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Solving Second Order Delay Differential Equations by Direct Two and Three Point One-Step Block Method

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Abstract

In this paper we present a two point and three point one-step block method for solving second order delay differential equations (DDEs). The one-step block method will solve directly the second order DDEs without reducing to first order equations. The two point and three point one-step block method will compute the solutions for the DDEs at two and three points simultaneously along the interval. These methods will solve the retarded type of DDE of single delay using constant step size. The P- stability and Q-stability are also discussed. The numerical results are presented to illustrate the performance of those block method for solving delay differential equations.

Mathematics Subject Classification: 65L06, 65L10

Keywords: Delay differential equations, Two point block, Direct method

1 Introduction

Delay differential equations are similar to ordinary differential equations, but their evolution involves past values of the state variable. The solution of delay differential equations requires knowledge of not only the current state, but also of the state at a certain time previously. Delay differential equations (DDEs) have numerous applications in science and engineering, for example in population dynamics and bioscience problems.

Generally, a DDE refer to both retarded type of DDE (RDE) and neutral type of DDE (NDDE). In this research, we only concerned with DDE of the retarded type (RDE). One-step block method is used to solve second order DDEs of the form

$$\begin{aligned} y''(x) &= f(x, y, y(x-\tau)), \qquad \text{for } a \le x \le b \\ y(x) &= \varphi(x), \qquad \text{for } x \le a \end{aligned} \tag{1.1}$$

which has one delay term. $\varphi(x)$ is the initial function, $\tau(x, y(x))$ is called the delay, $x-\tau(x, y(x))$ is called the delay argument and the value of $y(x-\tau(x, y(x)))$ is the solution of the delay term. The delay is called constant delay if it is a constant, it is called time dependent delay if the delay is function of time x and the delay is known as state dependent delay if it is a function of time x and $y(x)$.

It appears in the literature that there is wide interest in the numerical solution of DDE and many approaches have been adopted for solving particular equations. Such work can be found in Bellen and Zennaro [1], Al-Mutib [2], Evans and Raslan [3], El-Hawary and El-Shami [5], Oberle and Pesh [6], Martin and Garcia [8], Taiwo and Odetunde [10], and Radzi et al. [7].

Bellen and Zennaro [1] presented the algorithms based on the predictor-corrector version of the one-step collocation method at Gaussion points for non-stiff DDEs with time dependent delays. Ismail et al. [4] compared the numerical results based on Newton divided difference and In’t Hout Interpolations in solving delay differential equations. Several techniques have been proposed to approximate the delayed term. For instance, Barwell [11] implemented linear and quadratic interpolation whereas Radzi et al. [7] used Neville’s interpolation to compute the delay term. Radzi et al. [7] discussed the two and three point one-step block method for the treatment of first order delay differential equations. Numerical results are presented to show the performance based on those block methods for solving delay differential equations. The numerical results shown that the three point one-step method is more efficient compared to the two point one-step method. Martin and Garcia [8] developed variable step size multistep methods for higher order delay differential equations. Evans and

Raslan [3] developed a numerical method for linear and non linear higher order delay differential equations based on the Adomian decomposition method. While Taiwo and Odetunde [10] proposed a new decomposition method for the numerical solution of the second order delay differential equations.

The approach in this research is to extend the proposed block method in Mukhtar et al. [9] and Majid et al. [12] for solving equation (1.1) directly without reducing to system of first order DDEs using two point and three point one-step block method.

