SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION

SEPTEMBER 2014 SESSION

SUBJECT CODE	:	NMB20203
SUBJECT TITLE	:	MATHEMATICS FOR ENGINEERS 3
LEVEL	:	BACHELOR
TIME / DURATION	:	9.00 AM – 12.00 PM (3 HOURS)
DATE	:	2 JANUARY 2015

INSTRUCTIONS TO CANDIDATES

1. **Please read the instructions given in the question paper CAREFULLY.**
2. **This question paper is printed on both sides of the paper.**
3. **Please write your answers on the answer booklet provided.**
4. **Answer should be written in blue or black ink except for sketching, graphic and illustration.**
5. **This question paper consists of SIX (6) questions. Answer FIVE (5) questions only.**
6. **Answer all questions in English.**

THERE ARE 3 PAGES OF QUESTIONS AND 5 PAGES OF APPENDIX, EXCLUDING THIS PAGE.

INSTRUCTION: Answer <u>FIVE</u> (5) questions only.

Please use the answer booklet provided.

Question 1

a) If $r = \ 3t^2 + 3t \ \ i - \cos 3t \ \ j + 2e^{2t}\ k$, determine

i. $\frac{dr}{dt}$

ii. $\frac{d^2r}{dt^2}$

iii. $\frac{d^2r}{dt^2}$

At $t = 0$

(6 marks)

b) If $F = 3i + 2u^3 j - 3uk$ and $G = u^2\ i - 2u\ j + 4k$, determine

$$\int_0^2 F \times G \ \ du$$

(7 marks)

c) A moving particle starts at position $r\ \ 0 \ = \ 1, \frac{2}{9}, \frac{1}{24}$ with initial velocity $v\ \ 0 \ = \ 1, \frac{2}{3}, \frac{5}{6}$ and its acceleration is $a\ \ t \ = \ 4t, 2e^{3t}, \ \ 2t - 1 \ ^2$. Find its velocity and position vector at time, t .

(7 marks)

Question 2

a) Find the unit normal to the surface $\emptyset = 2x^4z^2 + x^2y^2 + xyz - 3$ at the point $(1,2,3)$.

(6 marks)

b) Let $r\ \ t \ = \ \bar{t}\ i + 2e^{2t} j - 3\sin(2t)\ k$, find $r'\ \ t$ and unit tangent vector at $t = 0$.

(6 marks)

c) Determine the directional derivative of $\emptyset = 2xz^2 + xe^{2y} + xyz$ at the point (1,0,3) in the direction of $A = 2i + 4j - 3k$.

(8 marks)

Question 3

a) If $\emptyset = x^2 \cos\ z\ \ + ye^{2x} - 2y^2$, find the value of $grad\ \emptyset$ and $grad\ \emptyset$ at point $(0,1,0)$.

(8 marks)

b) If $A = 2x^2 z\ i + xz\ j +\ \ y - z\ \ k$, find

i) $curl\ curl\ A$

ii) $div\ curl\ A$

(12 marks)

Question 4

If $F = x^2 y\ i + 2yz\ j + 3xz^2\ k$, evaluate:

a) the line integral $\int_c F \cdot dr$ along the curves c_1, c_2 and c_3 where the start and end point of each curve is given as (0,0,0) and (2,0,0) for c_1; (2,0,0) and (2,4,0) for c_2; and (2,4,0) and (2,4,8) for c_3.

(8 marks)

b) The line integral $\int_c F \cdot dr$ along the curve with parametric equations$x = t, y = 2t$ and $z = 3t$, between the points (0,0,0) and (1,2,3)

(12 marks)

Question 5

a) If $F = 3i + z\ j + 2y\ k$, determine the volume integral $F\ dV$ where V is the volume bounded by the planes z = 0, z = 3 and the surface $x^2 + y^2 = 4$.

(10 marks)

b) A vector field $F = y\ i + 2\ j + k$ exists over a surface defined by$x^2 + y^2 + z^2 = 9$, bounded by $x = 0$ and $y = 0z = 0$ in the first octant. Determine the first integral of F over the surface, $_S\ F \cdot dS$

(10 marks)

Question 6

Verify the Gauss divergence theorem for the vector field $F = xi + 2j + z^2k$ taken over the region bounded by the planes $z = 0,\ z = 4,\ x = 0,\ y = 0$ and the surface $x^2 + y^2 = 4$ in the first octant.

