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CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION

SEPTEMBER 2014 SESSION

SUBJECT CODE	:	**FEB23023**
SUBJECT TITLE	:	**ELECTROMAGNETISM**
LEVEL	:	**BACHELOR**
TIME / DURATION	:	**2.00 PM – 5.00 PM** **(3.0 HOURS)**
DATE	:	**11 JANUARY 2015**

INSTRUCTIONS TO CANDIDATES

1. **Please read the instructions given in the question paper CAREFULLY.**
2. **This question paper is printed on both sides of the paper.**
3. **Please write your answers on the answer booklet provided.**
4. **Answers should be written in blue or black ink except for sketching, graphic and illustration.**
5. **This question paper consists of TWO (2) sections. Section A and B. Answer all questions in Section A. For Section B, answer three (3) questions only.**
6. **Answer all questions in English.**
7. **Do not open the question paper until instructed to do so**

THERE ARE 4 PAGES OF QUESTIONS, EXCLUDING THIS PAGE AND APPENDIX.

SECTION A(Total: 40marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question1

In Cartesian coordinates, vector **A** is directed from (1, -1, -3) and ends at point (2, -1, 0). Vector **B** is directed from (4, -2, 1) to point (-1, 3, 2). Determine:

(a) The magnitude, *A* and unit vector, $\mathbf{a}$ (4 marks)

(b) Vector **B** (2 marks)

(c) The angle between vectors **A** and **B**,θ_{AB} (4 marks)

(d) A vector **C**whosemagnitude is 8 and whosedirection is perpendicular to both vectors **A** and **B** (6 marks)

(e) $(\mathbf{A} \times \boldsymbol{y}) \cdot \mathbf{z}$ (4 marks)

Question 2

(a) Find the directional derivative of $T = 2x^2 - 5y^2z$ along direction $\mathbf{l} = \mathbf{x}3 + \mathbf{y}2 - \mathbf{z}4$ and evaluate it at (2, -1, 3) (10 marks)

(b) Determine the divergence of the vector field $\mathbf{E} = \mathbf{x}3x^2 + \mathbf{y}2z + \mathbf{z}y^2z$ at (-2, 1, 3) (5 marks)

(c) Find the Laplacian of the scalar function $V = -3x^2y + 5y^3z$ (5 marks)

SECTION B(Total:60marks)

INSTRUCTION: Answer THREE (3) questions only.
Please use the answer booklet provided.

Question 3

(a) A circular cylinder of radius r = 5 cm is concentric with the z-axis and extends between z = - 3 cm and z = 3 cm. Determine the cylinder's volume. (5 marks)

(b) The spherical strip shown in **Figure 1** below is a section of a sphere of radius 3 cm. Find the area of the strip. (5 marks)

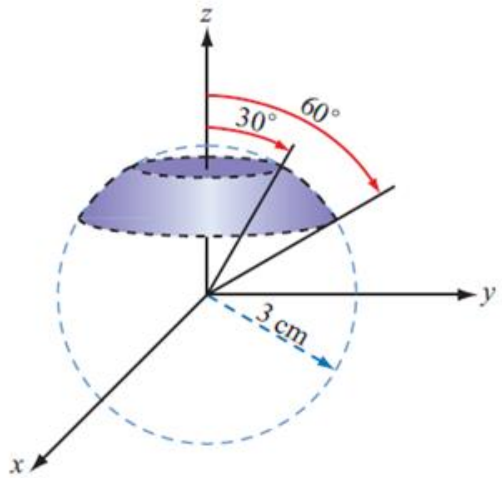

Figure 1

(c) Transform the vector **A** = $\mathbf{y}\ x^2 + y^2 + z^2\ \ - \mathbf{z}(x^2 + y^2)$ into spherical coordinates and evaluate it at (-1, 1, 2). (10 marks)

Question 4

(a) Find the total charge contained in a cylindrical volume defined by $r \leq 2\ m$ and $0 \leq z \leq 3\ m$ if $\rho_v = 20rz$ (mC/m^3) (6 marks)

(b) Three point charges, each with q = 3 nC, are located at the corners of a triangle in the x-y plane, with one corner at the origin, another at (2 cm, 0, 0) and the third at (0, 2 cm, 0). Calculate the force acting on the charge located at the origin. Take $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m (14 marks)

Question 5

(a) State Biot-Savart Law (2 marks)

(b) Two parallel, circular loops carrying a current, I = 20 A, each are arranged as shown in **Figure 2** below. The first loop is situated in the x-y plane with its center at the origin andthe second loop's center is at z = 2 m. If the two loops have the same radius a = 3 m, determine the magnetic field, **H** at:

(i) z = 0 (6 marks)

(ii) z = 1 m (6 marks)

(iii) z = 2 m (6 marks)