2 Formulation and Implementation

Most numerical methods for solving ordinary differential equations (ODEs)

$$y''(x) = f(x, y, y'), \quad y(a) = y_0, \; y'(a) = y'_0 \qquad x \in [a,b] \tag{2.1}$$

can be adapted to solve DDE. In this paper, we implement two point one-step block method proposed by Mukhtar et al. [9] and Majid et al. [12] to solve DDEs. The formulae of the one-step block methods are as follows:

Two point one-step block method:

$$\begin{aligned}
y'_{n+1} &= y'_n + \frac{h}{12}\left(5f_n + 8f_{n+1} - f_{n+2}\right)\\
y_{n+1} &= y_n + hy'_n + \frac{h^2}{24}\left(7f_n + 6f_{n+1} - f_{n+2}\right)\\
y'_{n+2} &= y'_{n+1} + \frac{h}{12}\left(-f_n + 8f_{n+1} + 5f_{n+2}\right)\\
y_{n+2} &= y_{n+1} + hy'_{n+1} + \frac{h^2}{24}\left(-f_n + 10f_{n+1} + 3f_{n+2}\right)
\end{aligned} \tag{2.2}$$

Three point one-step block method:

$$\begin{aligned}
y'_{n+1} &= y'_n + \frac{h}{24}(9f_n + 19f_{n+1} - 5f_{n+2} + f_{n+3})\\
y_{n+1} &= y_n + hy'_n + \frac{h^2}{360}(97f_n + 114f_{n+1} - 39f_{n+2} + 8f_{n+3})\\
y'_{n+2} &= y'_{n+1} + \frac{h}{24}(-f_n + 13f_{n+1} + 13f_{n+2} - f_{n+3})\\
y_{n+2} &= y_{n+1} + hy'_{n+1} + \frac{h^2}{360}(-8f_n + 129f_{n+1} + 66f_{n+2} - 7f_{n+3})\\
y'_{n+3} &= y'_{n+2} + \frac{h}{24}(f_n - 5f_{n+1} + 19f_{n+2} + 9f_{n+3})\\
y_{n+3} &= y_{n+2} + hy'_{n+2} + \frac{h^2}{360}(7f_n - 36f_{n+1} + 171f_{n+2} + 38f_{n+3})
\end{aligned} \tag{2.3}$$

We start by solving second order delay differential equations in the form of (1.1) in the interval $[a,b]$. We implement this method using C programming with simple iterations to generate the approximate y-values. The input of the program are the endpoints a and b, initial condition y_0, y'_0 and step size h. The output of the program are the values x which are used to calculate each y-approximation, y'-approximation, step size h, exact solution, y-exact, the error between approximation and exact solution, maximum error and execution time.

The algorithm for the 2PBDDE code is given as follows:

Step 1 : Set $h = \frac{b-a}{N}$;

Step 2 : Set the initial values x_0, y_0, $(x_0 - \tau)$, $(y_0 - \tau)$ and $f(x_0, y_0, (y_0 - \tau))$.

Step 3 : Calculate values of x_1 and x_2 using the direct Adam Bashford

Step 4 : Calculate the predictors values of y_1, y_2, y_1' and y_2'.

Step 5 : Calculate value of delay argument of the equation, $(x_i - \tau)$.

Step 6 : Locate the position of $(x_i - \tau)$. If $(x_i - \tau) \le x_0$, use the initial function to approximate the delay term $y(x_i - \tau)$ else approximate values of delay term using divided difference interpolation.

Step 7 : Calculate values of f_1 and f_2 using values of y_1, y_2, y'_1, y'_2, $y(x_1 - \tau)$ and $y(x_2 - \tau)$ obtain from **Step 6**.

Step 8 : Calculate the corrector values for y_1, y_2, y'_1 and y'_2 using the two point one - step block method formula:

$$y'_1 = y'_0 + \frac{h}{12}(5f_0 + 8f_1 - f_2)$$

$$y_1 = y_0 + hy'_0 + \frac{h^2}{24}(7f_0 + 6f_1 - f_2)$$

$$y'_2 = y'_1 + \frac{h}{12}(-f_0 + 8f_1 + 5f_2)$$

$$y_2 = y_1 + hy'_1 + \frac{h^2}{24}(-f_0 + 10f_1 + 3f_2)$$

Step 9 : Calculate the error.

Step 10: Update values of x_0, x_1, y_0, y_1, f_0 and f_1 for the next block. Go to **Step 2.**

Step 11: The procedure is complete.