(20 marks)

END OF QUESTION

APPENDIX 1 - Trigonometric Identities and Formulas

Fundamental Identities

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Formulas For Negatives

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

Addition Formulas

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Subtraction Formulas

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Half-Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\ldots\ldots\ldots\ldots = 1 - 2\sin^2\theta$$

$$\ldots\ldots\ldots\ldots = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Product-To-Sum Formulas

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

Sum-To-Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

APPENDIX 2–Table of Differentiation

Trigonometric Functions	Inverse Trigonometric Functions
$\frac{d}{dx}\left(\sin f(x)\right) = f'(x)\cos f(x)$	$\frac{d}{dx}\left(\sin^{-1}U\right) = \frac{1}{\sqrt{1-U^2}}\frac{dU}{dx}$, $\lvert U\rvert<1$
$\frac{d}{dx}\left(\cos f(x)\right) = -f'(x)\sin f(x)$	$\frac{d}{dx}\left(\cos^{-1}U\right) = \frac{-1}{\sqrt{1-U^2}}\frac{dU}{dx}$, $\lvert U\rvert<1$
$\frac{d}{dx}\left(\tan f(x)\right) = f'(x)\sec^2 f(x)$	$\frac{d}{dx}\left(\tan^{-1}U\right) = \frac{1}{1+U^2}\frac{dU}{dx}$
$\frac{d}{dx}\left(\csc f(x)\right) = -f'(x)\csc f(x)\cot f(x)$	$\frac{d}{dx}\left(\csc^{-1}U\right) = \frac{-1}{\lvert U\rvert\sqrt{U^2-1}}\frac{dU}{dx}$, $\lvert U\rvert>1$
$\frac{d}{dx}\left(\sec f(x)\right) = f'(x)\sec f(x)\tan f(x)$	$\frac{d}{dx}\left(\sec^{-1}U\right) = \frac{1}{\lvert U\rvert\sqrt{U^2-1}}\frac{dU}{dx}$, $\lvert U\rvert>1$
$\frac{d}{dx}\left(\cot f(x)\right) = -f'(x)\csc^2 f(x)$	$\frac{d}{dx}\left(\cot^{-1}U\right) = \frac{-1}{1+U^2}\frac{dU}{dx}$

Hyperbolic Functions	Inverse Hyperbolic Functions
$\frac{d}{dx}\left(\sinh U\right) = \cosh U\frac{dU}{dx}$	$\frac{d}{dx}\left(\sinh^{-1}U\right) = \frac{1}{\sqrt{1+U^2}}\frac{dU}{dx}$
$\frac{d}{dx}\left(\cosh U\right) = \sinh U\frac{dU}{dx}$	$\frac{d}{dx}\left(\cosh^{-1}U\right) = \frac{1}{\sqrt{U^2-1}}\frac{dU}{dx}$, $U>1$
$\frac{d}{dx}\left(\tanh U\right) = \operatorname{sech}^2 U\frac{dU}{dx}$	$\frac{d}{dx}\left(\tanh^{-1}U\right) = \frac{1}{1-U^2}\frac{dU}{dx}$, $\lvert U\rvert<1$
$\frac{d}{dx}\left(\operatorname{csch} U\right) = -\operatorname{csch} U\coth U\frac{dU}{dx}$	$\frac{d}{dx}\left(\operatorname{csch}^{-1}U\right) = \frac{-1}{\lvert U\rvert\sqrt{1+U^2}}\frac{dU}{dx}$, $U\neq 0$
$\frac{d}{dx}\left(\operatorname{sech} U\right) = -\operatorname{sech} U\tanh U\frac{dU}{dx}$	$\frac{d}{dx}\left(\operatorname{sech}^{-1}U\right) = \frac{-1}{U\sqrt{1-U^2}}\frac{dU}{dx}$, $0<U<1$
$\frac{d}{dx}\left(\coth U\right) = -\operatorname{csch}^2 U\frac{dU}{dx}$	$\frac{d}{dx}\left(\coth^{-1}U\right) = \frac{1}{1-U^2}\frac{dU}{dx}$, $\lvert U\rvert>1$

Exponential Function	Natural Logarithmic Function
$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$	$\frac{d}{dx}\left(\ln f(x)\right) = \frac{1}{f(x)}f'(x)$

APPENDIX 3–Table of Integration

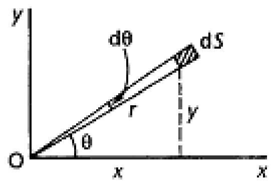
Trigonometric Functions Where $f(x) = ax + b$	Inverse Trigonometric Functions
$\int \cos f(x)\,dx = \frac{\sin f(x)}{f'(x)} + C$	$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \sin f(x)\,dx = -\frac{\cos f(x)}{f'(x)} + C$	$\int \frac{-1}{\sqrt{a^2 - x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \sec^2 f(x)\,dx = \frac{\tan f(x)}{f'(x)} + C$	$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \csc^2 f(x)\,dx = -\frac{\cot f(x)}{f'(x)} + C$	$\int \frac{-1}{\lvert x \rvert\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \sec f(x) \tan f(x)\,dx = \frac{\sec f(x)}{f'(x)} + C$	$\int \frac{1}{\lvert x \rvert\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \csc f(x) \cot f(x)\,dx = -\frac{\csc f(x)}{f'(x)} + C$	$\int \frac{-1}{a^2 + x^2}\,dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$

