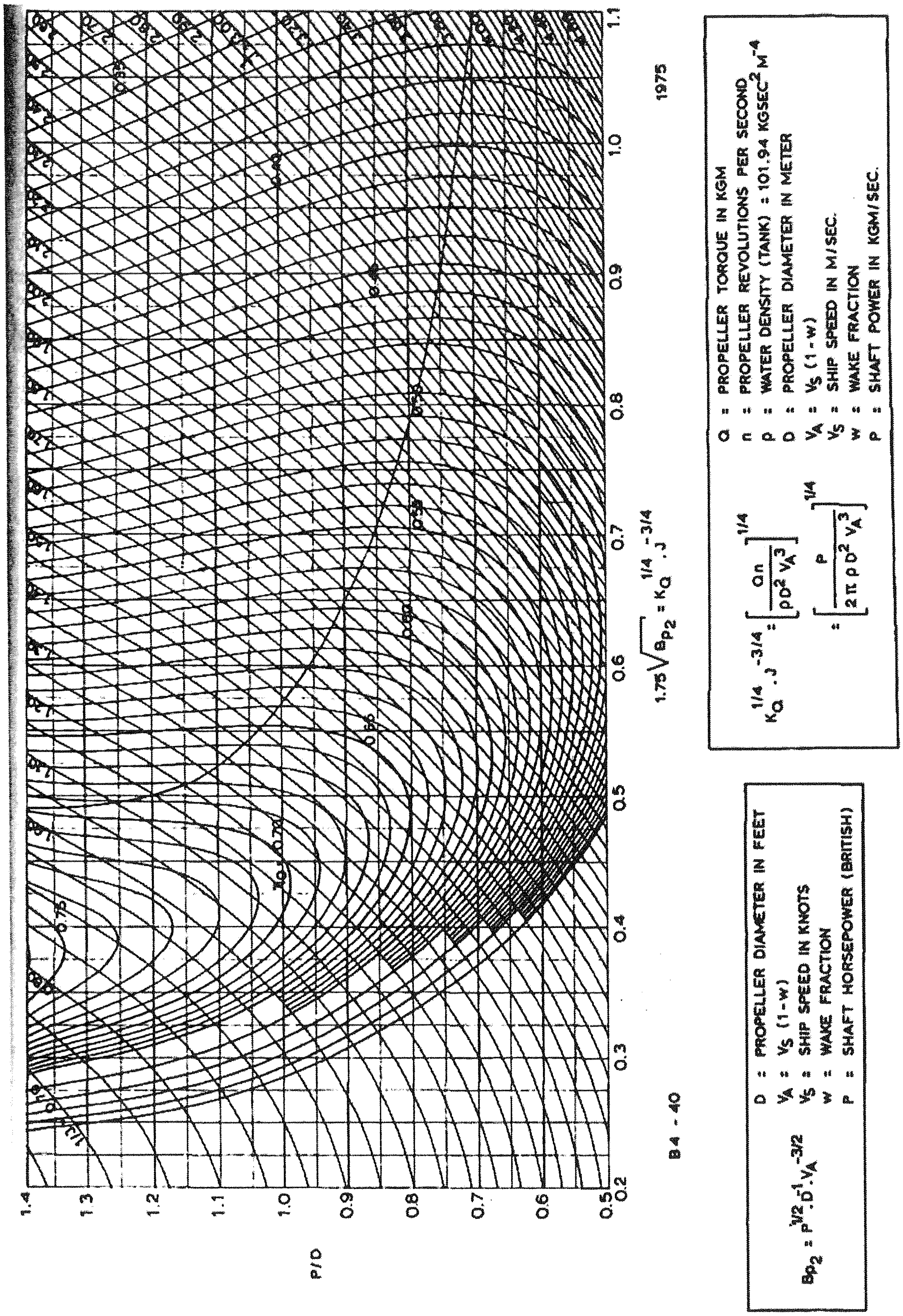
(5 marks)

(Total 20 marks)