The codes were implemented in *PECE* scheme where *P* and *C* denote the application of predictor and corrector respectively while *E* denote the evaluation of function f. Now, we described how the calculation of $y(\alpha)$ where $\alpha = x - \tau(x, y(x))$ is being carried out. The location of α is sought because the calculation of the delay term depends on this location. We should use the interpolation method which has either the same or higher order than the integration method in order to preserve the desired order of accuracy. Here the delay term is approximated using four points divided difference interpolation if values of y_{n+m} is obtained by two point block method. Otherwise, five points divided difference interpolation is applied if values of y_{n+m} is obtained by three point block method. In divided difference form, the interpolating polynomial can be written as

$$P_n(x) = y[x_0] + (x - x_0)y[x_0, x_1] + (x - x_0)(x - x_1)y[x_0, x_1, x_2] + \ldots$$
$$+ (x - x_0)\ldots(x - x_{n-1})y[x_0, x_1, \ldots, x_n]$$

where

$$y[x_0, x_1, \ldots, x_n] = \frac{y[x_1, x_2, \ldots, x_n] - y[x_0, x_1, \ldots, x_{n-1}]}{x_n - x_0}.$$

3 Stability Analysis

In this section, we will discuss the stability analysis of two point one-step block method for the numerical solution of delay differential equations (DDEs). We will find the P-stability and Q-stability of the method using the following test equations,

$$\begin{aligned} y'' &= \lambda y(x) + \mu y(x - \tau), \qquad & x \ge x_0 \\ y(x) &= \phi(x), & x \le x_0 \end{aligned} \tag{3.1}$$

and

$$\begin{aligned} y'' &= \mu y(x - \tau), \qquad & x \ge x_0 \\ y(x) &= \phi(x), & x \le x_0 \end{aligned} \tag{3.2}$$

where the parameters $\lambda, \mu \in R$. Consider a fixed step size h such that $x_n = x_0 + nh$ and $mh = \tau$, $m \in I^+$ and letting $H_1 = h^2\lambda$ and $H_2 = h^2\mu$,

Definition 4.1: For a fixed step size h and λ, $\mu \in R$ in (3.1), the region R_P in the complex $H_1 - H_2$ plane is called the P-stability region if for any $(H_1, H_2) \in R_P$, the numerical solution of (3.1) vanishes as $x_n \to \infty$.

Definition 4.2: For a fixed step size h and $\mu \in C$ in (3.2), the region R_Q in the complex H_2 plane is called the Q-stability region if for any $H_2 \in R_Q$, the numerical solution of (3.2) vanishes as $x_n \to \infty$.

3.1 P-Stability Analysis

The general form of two point and three point one-step block method can be written in the matrix form as

$$A_2 Y_{N+2} = A_1 Y_{N+1} + hB_2 Y_{N+2} + hB_1 Y_{N+1} + h^2 C_1 F_{N+1} \tag{3.3}$$

where

Two point one-step block method:

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix},\ A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ C_1 = \begin{bmatrix} 0 & \frac{5}{12} & \frac{8}{12} & \frac{-1}{12} \\ 0 & \frac{7}{24} & \frac{6}{24} & \frac{-1}{24} \\ 0 & \frac{-1}{12} & \frac{8}{12} & \frac{5}{12} \\ 0 & \frac{-1}{24} & \frac{10}{24} & \frac{3}{24} \end{bmatrix}$$

$$Y_{N+2} = \begin{bmatrix} y'_{n+1} \\ y_{n+1} \\ y'_{n+2} \\ y_{n+2} \end{bmatrix},\ Y_{N+1} = \begin{bmatrix} y'_{n-1} \\ y_{n-1} \\ y'_{n} \\ y_{n} \end{bmatrix}, F_{N+1} = \begin{bmatrix} f_{n-1} \\ f_{n} \\ f_{n+1} \\ f_{n+2} \end{bmatrix}.$$

Three point one-step block method:

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix},\ A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 0 & \frac{9}{24} & \frac{19}{24} & -\frac{5}{24} & \frac{1}{24} \\ 0 & 0 & \frac{97}{360} & \frac{114}{360} & -\frac{39}{360} & \frac{8}{360} \\ 0 & 0 & -\frac{1}{24} & \frac{13}{24} & \frac{13}{24} & \frac{-1}{24} \\ 0 & 0 & -\frac{8}{360} & \frac{129}{360} & \frac{6}{360} & -\frac{7}{360} \\ 0 & 0 & \frac{1}{24} & -\frac{5}{24} & \frac{19}{24} & \frac{9}{24} \\ 0 & 0 & \frac{7}{360} & -\frac{36}{360} & \frac{171}{360} & \frac{38}{360} \end{bmatrix}$$

$$Y_{N+2} = \begin{bmatrix} y'_{n+1} \\ y_{n+1} \\ y'_{n+2} \\ y_{n+2} \\ y'_{n+3} \\ y_{n+3} \end{bmatrix}, \ Y_{N+1} = \begin{bmatrix} y'_{n-2} \\ y_{n-2} \\ y'_{n-1} \\ y_{n-1} \\ y'_{n} \\ y_{n} \end{bmatrix}, \ F_{N+1} = \begin{bmatrix} f_{n-2} \\ f_{n-1} \\ f_{n} \\ f_{n+1} \\ f_{n+2} \\ f_{n+3} \end{bmatrix}.$$

The following result obtained when (3.3) is applied to (2.1)

$$A_2 Y_{N+2} = A_1 Y_{N+1} + hB_2 Y_{N+2} + hB_1 Y_{N+1} + h^2 C_1 \left(\lambda Y_{N+1} + \mu Y_{N+1-m}\right) \tag{3.4}$$

Rearranging we have

$$\left(A_1 + hB_1 + h^2 C_1 \lambda\right) Y_{N+1} + \left(-A_2 + hB_2\right) Y_{N+2} + h^2 \mu C_1 Y_{N+1-m} = 0 \tag{3.5}$$

The P-stability regions are sketched in the $H_1 - H_2$ plane by using the boundary locus technique where the boundary is determined by letting $\varsigma = 1$, $\varsigma = -1$ and $\varsigma = e^{i\theta}$ for $0 \le \theta \le 2\pi$. For $\varsigma = e^{i\theta}$, the boundary of the stability region is obtained by solving two simultaneous equations

$$\Re\left\{\pi_{2PB,m}\left(H_1, H_2; e^{i\theta}\right)\right\} \text{ and } \Im\left\{\pi_{2P!B,m}\left(H_1, H_2; e^{i\theta}\right)\right\} = 0 \tag{3.6}$$

The P-stability polynomial for the two point one-step block method is given by

$$\pi_{2PB,m}\left(H_1, H_2; \varsigma\right) = \det\left|\left(A_1 + hB_1 + H_1 C_1\right)\varsigma^{1+m} + \left(-A_2 + hB_2\right)\varsigma^{2+m} + H_2 C_1 \varsigma^1\right| \tag{3.7}$$

In this research, (3.7) is also applied to the three point one-step block method for finding the P-stability regions. Taking $m = 1$ for all cases, the regions for two point and three point one-step block method are depicted in Figure 1 and 2 respectively.

