CONFIDENTIAL

UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF MARINE ENGINERING TECHNOLOGY

FINAL EXAMINATION
JANUARY 2016 SEMESTER

COURSE CODE	:	LGB 20903
COURSE NAME	:	QUANTITATIVE ANALYSIS
PROGRAMME NAME	:	BACHELOR OF ENGINEERING TECHNOLOGY IN MARINE ELECTRICAL AND ELECTRONICS
DATE	:	18 MAY 2016
TIME	:	02.00 PM – 05.00 PM
DURATION	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please CAREFULLY read the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of TWO (2) sections; Section A and Section B.
4. Answer ALL FOUR (4) questions in Section A. For Section B, answer THREE (3) questions ONLY.
5. Please write your answers on answer sheet provided.
6. Answer all questions in English language ONLY.
7. FORMULA has been appended for your reference.

THERE ARE 5 PAGES OF QUESTIONS, INCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL FOUR questions.
Please use the answer booklet provided.

Question 1

(a) If given $g(z)=z+2$ and $f(z)=z+1$, show that the multiplication of $f(z)$ and $g(z)$ satisfies the Cauchy-Riemann which analytic in some region of complex variable.

(6 marks)

(b) Determine the residue of $f(z)=\dfrac{2z}{(z^2-1)}$ at each of its poles in the finite Z plane.

(4 marks)

Question 2

(a) Solve the following using table of Laplace transform.

i. $f(t)=3e^{-4t}\sin(3t)+5t-7e^{6t}t^3$

(3 marks)

ii. $f(t)=3t\cos(5t)-7e^{6t}\cosh(4t)$

(3 marks)

(b) Show that the Laplace transform of $f(t)=e^{-6t}$ is $F(s)=\dfrac{1}{s+6}$ by using the definition of Laplace transform.

(4 marks)

Question 3

Given the unit impulse response $g(t) = 5e^{6t}$. Define the convolution between the unit impulse responses to

(a) a step input function

(5 marks)

(b) an exponential function, $f(t) = e^{-2t}$

(5 marks)

Question 4

By using the table of Fourier transform, solve Fourier transform by transforming the following function.

(a) $f(t) = 9\sin c\left(\frac{3(t-4)}{7}\right)$

(5 marks)

(b) $f(t) = 6rect(t)$ for $f(3t-6)$

(5 marks)

SECTION B (Total Marks: 80 marks)

INSTRUCTION: Answer only THREE questions (60 marks).
Please use the answer booklet provided.

Question 5

(a) Integrate of $f(z)=\dfrac{4z^3-20}{16z-i}$ using Cauchy Integral formula.

(4 marks)

(b) Given the contour integral $\oint_c \dfrac{3z^2+2}{(z-1)(z^2+9)}dz$ if C is:

i. Determine the residue for each poles in the finite plane.

(10 marks)

ii. Define Cauchy residue of the contour at circle $|z|=1.5$ and $|z|=15.5$

(6 marks)

Question 6

(a) Given the differential equation of the system is $f'(t)+9f(t)=1$ subject to the initial conditions $f(0)=1$

(10 marks)

(b) Solve the inverse of Laplace transform of the function $F(s)=\dfrac{5s}{s^2+2s+26}$.

(10 marks)

Question 7

Convolve the following two functions as shown in Figure 1.

(20 marks)

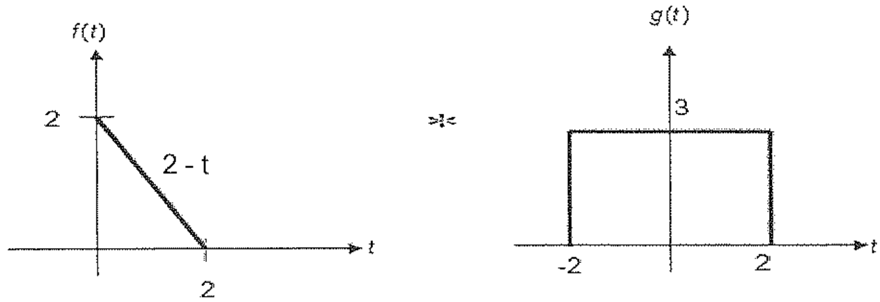

Figure 1

Question 8

(a) Determine Fourier Transform of convolution $f(t) = 6rect(5t)$ and $x^{'}(t)$ if given $x(t) = 2\delta(t-9)$.

(10 marks)

(b) Define the definition of Fourier transform of $x(t) = t$ for the range of $0 \leq t < \infty$.