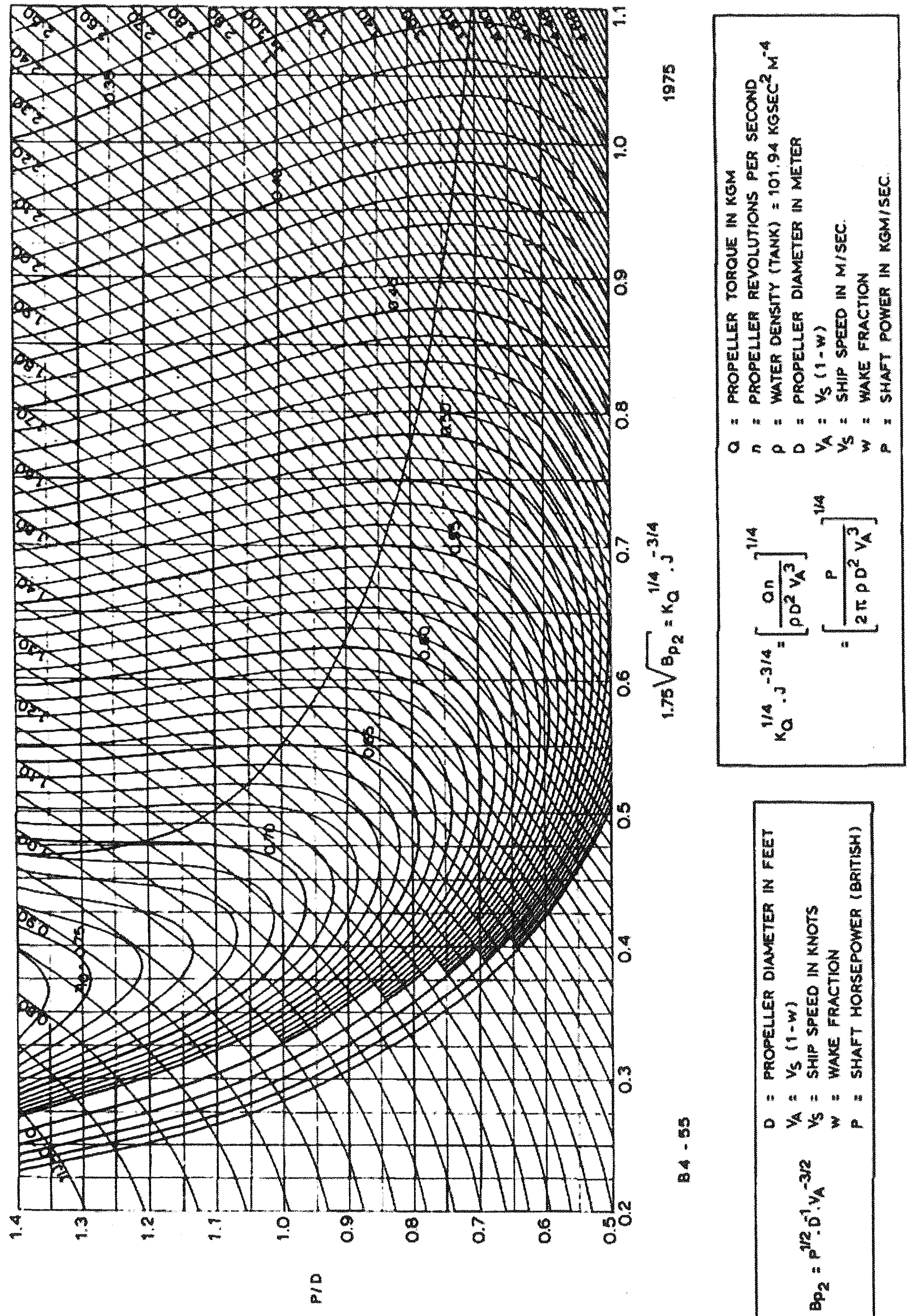
END OF QUESTION



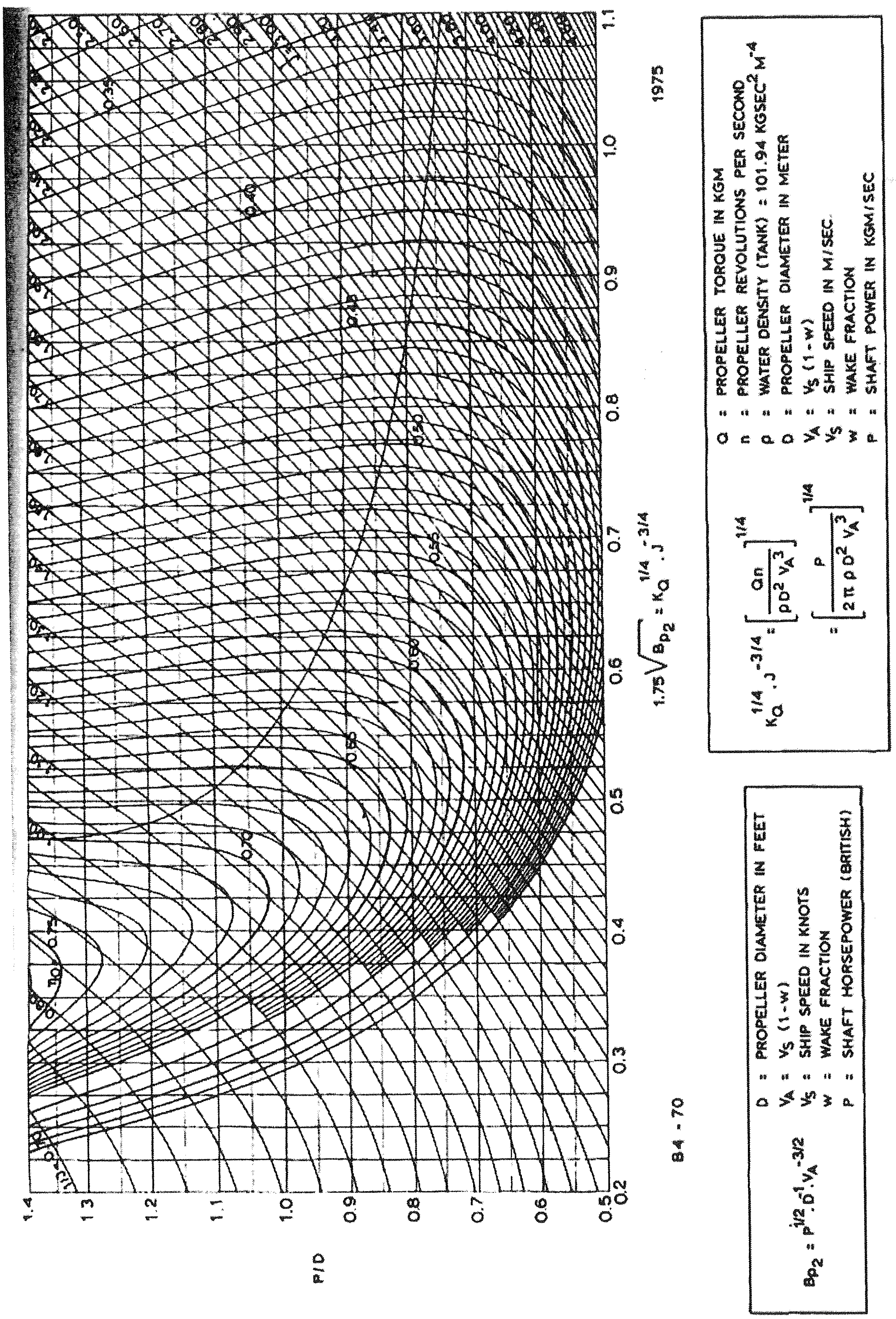
Annex 1 Gawn series K_T-K_Q-J chart for 3 blades propeller with BAR 0.95