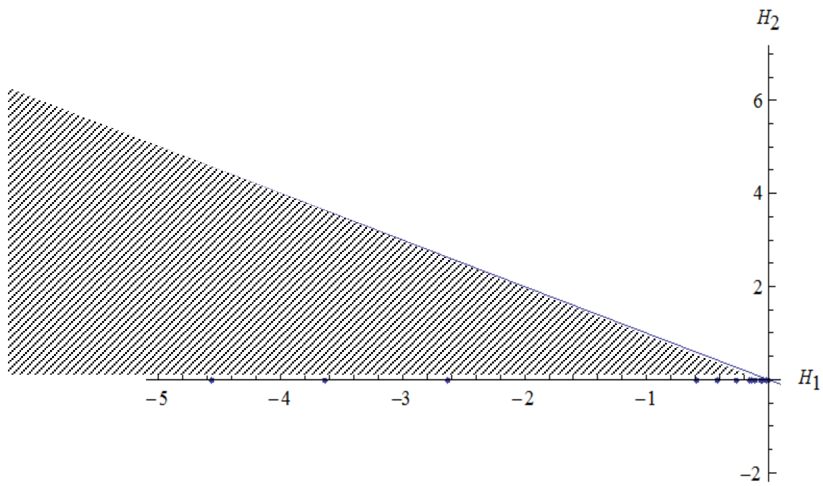

Figure 1: P-stability region for two point one-step block method

The P-stability regions of those methods lie inside the open ended regions given in Figure 1 and 2. From the figures, it is observed that the regions are about the same when the method is implemented in two point and three point one-step block method.

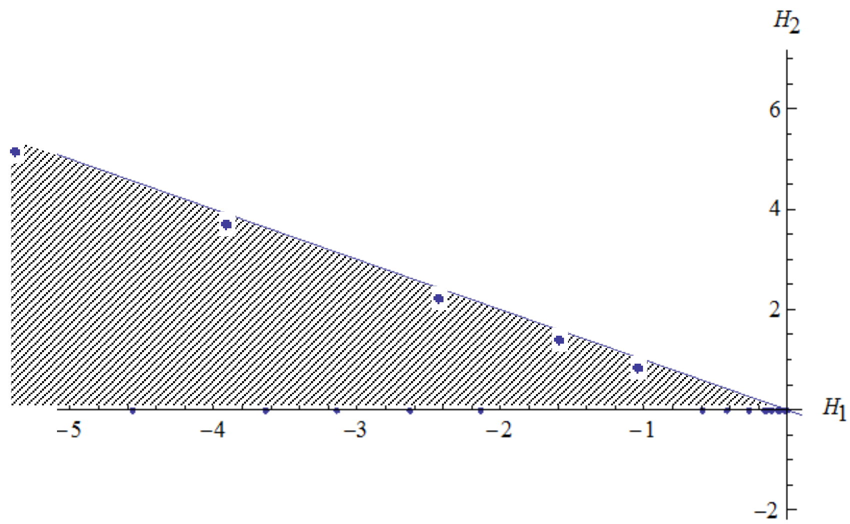

Figure 2: P-stability region for three point one-step block method

3.2 Q-Stability Analysis

The Q-stability regions are sketched in the complex H_2-plane. By using the boundary locus technique, the regions are determined by solving

$$\psi_{2P1B,m}\left(H_2; e^{i\theta}\right) = 0. \tag{3.8}$$

The general Q-stability polynomial for the two point one-step block method, $\psi_{2P1B,m}(H_2;\varsigma)$ is given by

$$\pi_{2PB,m}(H_1, H_2;\varsigma) = \det\left|(A_1 + hB_1)\varsigma^{1+m} + (-A_2 + hB_2)\varsigma^{2+m} + H_2 C_1 \varsigma^1\right|. \tag{3.9}$$

Eqn. (3.9) is also applied to the three point one-step block method for finding the Q-stability region. The Q-stability regions where the polynomial satisfies the root condition is sketched in the complex H_2-plane for two point and three point one-step block method are shown in Figure 3 and 4 respectively. The value of m is also considered as 1.