(10 marks)

END OF QUESTIONS

INTEGRATION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int \cos x\,dx = \sin x + c$	$\int \cos f(x)dx = \frac{\sin f(x)}{f'(x)} + c$
$\int \sin x\,dx = -\cos x + c$	$\int \sin f(x)dx = \frac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x\,dx = \tan x + c$	$\int \sec^2 f(x)dx = \frac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x\,dx = \sec x + c$	$\int \sec f(x) \tan f(x)dx = \frac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x\,dx = -\csc x + c$	$\int \csc f(x) \cot f(x)dx = \frac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x\,dx = -\cot x + c$	$\int \csc^2 f(x)dx = \frac{-\cot f(x)}{f'(x)} + c$
$\int \tan x\,dx = \ln\lvert\sec x\rvert + c$	$\int \tan x\,dx = \frac{\ln\lvert\sec f(x)\rvert}{f'(x)} + c$
$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + c$	$\int \sec x\,dx = \frac{\ln\lvert\sec f(x) + \tan f(x)\rvert}{f'(x)} + c$
$\int \cot x\,dx = \ln\lvert\sin x\rvert + c$	$\int \cot x\,dx = \frac{\ln\lvert\sin f(x)\rvert}{f'(x)} + c$
$\int \csc x\,dx = -\ln\lvert\csc x + \cot x\rvert + c$	$\int \csc x dx = \frac{-\ln\lvert\csc f(x) + \cot f(x)\rvert}{f'(x)} + c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int e^x dx = e^x + c$	$\int e^{f(x)}\,dx = \frac{e^{f(x)}}{f'(x)} + c$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int \frac{1}{x}dx = \ln\lvert x\rvert + c$	$\int \frac{1}{f(x)}dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + c$

DIFFERENTIATION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$

TRIGONOMETRY IDENTITIES

FUNDAMENTAL IDENTITIES	FORMULAS FOR NEGATIVES
$\csc\theta = \dfrac{1}{\sin\theta}$	$\sin(-\theta) = -\sin\theta$
$\sec\theta = \dfrac{1}{\cos\theta}$	$\cos(-\theta) = \cos\theta$
$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$	$\tan(-\theta) = -\tan\theta$
$\cos^2\theta = \dfrac{1}{2}(1+\cos 2\theta)$	$\csc(-\theta) = -\csc\theta$
$\sin^2\theta + \cos^2\theta = 1$	$\sec(-\theta) = \sec\theta$
$1+\tan^2\theta = \sec^2\theta$	$\cot(-\theta) = -\cot\theta$
$1+\cot^2\theta = \csc^2\theta$	$\sin^2\theta = \dfrac{1}{2}(1-\cos 2\theta)$

ADDITION FORMULAS	SUBTRACTION FORMULAS
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$	$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

HALF-ANGLE FORMULAS	DOUBLE-ANGLE FORMULAS
$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$	$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\ldots\ldots\ldots = 1 - 2\sin^2\theta$ $\ldots\ldots\ldots = 2\cos^2\theta - 1$
$\tan\dfrac{\theta}{2} = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$	$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

PRODUCT-TO-SUM FORMULAS	SUM-TO-PRODUCT FORMULAS
$\sin\alpha\cos\beta = \dfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$	$\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$
$\cos\alpha\sin\beta = \dfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$	$\sin\alpha - \sin\beta = 2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$
$\cos\alpha\cos\beta = \dfrac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$	$\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

COMPLEX ANALYSIS

$$\int_{\Gamma} f(z)dz = \int_{a}^{b} f(\Gamma(t))\Gamma'(t)dt$$

$$\oint_{c} \frac{f(z)}{z - z_0} dz = 2\pi i f(z_0)$$

Residue at pole z $= \lim_{z \to z_0} [(z - z_0)f(z)]$

Residue at pole z $= \frac{1}{(m-1)!} \lim_{z \to z_0} \left\{ \frac{d^{m-1}}{dz^{m-1}} [(z - z_0)^m f(z)] \right\}$

$$\oint_{c} f(z)dz = 2\pi j [\text{sum of residue of } f(z) \text{ at poles inside } C]$$