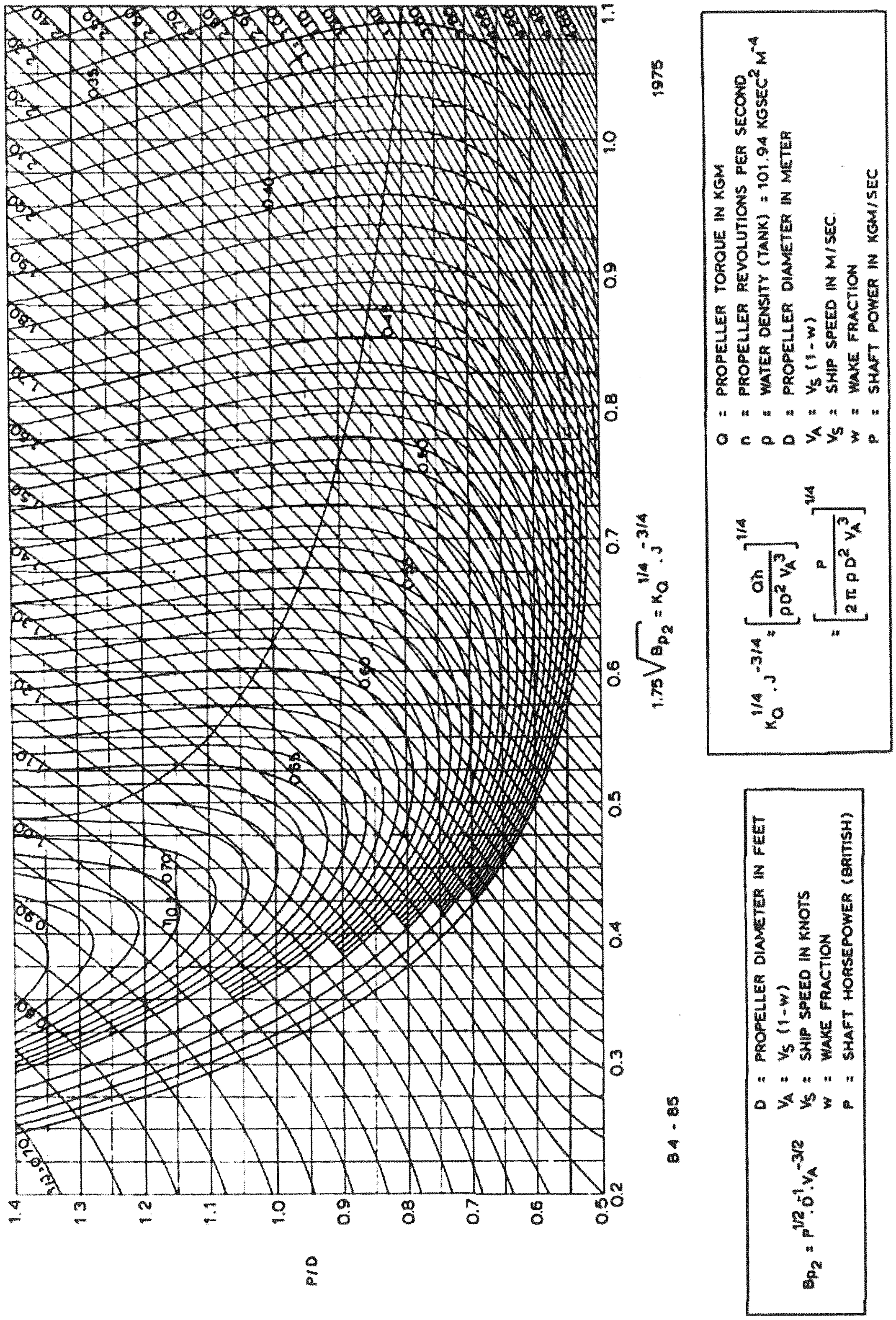
Annex 2 B_P-δ chart for 4 blades propeller with BAR 0.40