Below is the Q-stability polynomial and the stability regions for two point one-step block method:

$$\Psi_{2PB,m}(H_2;\varsigma) = \frac{1}{18}h^3\varsigma^{5+m} + \frac{1}{9}h^2\varsigma^{5+2m} - \frac{1}{9}h^2\varsigma^{6+2m} + \frac{1}{24}h\varsigma^{5+3m} + \frac{1}{4}ht^{6+3m}$$
$$-\frac{13}{8}h\varsigma^{7+3m} + \varsigma^{6+4m} - 2\varsigma^{7+4m} + \varsigma^{8+4m}$$

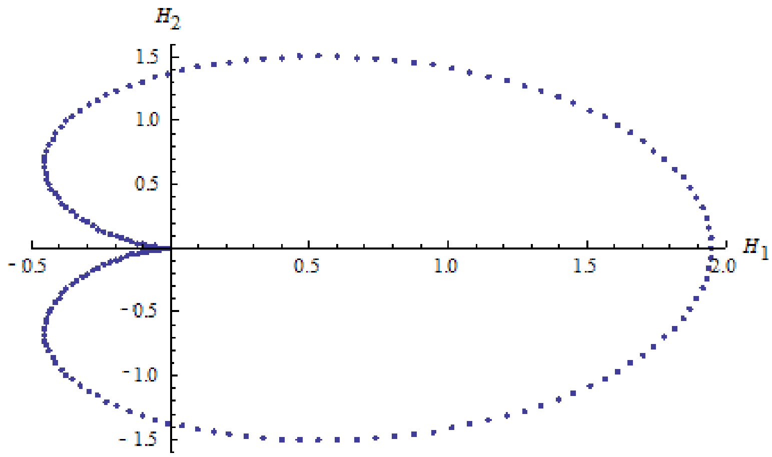

Figure 3: Q-stability region for two point one-step block method

The stability polynomial and Q-stability of the three point one-step block method is given as follows

$$\begin{aligned}\Psi_{3PB,m}(H_2;\varsigma)=&\frac{337}{8640}h^4\varsigma^{8+2m}-\frac{1}{60}h^3\varsigma^{8+3m}+\frac{45}{64}h^3\varsigma^{9+3m}-\frac{1}{8640}h^2\varsigma^{8+4m}\\&-\frac{701}{4320}h^2t^{9+4m}+\frac{3499}{1728}h^2t^{10+4m}+\frac{3}{20}ht^{9+5m}-\frac{63}{40}ht^{10+5m}\\&-\frac{123}{40}ht^{11+5m}+t^{10+6m}-2t^{11+6m}+t^{12+6m}\end{aligned}$$

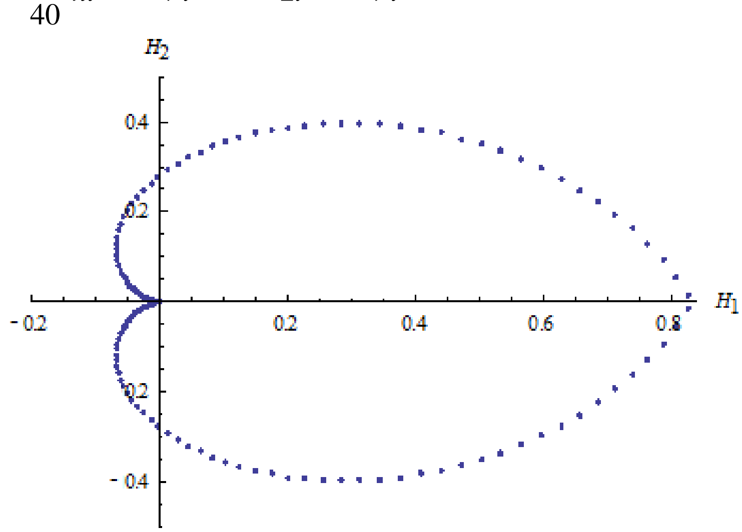

Figure 4: Q-stability region for three point one-step block method

The Q-stability regions of those methods lie inside the bounded regions given in Figure 3 and 4.