Property	$x(t)$	$X(f)$	$X(\omega)$
Similarity	$X(t)$	$x(-f)$	$2\pi x(-\omega)$
Time Scaling	$x(\alpha t)$	$\frac{1}{\lvert\alpha\rvert}X\left(\frac{f}{\alpha}\right)$	$\frac{1}{\lvert\alpha\rvert}X\left(\frac{\omega}{\alpha}\right)$
Folding	$x(-t)$	$X(-f)$	$X(-\omega)$
Time Shift	$x(t-\alpha)$	$e^{-j2\pi f\alpha}X(f)$	$e^{-j\omega\alpha}X(\omega)$
Frequency Shift	$e^{j2\pi\alpha t}x(t)$	$X(f-\alpha)$	$X(\omega-2\pi\alpha)$
Convolution	$x(t)\star h(t)$	$X(f)H(f)$	$X(\omega)H(\omega)$
Multiplication	$x(t)h(t)$	$X(f)\star H(f)$	$\frac{1}{2\pi}X(\omega)\star H(\omega)$
Modulation	$x(t)\cos(2\pi\alpha t)$	$0.5[X(f+\alpha)+X(f-\alpha)]$	$0.5[X(\omega+2\pi\alpha)+X(\omega-2\pi\alpha)]$
Derivative	$x'(t)$	$j2\pi fX(f)$	$j\omega X(\omega)$
Times-t	$-j2\pi tx(t)$	$X'(f)$	$2\pi X'(\omega)$
Integration	$\int_{-\infty}^{t} x(t)\,dt$	$\frac{1}{j2\pi f}X(f)+0.5X(0)\delta(f)$	$\frac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$
Conjugation	$x^*(t)$	$X^*(-f)$	$X^*(-\omega)$
Correlation	$x(t)\star\star y(t)$	$X(f)Y^*(f)$	$X(\omega)Y^*(\omega)$
Autocorrelation	$x(t)\star\star x(t)$	$X(f)X^*(f)=\lvert X(f)\rvert^2$	$X(\omega)X^*(\omega)=\lvert X(\omega)\rvert^2$

Fourier Transform Theorems	
Central ordinates	$x(0)=\int_{-\infty}^{\infty} X(f)\,df=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\,d\omega \qquad X(0)=\int_{-\infty}^{\infty} x(t)\,dt$
Parseval's theorem	$E=\int_{-\infty}^{\infty} x^2(t)\,dt=\int_{-\infty}^{\infty} \lvert X(f)\rvert^2\,df=\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2\,d\omega$
Plancherel's theorem	$\int_{-\infty}^{\infty} x(t)y^*(t)\,dt=\int_{-\infty}^{\infty} X(f)Y^*(f)\,df=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)Y^*(\omega)\,d\omega$

FOURIER TRANSFORM

Entry	$x(t)$	$X(f)$	$X(\omega)$
1	$\delta(t)$	1	1
2	$\text{rect}(t)$	$\text{sinc}(f)$	$\text{sinc}\left(\frac{\omega}{2\pi}\right)$
3	$\text{tri}(t)$	$\text{sinc}^2(f)$	$\text{sinc}^2\left(\frac{\omega}{2\pi}\right)$
4	$\text{sinc}(t)$	$\text{rect}(f)$	$\text{rect}\left(\frac{\omega}{2\pi}\right)$
5	$\cos(2\pi\alpha t)$	$0.5[\delta(f+\alpha)+\delta(f-\alpha)]$	$\pi[\delta(\omega+2\pi\alpha)+\delta(\omega-2\pi\alpha)]$
6	$\sin(2\pi\alpha t)$	$j0.5[\delta(f+\alpha)-\delta(f-\alpha)]$	$j\pi[\delta(\omega+2\pi\alpha)-\delta(\omega-2\pi\alpha)]$
7	$e^{-\alpha t}u(t)$	$\frac{1}{\alpha+j2\pi f}$	$\frac{1}{\alpha+j\omega}$
8	$te^{-\alpha t}u(t)$	$\frac{1}{(\alpha+j2\pi f)^2}$	$\frac{1}{(\alpha+j\omega)^2}$
9	$e^{-\alpha\|t\|}$	$\frac{2\alpha}{\alpha^2+4\pi^2 f^2}$	$\frac{2\alpha}{\alpha^2+\omega^2}$
10	$e^{-\pi t^2}$	$e^{-\pi f^2}$	$e^{-\omega^2/4\pi}$
11	$\text{sgn}(t)$	$\frac{1}{j\pi f}$	$\frac{2}{j\omega}$
12	$u(t)$	$0.5\delta(f)+\frac{1}{j2\pi f}$	$\pi\delta(\omega)+\frac{1}{j\omega}$
13	$e^{-\alpha t}\cos(2\pi\beta t)u(t)$	$\frac{\alpha+j2\pi f}{(\alpha+j2\pi f)^2+(2\pi\beta)^2}$	$\frac{\alpha+j\omega}{(\alpha+j\omega)^2+(2\pi\beta)^2}$
14	$e^{-\alpha t}\sin(2\pi\beta t)u(t)$	$\frac{2\pi\beta}{(\alpha+j2\pi f)^2+(2\pi\beta)^2}$	$\frac{2\pi\beta}{(\alpha+j\omega)^2+(2\pi\beta)^2}$
15	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$
16	$x_p(t)=\sum_{k=-\infty}^{\infty}X[k]e^{j2\pi kf_0t}$	$\sum_{k=-\infty}^{\infty}X[k]\delta(f-kf_0)$	$\sum_{k=-\infty}^{\infty}2\pi X[k]\delta(\omega-k\omega_0)$

