CONFIDENTIAL

UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF INDUSTRIAL TECHNOLOGY

FINAL EXAMINATION
JANUARY 2016 SEMESTER

COURSE CODE	:	JGB10203
COURSE TITLE	:	ENGINEERING MATHEMATICS 1
PROGRAMME LEVEL	:	BACHELOR
DATE	:	23 MAY 2016
TIME		9.00 AM – 12.00 PM
DURATION	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. This question paper consists of TWO (2) sections.
4. Answer ALL questions in Section A. Choose THREE (3) questions from Section B.
5. Please write your answers on the answer booklet provided.
6. Please answer all questions in English only.
7. Related formulae attached as reference.

THERE ARE 3 PAGES OF QUESTIONS EXCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question 1

(a) Determine the first five terms of the sequence

$$a_n = \frac{(-1)^n(4n+1)}{2^n+1}$$

(5 marks)

(b) Identify a formula (the Rule) for the general term a_n of the given sequence

$$\left\{\frac{1}{5}, -\frac{3}{8}, \frac{9}{11}, -\frac{27}{14}, \frac{81}{17}, \dots, \right\}$$

(5 marks)

Question 2

These questions are related to differentiation.

(a) Find the first and second derivatives of the function

i. $f(x) = 10 + 3x - x^3 + 2x^4$

(2 marks)

ii. $f(x) = e^{5x}$

(2 marks)

(b) Calculate derivative of given function

$$f(x) = (x^3 - 13)\ \sin(1 - 5x)$$

(6 marks)

Question 3

Given vectors $a = (1, -2,\ 3)$, $b = (-4,\ 5,\ 6)$ and $c = (-7,\ 8,\ 9)$.

(a) Identify scalar product of vectors a and b.

(2 marks)

(b) Find magnitude of the vector $d = -4a + 2b$

(3 marks)

(c) Find the angle between the vectors a and c.

(5 marks)

Question 4

Given complex numbers $z_1 = 2 + 2j$, $z_2 = 1 - j$ and $z_3 = -2 + j$

(a) Identify $z_1 + z_2$ and $z_3 - z_1$

(2 marks)

(b) Find trigonometric forms of z_1.

(3 marks)

(c) Calculate z_1^{10} by applying De Moivre's formula.

(5 marks)

SECTION B (Total: 60 marks)

INSTRUCTION: Answer THREE (3) questions.
Please use the answer booklet provided.

Question 1

Presented questions are related to Series.

(a) Test whether the series $\sum_{k=1}^{\infty} \frac{(5)^n}{(n+1)!}$ is convergent or divergent (Apply Ratio Test)

(10 marks)

(b) Investigate given series for convergence $\sum_{k=1}^{\infty} (-1)^{2n-1} \left(\frac{n}{4n^2+3}\right)$ (Use Alternating Series Test).

(10 marks)

Question 2

(a) Use logarithmic differentiation to find the derivative of the given function.

$$y = (2x - 5)(2 + x^3)\,(1 - 4x)^2$$

(10 marks)

(b) Find $\frac{dy}{dx}$ by implicit differentiation then determine the equation of the tangent line and normal to the graph of the function defined by the given equation at the indicated point.
$x^2 + xy = y^2 - 1$ at $P(1, -1)$

(10 marks)

Question 3

These questions are related to integration

(a) Evaluate given integral by using partial fractions

$$\int \frac{x+4}{x^2-10x+16}\,dx$$

(10 marks)

(b) Calculate the given integral by using integration by parts

$$\int x^2 \sin(x+5)\,dx$$

(10 marks)

Question 4

First order differential equations

(a) Solve given first order differential equation by separating the variables

$$3x^3 + (y-2)^2\frac{dy}{dx} = 0$$

(8 marks)

(b) Solve the following homogeneous equation by substituting $y = vx$

$$(4y+3x)\frac{dy}{dx} = 3x - y$$

(12 marks)

END OF EXAMINATION PAPER

JGB 10203 ENGINEERING MATHEMATICS 1
FORMULAE FOR FINAL EXAM

SEQUENCES AND SERIES

1. If $\lim_{n\to\infty} a_n$ exists we say the sequence a_n is **converges** (or convergent), otherwise we say the sequence **diverges** (or divergent).
2. A sequence $\{a_n\}$ is called **increasing** if $a_n < a_{n+1}$ for all $n \geq 1$, that is $a_1 < a_2 < a_3 \ldots < a_n < a_{n+1} \ldots$. It is called **decreasing** if $a_n > a_{n+1}$.