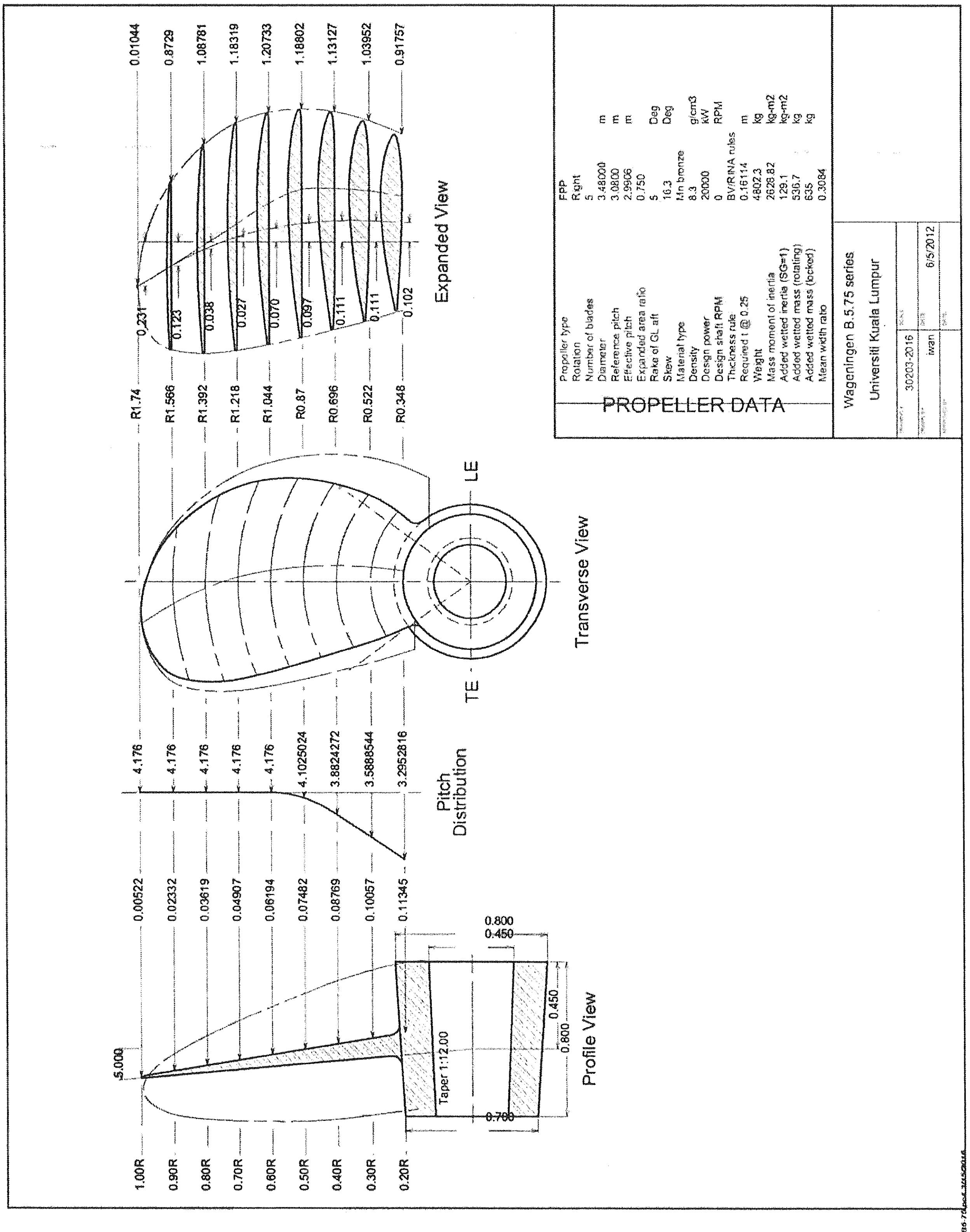
Annex 3 *B_P- δ* chart for 4 blades propeller with BAR 0.55



Annex 4 *B_P-δ* chart for 4 blades propeller with BAR 0.70



Annex 5 B_P- δ chart for 4 blades propeller with BAR 0.85



Annex 6 Propeller drawing for a 5 bladed propeller.

USEFUL FORMULA & DATA

Density

Fresh water = 1000 kg/m³
Sea water = 1025 kg/m³
Air at 15°C = 1.225 kg/m³

Kinematic viscosity

Fresh water at 15°C = 1.139×10^{-6} m³/s
Sea water at 15°C = 1.183×10^{-6} m³/s
Fresh water at 24°C = 9.131×10^{-7} m³/s

Granville Line Formulation

$$C_{FO} = \frac{0.0776}{(\log_{10} \mathrm{Re} - 1.88)^2} + \frac{60}{\mathrm{Re}}$$