	$f(t)$	$F(s) = \mathfrak{L}[f(t)](s)$
26.	$\sinh(at) - \sin(at)$	$\frac{2a^3}{s^4 - a^4}$
27.	$\cosh(at) - \cos(at)$	$\frac{2a^2 s}{s^4 - a^4}$
28.	$\frac{1}{\sqrt{\pi t}} e^{at}(1 + 2at)$	$\frac{s}{(s-a)^{3/2}}$
29.	$J_0(at)$	$\frac{1}{\sqrt{s^2 + a^2}}$
30.	$J_n(at)$	$\frac{1}{a^n} \frac{\left(\sqrt{s^2+a^2} - s\right)^n}{\sqrt{s^2 + a^2}}$
31.	$J_0(2\sqrt{at})$	$\frac{1}{s} e^{-a/s}$
32.	$\frac{1}{t}\sin(at)$	$\tan^{-1}\left(\frac{a}{s}\right)$
33.	$\frac{2}{t}[1 - \cos(at)]$	$\ln\left(\frac{s^2 + a^2}{s^2}\right)$
34.	$\frac{2}{t}[1 - \cosh(at)]$	$\ln\left(\frac{s^2 - a^2}{s^2}\right)$
35.	$\frac{1}{\sqrt{\pi t}} - a e^{a^2 t} \operatorname{erfc}\left(\frac{a}{\sqrt{t}}\right)$	$\frac{1}{\sqrt{s} + a}$
36.	$\frac{1}{\sqrt{\pi t}} + a e^{a^2 t} \operatorname{erf}\left(\frac{a}{\sqrt{t}}\right)$	$\frac{\sqrt{s}}{s - a^2}$
37.	$e^{a^2 t} \operatorname{erf}(a\sqrt{t})$	$\frac{a}{\sqrt{s}(s - a^2)}$
38.	$e^{a^2 t} \operatorname{erfc}(a\sqrt{t})$	$\frac{1}{\sqrt{s}(\sqrt{s} + a)}$
39.	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{1}{s} e^{-a\sqrt{s}}$
40.	$\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\frac{1}{\sqrt{s}} e^{-a\sqrt{s}}$
41.	$\frac{1}{\sqrt{\pi(t+a)}}$	$\frac{1}{\sqrt{s}} e^{as} \operatorname{erfc}(\sqrt{as})$
42.	$\frac{1}{\pi t}\sin(2a\sqrt{t})$	$\operatorname{erf}\left(\frac{a}{\sqrt{s}}\right)$
43.	$f\left(\frac{t}{a}\right)$	$aF(as)$
44.	$e^{bt/a} f\left(\frac{t}{a}\right)$	$aF(as - b)$
45.	$\delta_\epsilon(t)$	$\frac{e^{-\epsilon s}(1 - e^{-\epsilon s})}{\epsilon s}$
46.	$\delta(t - a)$	e^{-as}
47.	$L_n(t)$ (Laguerre polynomial)	$\frac{1}{s}\left(\frac{s-1}{s}\right)^n$

LAPLACE TRANSFORM

	$f(t)$	$F(s) = \mathfrak{L}[f(t)](s)$
1.	1	$\frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	$t^n (n = 1, 2, 3, \cdots)$	$\frac{n!}{s^{n+1}}$
4.	$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
5.	e^{at}	$\frac{1}{s-a}$
6.	te^{at}	$\frac{1}{(s-a)^2}$
7.	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
8.	$\frac{1}{a-b}(e^{at} - e^{bt})$	$\frac{1}{(s-a)(s-b)}$
9.	$\frac{1}{a-b}(ae^{at} - be^{bt})$	$\frac{s}{(s-a)(s-b)}$
10.	$\frac{(c-b)e^{at} + (a-c)e^{bt} + (b-a)e^{ct}}{(a-b)(b-c)(c-a)}$	$\frac{1}{(s-a)(s-b)(s-c)}$
11.	$\sin(at)$	$\frac{a}{s^2+a^2}$
12.	$\cos(at)$	$\frac{s}{s^2+a^2}$
13.	$1 - \cos(at)$	$\frac{a^2}{s(s^2+a^2)}$
14.	$at - \sin(at)$	$\frac{a^3}{s^2(s^2+a^2)}$
15.	$\sin(at) - at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
16	$\sin(at) + at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
17.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$
18.	$t\cos(at)$	$\frac{(s^2-a^2)}{(s^2+a^2)^2}$
19.	$\frac{\cos(at) - \cos(bt)}{(b-a)(b+a)}$	$\frac{s}{(s^2+a^2)(s^2+b^2)}$
20.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
21.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
22.	$\sinh(at)$	$\frac{a}{s^2-a^2}$
23.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
24.	$\sin(at)\cosh(at) - \cos(at)\sinh(at)$	$\frac{4a^3}{s^4+4a^4}$
25.	$\sin(at)\sinh(at)$	$\frac{2a^2 s}{s^4+4a^4}$