Hyperbolic Functions Where $f(x) = ax + b$	Inverse Hyperbolic Functions
$\int \cosh f(x)\,dx = \frac{\sinh f(x)}{f'(x)} + C$	$\int \frac{1}{\sqrt{a^2 + x^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad a > 0$
$\int \sinh f(x)\,dx = \frac{\cosh f(x)}{f'(x)} + C$	$\int \frac{-1}{\sqrt{x^2 - a^2}}\,dx = \cosh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad x > a$
$\int \operatorname{sech}^2 f(x)\,dx = \frac{\tanh f(x)}{f'(x)} + C$	$\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C \; , \; \lvert x \rvert < a$
$\int \operatorname{csch}^2 f(x)\,dx = -\frac{\coth f(x)}{f'(x)} + C$	$\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C \; , \; \lvert x \rvert > a$
$\int \operatorname{sech} f(x) \tanh f(x)\,dx = \frac{-\operatorname{sech} f(x)}{f'(x)} + C$	$\int \frac{1}{x\sqrt{a^2 + x^2}}\,dx = -\frac{1}{a}\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + C \quad , \quad 0 < x < a$
$\int \operatorname{csch} f(x) \coth f(x)\,dx = -\frac{\operatorname{csch} f(x)}{f'(x)} + C$	$\int \frac{1}{x\sqrt{a^2 - x^2}}\,dx = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right) + C \quad , \quad 0 < x < a$

Exponential Function Where $f(x) = ax + b$	**Form** $\frac{1}{f(x)}$, where $f(x) = ax + b$
$\int e^{f(x)}\,dx = \frac{e^{f(x)}}{f'(x)} + C$	$\int \frac{1}{f(x)}\,dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + C$

Vector Calculus Formulaes

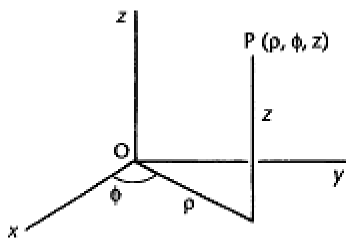
Plane Polar Coordinates (r, θ)



$x = r\cos\theta; \quad y = r\sin\theta$

$dS = r\,dr\,d\theta$

Cylindrical Polar Coordinates (ρ, φ, z)



$x = \rho\cos\phi$

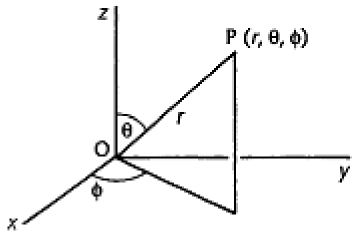
$y = \rho\sin\phi$

$z = z$

$dS = \rho\,d\phi\,dz$

$dV = \rho\,d\rho\,d\phi\,dz$

Spherical Polar Coordinates (r, θ, φ)



$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$dS = r^2\sin\theta\,d\theta\,d\phi$

$dV = r^2\sin\theta\,dr\,d\theta\,d\phi$

a) Scalar Field

$$\int_c V\,d\mathbf{r}$$

$$d\mathbf{r} = \mathbf{i}\,dx + \mathbf{j}\,dy + \mathbf{k}\,dz$$

b) Vector Field

$$\mathbf{F}: \quad \int_c \mathbf{F}\cdot d\mathbf{r}$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

$$d\mathbf{r} = \mathbf{i}\,dx + \mathbf{j}\,dy + \mathbf{k}\,dz$$

$$\mathbf{F}\cdot d\mathbf{r} = F_x\,dx + F_y\,dy + F_z\,dz$$

c) Volume Integral

$$\int_V \mathbf{F}\,dV = \int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2} \mathbf{F}\,dz\,dy\,dx$$

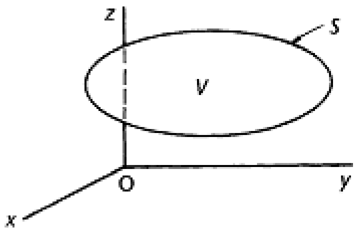
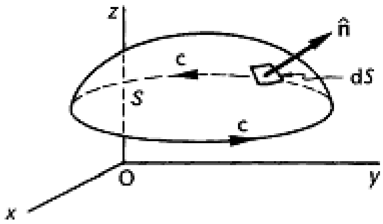
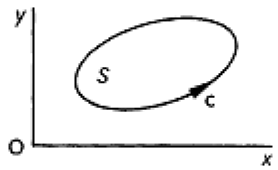
d) Surface Integral

i). Scalar Field

$$\int_S V\,d\mathbf{S} = \int_S V\hat{\mathbf{n}}\,dS; \qquad \hat{\mathbf{n}} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{\text{grad}\,\phi}{|\text{grad}\,\phi|}$$

ii). Vector Field

$$\int_S \mathbf{F}\cdot d\mathbf{S} = \int_S \mathbf{F}\cdot\hat{\mathbf{n}}\,dS; \qquad \hat{\mathbf{n}} = \frac{\nabla\phi}{|\nabla\phi|}$$

Gauss's Theorem  Closed surface S enclosing a region V in a vector field $\mathbf{F}$. $\int_V \operatorname{div} \mathbf{F}\, dV = \int_S \mathbf{F} \cdot d\mathbf{S}$	**Stoke's Theorem**  An open surface S bounded by a simple closed curve c, then $\int_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \oint_c \mathbf{F} \cdot d\mathbf{r}$
Green's Theorem  $\iint_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\, dy = \oint_c (P\, dx + Q\, dy)$	