Hughes Line Formulation

$$C_{FO} = \frac{0.066}{(\log_{10} \mathrm{Re} - 2.03)^2}$$

ATTC Line Formulation

$$\frac{0.242}{\sqrt{C_F}} = \log_{10}(\mathrm{Re}.C_F)$$

ITTC 1957 Model Ship Correlation Formulation

$$C_F = \frac{0.075}{(\log_{10} \mathrm{Re} - 2)^2}$$

Non-dimensional coefficient for total resistance

$$C_T = \frac{R_T}{\frac{1}{2}\rho S V^2}$$

Froude's approach

$$R_T = R_F + R_R$$
$$C_T = C_F + C_R$$

Hughes's approach

$$R_T = R_V + R_W$$
$$C_T = C_V + C_W$$
$$= C_F(1+k) + C_W$$

ITTC 1978 Resistance Prediction – Updated 2014

The total ship resistance coefficient without bilge keels is given by;

$$C_{TS} = C_{FS}(1+k) + C_R + \Delta C_F + C_A + C_{AAS}$$

where;

C_{FS} = frictional coefficient of ship according to the ITTC 1957 ship model correlation line

C_R = residual resistance calculated from the total and viscous resistance of the model

$$= C_{TM} - (1+k)C_{FM}$$

Bilge keels can be allowed for by multiplying the C_{FS} and C_A terms by the ratio

$\frac{S + S_{BK}}{S}$, S_{BK} = surface area of the bilge keels

The correlation allowance is calculated from

$$C_A = (5.68 - 0.6\log\mathrm{Re}) \times 10^{-03}$$

The roughness allowance is calculated from

$$\Delta C_F = 0.044\left[\left(\frac{k_s}{L_{WL}}\right)^{\frac{1}{3}} - 10.\mathrm{Re}^{-\frac{1}{3}}\right] + 0.000125$$

k_s can be taken as 150×10^{-6} m.

Air resistance is calculated from

$$C_{AAS} = C_{DA}\frac{\rho_A . A_{VS}}{\rho_S . S_S}$$

A_{VS} = transverse projected area of ship above the waterline

Propeller scale effect correction

$$K_{TOS} = K_{TO} - \Delta K_T$$
$$K_{QOS} = K_{QO} - \Delta K_Q$$

$$\Delta K_T = -\Delta C_D \times 0.3 \frac{P}{D} . \frac{cZ}{D}$$

$$\Delta K_Q = \Delta C_D \times 0.25 \frac{cZ}{D}$$
$$\Delta C_D = C_{DM} - C_{DS}$$

$$C_{DM} = 2\left(1 + 2\frac{t}{c}\right)\left[\frac{0.044}{R_{nco}^{1/6}} - \frac{5}{R_{nco}^{2/3}}\right]$$

$$C_{DS} = 2\left(1 + 2\frac{t}{c}\right)\left[1.89 + 1.62 \log_{10} \frac{c}{k_p}\right]^{-2.5}$$

$$k_p = 30 \times 10^{-6} m$$

$$R_{nco} = \frac{c\sqrt{V_A^2 + (2\pi nr)^2}}{\upsilon}$$

Emerson Blockage correction

$$\frac{\Delta V}{V} = 1.65 \frac{m_3}{1 - m_3 - F_{nh}^2}$$
$$m_1 = \frac{A_M}{A}$$
$$m_2 = \frac{\nabla}{A \times L}$$
$$m_3 = \frac{m_1 + m_2}{2}$$
$$F_{nh} = \frac{V}{\sqrt{gh}}$$

A_M : midship sectional area f model

A : tank cross sectional area

∇:model volume displacement

L : model length

V : model speed before correction

Schuster Blockage correction

$$\frac{\Delta V}{V} = \frac{m_1}{1 - m_1 - F_{nh}^2} + \left(1 - \frac{R_V}{R_T}\right) . \frac{2}{3} . F_{nh}^{10}$$

$$m_1 = \frac{A_M}{A_T}$$

where A_M is the maximum sectional area of the model and A_T is the maximum sectional area of the full scale vessel.

Ship Flow of Transmission of Power

$$P_E = R_T V_S$$
$$P_T = T . V_A$$
$$K_T = \frac{T}{\rho n^2 D^4}$$
$$K_Q = \frac{Q}{\rho n^2 D^5}$$
$$J = \frac{V_A}{nD}$$
$$\eta_O = \frac{K_T}{K_Q} . \frac{J}{2\pi}$$
$$\eta_H = \frac{P_E}{P_T} = \frac{R_T . V_S}{T . V_A}$$
$$\eta_B = \frac{P_T}{P_D}$$
$$\eta_D = QPC = \frac{P_E}{P_D} = \eta_R \eta_O \eta_H = \frac{(1 - t)}{(1 - w)} \eta_B$$

