UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF INDUSTRIAL TECHNOLOGY

FINAL EXAMINATION
JANUARY 2016 SEMESTER

COURSE CODE	:	JGB10403
COURSE TITLE	:	ENGINEERING MATHEMATICS 2
PROGRAMME LEVEL	:	BACHELOR
DATE	:	31 MAY 2016
TIME	:	2.30 PM – 5.30 PM
DURATION	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. This question paper consists of TWO (2) sections.
4. Answer ALL questions in Section A. Choose THREE (3) questions from Section B.
5. Please write your answers on the answer booklet provided.
6. Please answer all questions in English only.
7. Related formulae attached as reference.

THERE ARE 3 PAGES OF QUESTIONS EXCLUDING THIS PAGE.

SECTION A (Total 40 marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question 1

Siti Hajah has a total of RM 75,000 to be invested in two municipal bonds that have yields of 8.2 % and 10.2 % interest per year, respectively. If the interest Siti Hajah receives from the bonds in a year is RM 6750, how much does she have invested in each bond? Make the system of linear equations from given problem and solve the system by using Cramer's Rule.

(10 marks)

Question 2

Calculate first and second order partial derivatives.

(a) $z = x^3y^3 - 2xy + 13$

(4 marks)

(b) $z = e^{-3x}\cos 2y - e^{-2y}\sin 3x$.

(6 marks)

Question 3

(a) Find the Laplace transform of given function $f(t) = 3e^{-4t}$, where $f(t)$ defined for all $t \geq 0$.

(5 marks)

(b) Identify Inverse Laplace transform of given functions

i. $F(s) = \frac{2}{s-5}$

(2 marks)

ii. $F(s) = -\frac{7}{4s-3}$

(3 marks)

Question 4

Determine the area under the curve $y = -x^2 + 5x + 6$, between $x = 0$ and $x = 3$. Sketch the area in graphs.

(10 marks)

SECTION B (Total 60 marks)

INSTRUCTION: Answer THREE (3) questions only.
Please use the answer booklet provided.

Question 1

These questions are related to Multiple Integrals.

(a) Given straight line $y = x - 1$ and parabola $y^2 = 2x + 6$. Sketch the graph of both functions then identify limit points for both variables. Evaluate the area of region bounded by given functions, by using double integral: where D is the bounded region.

$$\iint_D xy \, dx \, dy$$

(10 marks)

(b) A solid is enclosed by the plane $z = 0$, the planes $x = -1, x = 3, y = 2, y = 5$ and the surface $z = 4x - 5y$. Find the volume of the solid.

(10 marks)

Question 2

These questions are related to hyperbolic functions.

(a) Given that $\sinh(x) = \frac{1}{4}$ and $\cosh^{-1}(\frac{6}{5}) = y$, then calculate the value of $z = -4x + 7y$.

(10 marks)

(b) Express hyperbolic functions $\cosh 2x$ in exponential form and hence solve for real values of x, the equation:

$$1 - 3\sinh x = \cosh 2x$$

(10 marks)

Question 3

Solve linear nonhomogeneous constant-coefficient second order differential equation:

$$-3y'' - y' + 4y = 1 + \cos 3x$$

And find particular solutions with conditions $f(0) = 0$, $f'(0) = 0$.

(20 marks)

Question 4

Given simultaneous equations with four variables:

$$\begin{cases} -2x-3y+4z-w=-5 \\ x-2y+3z+4w=25 \\ -x+y-2z+3w=11 \\ 3x+y+z-2w=-3 \end{cases}$$

(a) Evaluate the fourth order determinant.

(10 marks)

(b) Apply Gaussian elimination method to solve the system.

(10 marks)

END OF EXAMINATION PAPER

JGB 10403 ENGINEERING MATHEMATICS 2
FORMULAE FOR FINAL EXAM

SECOND ORDER DIFFERNTIAL EQUASTIONS

Solution of the equation of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$

1. Auxiliary equation: $am^2 + bm + c = 0$;
2. Types of solutions:
   a. Real and different roots (two roots) $m = m_1$ and $m = m_2$: $y = Ae^{m_1x} + Be^{m_2x}$;
   b. Real and equal roots (one root) $m = m_1$: $y = e^{m_1x}(A + Bx)$
   c. Complex roots $m = \alpha \pm j\beta$: $y = e^{\alpha x}(Acos\beta x + sin\beta x)$
3. Particular Integral (PI)

$f(x) = k$	Assume $y = C$
$f(x) = kx$ ...	$y = Cx + D$
$f(x) = kx^2$ ...	$y = Cx^2 + Dx + E$
$f(x) = ksin\, x$ (or $kcos\, x$)	$y = Ccos\, x + Dsin\, x$
$f(x) = ksin\, hx$ (or $kcos\, hx$)	$y = Ccosh\, x + Dsinh\, x$
$f(x) = e^{kx}$	$y = Ce^{kx}$

DIFFERENTIATION

a) General formulas:

1. $\frac{d}{dx}(C) = 0,\ C - constant$;
2. $\frac{d}{dx}(Cf(x)) = C\frac{d}{dx}f(x)$;
3. $\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x))$;
4. $\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x))$
5. $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ Power Rule
6. $\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$ Product rule
7. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{g^2(x)}$ Quotient rule
8. $\frac{d}{dx}f(g(x)) = \frac{d}{dx}f(g(x))\frac{d}{dx}(g(x))$ Chain Rule

b) Derivatives of some functions

Function	Derivative of function	Function	Derivative of function
$y=e^x$	$\frac{dy}{dx}=e^x$	$y=\sec x$	$\frac{dy}{dx}=\sec x\tan x$
$y=a^x$	$\frac{dy}{dx}=a^x\ln a$	$y=\csc x$	$\frac{dy}{dx}=-\csc x\cot x$
$y=\ln x$	$\frac{dy}{dx}=\frac{1}{x}$	$y=\sin^{-1}x$	$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$
$y=\log_a x$	$\frac{dy}{dx}=\frac{1}{x\ln a}$	$y=\cos^{-1}x$	$\frac{dy}{dx}=-\frac{1}{\sqrt{1-x^2}}$
$y=\sin x$	$\frac{dy}{dx}=\cos x$	$y=\tan^{-1}x$	$\frac{dy}{dx}=\frac{1}{1+x^2}$
$y=\cos x$	$\frac{dy}{dx}=-\sin x$	$y=\cot^{-1}x$	$\frac{dy}{dx}=-\frac{1}{1+x^2}$
$y=\tan x$	$\frac{dy}{dx}=\sec^2 x$	$y=\sec^{-1}x$	$\frac{dy}{dx}=\frac{1}{x\sqrt{x^2-1}}$
$y=\cot x$	$\frac{dy}{dx}=-\csc^2 x$	$y=\csc^{-1}x$	$\frac{dy}{dx}=-\frac{1}{x\sqrt{x^2-1}}$

INTEGRATION

$\int u\,dv=uv-\int v\,du$ (integration by part)	$\int dx=x+C$, (where C is constant)
$\int k\,dx=kx+C$, (k is any real number)	$\int x^n dx=\frac{x^{n+1}}{n+1}+C,\ n\neq -1$
$\int\frac{dx}{x}=\ln\lvert x\rvert+C$	$\int e^x dx=e^x+C$
$\int a^x dx=\frac{a^x}{\ln a}+C$	$\int\sin x\,dx=-\cos x+C$
$\int\cos x\,dx=\sin x+C$	$\int\sec^2 x\,dx=\tan x+C$
$\int\csc^2 x\,dx=-\cot x+C$	$\int\sec x\tan x\,dx=\sec x+C$
$\int\csc x\cot x\,dx=-\csc x+C$	$\int\tan x\,dx=\ln\lvert\sec x\rvert+C$
$\int\cot x\,dx=\ln\lvert\sin x\rvert+C$	$\int\sec x\,dx=\ln\lvert\sec x+\tan x\rvert+C$
$\int\csc x\,dx=\ln\lvert\csc x-\cot x\rvert+C$	$\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\frac{u}{a}+C,\ a>0$
$\int\frac{1}{a^2+x^2}dx=\frac{1}{a}\tan^{-1}\frac{u}{a}+C$	$\int\frac{1}{x\sqrt{x^2-a^2}}dx=\frac{1}{a}\sec^{-1}\frac{u}{a}+C$

INTEGRATION APPLICATION

1. Area under curve

$$A = \int_a^b y dx$$

2. Area between curves

$$A = \int_a^b (f(x) - g(x)) dx$$

3. Definite integral

$$\int_a^b y dx = [F(x)]_a^b = F(b) - F(a)$$

where $y = f(x) = F'(x)$

4. Parametric equations

$$x = f(t),\ y = F(t),\quad \int_a^b F(t) \frac{dx}{dt} dt.$$

5. Mean values

$$M = \frac{1}{b-a} \int_a^b y dx$$

6. Volumes of solids of revolution (about X-axis)

$$V = \int_a^b \pi y^2 dx$$

7. Parametric equations (volume, about X-axis)

$$V = \int_{t_1}^{t_2} \pi y^2 \frac{dx}{dt} dt$$

8. Volumes of solids of revolution (about Y-axis)

$$V = \int_a^b 2\pi x y dx$$

9. Centroid of plane figure

$$\bar{x} = \frac{\int_a^b xy dx}{\int_a^b y dx},\quad \bar{y} = \frac{\frac{1}{2}\int_a^b y^2 dx}{\int_a^b y dx}$$

10. Centres of gravity of solids of revolution

$$\bar{x} = \frac{\int_a^b x y^2 dx}{\int_a^b y^2 dx},\quad \bar{y} = 0$$

11. Lengths of curves $y = f(x)$,

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

12. Lengths of curves (Parametric equations) $x = f(t)$, $y = F(t)$,

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

13. Surface of revolution $y = f(x)$,

$$A = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

14. Surface (Parametric equations) $x = f(t),\ y = F(t)$,

$$A = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

LAPLACE TRANSFORM

1. The Laplace transform of $f(t)$ is

$$L\{f(x)\} = \int_{t=0}^{\infty} e^{-st} f(t)dt$$

2. The inverse Laplace transform of F(s) is $f(t) = L^{-1}\{F(s)\}$.
3. Table of Laplace transform:

$f(t) = L^{-1}\{F(s)\}$	$F(s) = L\{f(x)\}$
k	$\frac{k}{s}, \quad s > 0$
e^{-kt}	$\frac{1}{s+k}, \quad s > -k$
te^{-kt}	$\frac{1}{(s+k)^2}, \quad s > -k$
t	$\frac{1}{s^2}, \quad s > 0$
t^n	$\frac{n!}{s^{n+1}}, \quad s > 0$