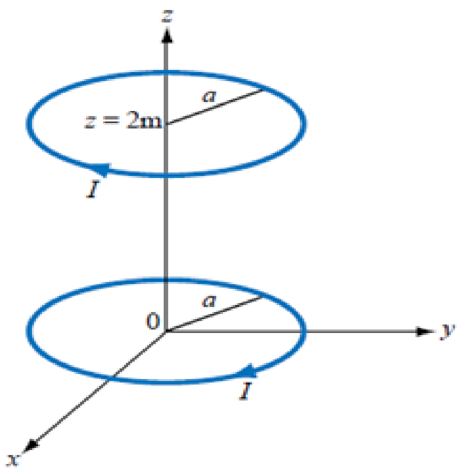

Figure 2

Question 6

(a) Define Lenz's Law (2 marks)

(b) An inductor is formed by winding, N = 10 turns of a thin conducting wire into a circular loop of radius, a = 10 cm. The inductor loop is in the x-y plane with its center at the origin, and connected to a resistor, R = 1 kΩ as shown in **Figure 3** below. In the presence of a magnetic field, $\mathbf{B} = 0.2\ \ \mathbf{y}2 + \mathbf{z}3\ \ \ \sin 1000t$, determine:

(i) The magnetic flux linking a single turn of the inductor, ϕ (6 marks)

(ii) The transformer emf, V_{emf}^{tr} (5 marks)

(iii) The polarity of V_{emf}^{tr} at t = 0 (3 marks)

(iv) The induced current in the circuit (4 marks)

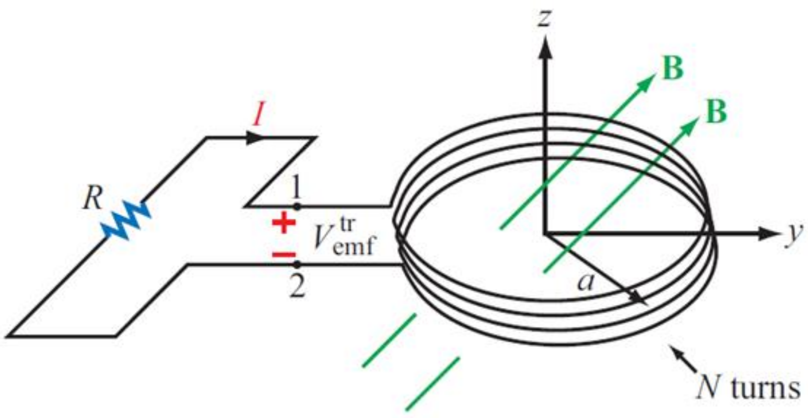

Figure 3

END OF QUESTION PAPER

APPENDIX

Table 1: Summary of vector relations

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$ds_x = \hat{\mathbf{x}}\,dy\,dz$ $ds_y = \hat{\mathbf{y}}\,dx\,dz$ $ds_z = \hat{\mathbf{z}}\,dx\,dy$	$ds_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $ds_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $ds_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$ds_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $ds_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $ds_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Table 2: Coordinate transformation relations

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

FORMULA

Vector analysis

Gradient of a scalar T:

$$\nabla T = \text{grad } T = \hat{\mathbf{x}}\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\frac{\partial T}{\partial z}$$

Directional derivative of T along **a**$_l$:

$$\frac{dT}{dl} = \nabla T \cdot \hat{\mathbf{a}}_l .$$

Divergence of a vector **E**:

$$\nabla \cdot \mathbf{E} = \text{div } \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

Laplacian of a scalar *V:*

$$\nabla^2 V = \nabla \cdot (\nabla V) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} .$$

Electrostatics

Charge distributions:

$$Q = \int_l \rho_l \, dl$$

$$Q = \int_S \rho_s \, ds$$

$$Q = \int_V \rho_v \, dV \qquad \text{(C)}$$

Electric field due to multiple charges:

$$\mathbf{E} = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{N} \frac{q_i(\mathbf{R} - \mathbf{R}_i)}{|\mathbf{R} - \mathbf{R}_i|^3} \qquad \text{(V/m).}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \simeq (1/36\pi) \times 10^{-9} \qquad \text{(F/m)}$$

Magnetostatics

Magnetic field of a loop, **H:**

$$\mathbf{H} = \hat{\mathbf{z}}\,\frac{I a^2}{2|z|^3} \qquad (\text{at } |z| \gg a)$$

$$\mathbf{H} = \hat{\mathbf{z}}\,\frac{I a^2}{2(a^2 + z^2)^{3/2}} \qquad (\text{A/m}).$$ at (0, 0, z) with a > z

$$\mathbf{H} = \hat{\mathbf{z}}\,\frac{I}{2a} \qquad (\text{at } z = 0).$$

Maxwell's equation for time-varying field

Magnetic flux:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} \qquad (\text{Wb})$$

Transformer emf:

$$V_{\text{emf}}^{\text{tr}} = -N\,\frac{d\Phi}{dt} \qquad (\text{V})$$