Propeller Torque $Q = \frac{P_D}{2\pi n}$

Propeller Thrust $T = \frac{P_T}{V_A}$

$$\eta_R = \frac{\eta_B}{\eta_O} = \frac{P_T}{P_D} \cdot \frac{P_{DO}}{P_{TO}}$$

Thrust identity; $P_T = P_{TO}; \therefore \eta_R = \frac{P_{DO}}{P_D}$

Torque identity; $P_D = P_{DO}; \therefore \eta_R = \frac{P_T}{P_{TO}}$

$$\frac{P_E}{P_B} = \eta_S \eta_R \eta_O \eta_H$$

$$t = \frac{T - R_T}{T} \text{ or } \frac{R_T}{T} = 1 - t$$

$$\omega = \frac{V_S - V_A}{V_S} \text{ or } \frac{V_A}{V_S} = 1 - \omega$$

Apparent Slip $= \left(1 - \frac{V}{Pn}\right)$

True Propeller Slip $= \left(1 - \frac{V_A}{Pn}\right)$

Propeller Design using Charts and Polynomials

Known power, RPM and advance velocity

$$\frac{K_Q}{J^5} = \frac{Q}{\rho n^2 D^5}\left(\frac{nD}{V_A}\right)^5 = \frac{Qn^3}{\rho V_A^5}$$

Known power, diameter and advance velocity

$$\frac{K_Q}{J^3} = \frac{Q}{\rho n^2 D^5}\left(\frac{nD}{V_A}\right)^3 = \frac{Qn}{\rho D^2 V_A^3} = \frac{P_D}{2\pi\rho D^2 V_A^3}$$

Known thrust, diameter and advance velocity

$$\frac{K_T}{J^2} = \frac{T}{\rho n^2 D^4}\left(\frac{nD}{V_A}\right)^2 = \frac{T}{\rho V_A^2 D^2}$$

Known thrust, RPM and advance velocity

$$\frac{K_T}{J^4} = \frac{T}{\rho n^2 D^4}\left(\frac{nD}{V_A}\right)^4 = \frac{Tn^2}{\rho V_A^4}$$

Cavitation Considerations

Burrill's Method

$$V_R = \left[(0.7\pi nD)^2 + V_A^2\right]^{\frac{1}{2}}$$

$$P_O = P_{atm} + \rho g h$$

Atmospheric pressure

$$P_{atm} = 101300 N/m^2$$

Vapour pressure of water

$P_V = 1700\ N/m^2$ at 15°C

$$\sigma = \frac{P_O - P_V}{q_T}$$

$$q_T = 0.5\rho V_R^2$$

$$\frac{A_P}{A_D} = \left[1.067 - 0.229\frac{P}{D}\right]$$

$$\tau_C = \frac{T}{\frac{1}{2}\rho A_P V_{R(0.7)}^2}$$

Keller's method

$$\frac{A_E}{A_O} = \frac{(1.3 + 0.3Z)T}{(P_O - P_V)D^2} + K$$

Propeller Lifting Line Theory

$$\tan\beta = \frac{V_A}{2\pi nr} = \frac{J}{\pi x}$$

$$\eta_i = \frac{\tan\beta}{\tan\beta_i}$$

$$\therefore \tan\beta_i = \frac{\tan\beta}{\eta_i}$$

$$a' = \frac{\tan^2\beta_i(1-\eta_i)}{1+\tan^2\beta_i}$$

$$k'T_i = \pi^3 k x^3 a'(1-a')$$

$$k'Q_i = k'T_i \frac{\lambda_i}{2}$$

$$K'T = k'T_i(1-\varepsilon\tan\beta_i)$$

Wake fraction integration

$$w_T = \frac{\int_{r_B}^{R} w'_T \cdot 2\pi r \cdot dr}{\int_{r_B}^{R} 2\pi r \cdot dr} = \frac{\int_{r_B}^{R} w'_T \cdot r \cdot dr}{\frac{1}{2}\left(R^2 - r_B^2\right)}$$

where;

$$w'_T = \frac{1}{2\pi}\int_0^{2\pi} w''_T \cdot d\theta$$

and R is the propeller radius and r_B is the boss radius.