5 Numerical results

In order to study the efficiency of the developed codes, we present some numerical results for the following problems:

Problem 1

$$y''(x) = -\frac{1}{2}y(x) + \frac{1}{2}y(x-\pi), \qquad 0 \le x \le \pi$$
$$y(x) = 1 - \sin(x), \qquad -\pi \le x \le 0$$
$$y(0) = 1, \quad y'(0) = -1$$

Solution:

$$y(x) = 1 - \sin(x)$$

Problem 2

$$y''(x) = y(x-\pi), \qquad 0 \le x \le \pi$$
$$y(x) = \sin(x), \qquad -\pi \le x \le 0$$
$$y(0) = 0, \quad y'(0) = 1$$

Solution:

$$y(x) = \sin(x)$$

Problem 3

$$y''(x) = -y(x) + y(x-1), \qquad 0 \le x \le 2$$
$$y(x) = x^2 + 3x + 2, \qquad -1 \le x \le 0$$
$$y(0) = 2, \quad y'(0) = 0$$

Solution:

$$y(x) = 4\cos(x) - \sin(x) + x^2 + x - 2 \qquad 0 \le x \le 1$$
$$y(x) = -2t\cos t\sin 1 + \frac{t\cos t\cos 1}{2} + t^2 - 4 + 2\cos t\cos 1 + 2t\sin t\cos 1 - t$$
$$+\frac{\cos t\sin 1}{2} + \frac{t}{2}\sin t\sin 1 + \left(\frac{3\cos 1}{2} + 4\right)\cos t + \left(\frac{-\cos 1}{2} + \frac{7\sin 1}{2} - 1\right)\sin t \qquad 1 \le x \le 2$$

The following notations are used in Table 1-3:

h	Step size used
MTD	Method employed
MAXE	Magnitude of the maximum error of the computed solution
FCN	Total function calls
TIME	Execution times in microseconds
2PBDDE	Two point one-step block method for solving second order DDE directly
3PBDDE	Three point one-step block method for solving second order DDE directly

The tables below shown the numerical results for the three given problems when solved using the proposed block methods. The codes were written in C language.

Table 1: Comparison between the 2PBDDE and 3PBDDE for solving Problem 1

h	METHOD	MAXE	FCN.	TIME (μ s)
$\frac{\pi}{30}$	2PBDDE	1.211031e-04	75	90
	3PBDDE	2.414116e-04	70	82
$\frac{\pi}{300}$	2PBDDE	1.173845e-07	750	194
	3PBDDE	2.337067e-07	700	162
$\frac{\pi}{3000}$	2PBDDE	9.798482e-11	7500	1118
	3PBDDE	1.781091e-11	7000	770

Table 2: Comparison between the 2PBDDE and 3PBDDE for solving Problem 2

h	METHOD	MAXE	FCN	TIME (μ s)
$\frac{\pi}{30}$	2PBDDE	1.968976e-06	75	96
	3PBDDE	2.900691e-06	70	85
$\frac{\pi}{300}$	2PBDDE	6.104956e-10	750	202
	3PBDDE	3.481216e-10	700	173
$\frac{\pi}{3000}$	2PBDDE	8.203267e-10	7500	1320
	3PBDDE	8.202990e-10	7000	823

Table 3: Cmparison between the 2PBDDE and 3PBDDE for solving Problem 3

h	METHOD	MAXE	FCN	TIME (μ s)
0.1	2PBDDE	3.529679e-06	50	245
	3PBDDE	7.134256e-06	47	181
0.01	2PBDDE	2.167619e-09	500	375
	3PBDDE	5.332341e-09	467	329
0.001	2PBDDE	4.581639e-12	5000	2130
	3PBDDE	8.416190e-12	4667	1673

The numerical results in Table 1-3 clearly indicate that 3PBDDE performs better in terms of total function calls and execution times compared to 2PBDDE. The total function calls for 3PBDDE is less than 2PBDDE when solving all tested problems. The computational cost at smaller step size obviously decreases when the codes are implemented in 3PBDDE compared to 2PBDDE. Both of the methods achieved the desired accuracy in all tested problems.

6 Conclusions

In this paper, we have presented the numerical solution of DDEs using two and three point one-step block method. Hence, we have shown the efficiency of the proposed methods are suitable for solving second order DDEs directly.

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