3. **The Ratio Test** (for series).
   a. If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$ then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore **convergent**).
   b. If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$ then the series $\sum_{n=1}^{\infty} a_n$ is **divergent.**
   c. If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L = 1$ then the Ration test is inconclusive.
4. **The Alternating series Test**

If the **Alternating series** $\sum_{n=1}^{\infty}(-1)^{n-1}b_n = b_1 - b_2 + b_3 - b_4 + b_5 - \cdots,$ where $b_n > 0$ satisfies i) $b_{n+1} \leq b_n$, ii) $\lim_{n\to\infty} b_n = 0$ then the series is **convergent.**

5. **Taylor and Maclaurin Series**

$$f(x) = \sum_{n=1}^{\infty} \frac{f^{(n)}(x)}{n!}(x-a)^n = f(a) + \frac{f'(x)}{1!}(x-a) \quad + \frac{f''(x)}{2!}(x-a)^2 + \cdots$$

$$f(x) = \sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n = f(0) + \frac{f'(0)}{1!}x \quad + \frac{f''(0)}{2!}(x)^2 + \frac{f'''(0)}{3!}(x)^3 \ldots$$

COMPLEX NUMBERS

1. **Complex number** $z = a + bj$, where $j^2 = -1, a$ and b are real numbers.
2. **Trigonometric form** of complex number $z = r(\cos\theta + j\sin\theta)$, where $r = \sqrt{a^2+b^2}, \theta = tan^{-1}(\frac{b}{a})$.
3. **Exponential form** of complex number $z = re^{j\theta}$.
4. Power of complex number (**Moivre's formula**) $z^n = r^n(\cos(n\theta) + j\sin(n\theta))$.
5. Roots of a complex number $w_k = z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n} + \frac{360\,k}{n}\right) + j\sin\left(\frac{\theta}{n} + \frac{360\,k}{n}\right)\right)$ where $k = 1,\ 2,\ 3, \ldots, n-1$.

VECTORS

1. **Scalar product** (dot product) of two vectors: $a \cdot b = |a||b|cos\theta$,
2. $\vec{a} = a_1 i + a_2 j + a_3 k$ and $\vec{b} = b_1 i + b_2 j + b_3 k$ then $a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$.
3. **Vector product** (cross product) of two vectors $\vec{a}$ and $\vec{b}$: $\vec{a} = a_1 i + a_2 j + a_3 k$ and $\vec{b} = b_1 i + b_2 j + b_3 k$: $a \times b = |a||b|sin\theta$, or

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2 b_3 - a_3 b_2)i + (a_1 b_3 - a_3 b_1)j + (a_1 b_2 - a_2 b_1)k\ .$$

FIRST ORDER DIFFERNTIAL EQUASTIONS

1. By **direct integration:** $\frac{dy}{dx} = f(x)$ gives $y = \int f(x)dx$.
2. By **separating variables:** $F(y)\frac{dy}{dx} = f(x)$ gives $\int F(y)dy = \int f(x)dx$.
3. Homogeneous equations: **substituting $y = vx$** gives $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

DIFFERENTIATION

a) General formulas:

1. $\frac{d}{dx}(C) = 0,\ C - constant$;
2. $\frac{d}{dx}(Cf(x)) = C\frac{d}{dx}f(x)$;
3. $\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x))$;
4. $\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x))$
5. $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ Power Rule
6. $\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$ Product rule
7. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{g^2(x)}$ Quotient rule
8. $\frac{d}{dx}f(g(x)) = \frac{d}{dx}f(g(x))\frac{d}{dx}(g(x))$ Chain Rule

b) **Differentiation table**

Function	Derivative of function	Function	Derivative of function
$y=e^x$	$\frac{dy}{dx}=e^x$	$y=\sec x$	$\frac{dy}{dx}=\sec x\tan x$
$y=a^x$	$\frac{dy}{dx}=a^x\ln a$	$y=\csc x$	$\frac{dy}{dx}=-\csc x\cot x$
$y=\ln x$	$\frac{dy}{dx}=\frac{1}{x}$	$y=\sin^{-1}x$	$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$
$y=\log_a x$	$\frac{dy}{dx}=\frac{1}{x\ln a}$	$y=\cos^{-1}x$	$\frac{dy}{dx}=-\frac{1}{\sqrt{1-x^2}}$
$y=\sin x$	$\frac{dy}{dx}=\cos x$	$y=\tan^{-1}x$	$\frac{dy}{dx}=\frac{1}{1+x^2}$
$y=\cos x$	$\frac{dy}{dx}=-\sin x$	$y=\cot^{-1}x$	$\frac{dy}{dx}=-\frac{1}{1+x^2}$
$y=\tan x$	$\frac{dy}{dx}=\sec^2 x$	$y=\sec^{-1}x$	$\frac{dy}{dx}=\frac{1}{x\sqrt{x^2-1}}$
$y=\cot x$	$\frac{dy}{dx}=-\csc^2 x$	$y=\csc^{-1}x$	$\frac{dy}{dx}=-\frac{1}{x\sqrt{x^2-1}}$

c) Differentiation of parametric functions.

If function given in parametric form $y=y(t), x=x(t)$ then

$$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx} \text{ first order and } \frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx} \text{ second order}$$

d) **Logarithmic differentiation**

$$y=uvw$$

$$\ln(y)=\ln(uvw)$$

$$\ln(y)=\ln(u)+\ln(v)+\ln(w)$$

$$\frac{d}{dx}\ln(y)=\frac{d}{dx}(\ln(u)+\ln(v)+\ln(w))$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{u}\frac{du}{dx}+\frac{1}{v}\frac{dv}{dx}+\frac{1}{w}\frac{dw}{dx}$$

$$\frac{dy}{dx}=uvw\left(\frac{1}{u}\frac{du}{dx}+\frac{1}{v}\frac{dv}{dx}+\frac{1}{w}\frac{dw}{dx}\right)$$

e) **Implicit Differentiation**

$f(x,y)=0$ differentiate given implicit function (both sides of equation) with respect to x considering y as a function.

INTEGRATION

a) **Integration of the forms** $\int f(x)f'(x)\,dx$ *and* $\int \frac{f'(x)}{f(x)}dx$

$$\int f(x)f'(x)\,dx = \begin{bmatrix} u = f(x), \\ du = f'(x)dx \end{bmatrix} = \int u\,du = \frac{u^2}{2} + C = \frac{f^2(x)}{2} + C$$

$$\int \frac{f'(x)}{f(x)}dx = \begin{bmatrix} u = f(x), \\ du = f'(x)dx \end{bmatrix} = \int \frac{du}{u} = \ln u + C = \ln f(x) + C$$

b) **Integration table**

$\int udv = uv - \int vdu$ (integration by part)	$\int dx = x + C$, (where C is constant)
$\int k\,dx = kx + C$, (k is any real number)	$\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int e^x dx = e^x + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \sin x dx = -\cos x + C$
$\int \cos x dx = \sin x + C$	$\int \sec^2 x dx = \tan x + C$
$\int \csc^2 x dx = -\cot x + C$	$\int \sec x \tan x dx = \sec x + C$
$\int \csc x \cot x dx = -\csc x + C$	$\int \tan x dx = \ln\|\sec x\| + C$
$\int \cot x dx = \ln\|\sin x\| + C$	$\int \sec x dx = \ln\|\sec x + \tan x\| + C$
$\int \csc x dx = \ln\|\csc x - \cot x\| + C$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{u}{a} + C,\ a > 0$
$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$	$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$
$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a}\ln\left\|\frac{x+a}{x-a}\right\| + C$	$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$