CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR
MFI/MICET/BMI

FINAL EXAMINATION
JANUARY 2014 SESSION

SUBJECT CODE	:	NCB 10303
SUBJECT TITLE	:	MATHEMATICS FOR ENGINEERS 2
LEVEL	:	BACHELOR
TIME / DURATION	:	9.00 am – 12.00 noon (3 HOURS)
DATE	:	02 JUNE 2014

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of SIX (6) questions. Answer FIVE (5) questions only.
6. Answer all questions in English.
7. Appendix is attached.

THERE ARE 4 PAGES OF QUESTIONS AND 4 PAGES OF APPENDIX, EXCLUDING THIS PAGE.

INSTRUCTION: Answer FIVE (5) questions only.
Please use the answer booklet provided.

Question 1

a) Find the particular solution for the 1st order linear differential equation below by the integrating factor.

$$x\frac{dy}{dx}+3y=2x^2, \qquad y(1)=1$$

(5 marks)

b) The reaction to form an iodine molecule is a second-order kinetics rate of loss of iodine atoms and follows the 1st order ODE equation,

$$\frac{dI}{dt}=-kI^2$$

At 23°C in the gas phase this reaction has a rate constant $k=7.0\times10^{9}\ L\,mol^{-1}s^{-1}$.

Using the separation of variables method solve the 1st order ODE to find I after 1.5 s if the boundary condition is $I_0=6.72\times10^{-3} mol\ L^{-1}$ at $t=0\ s$.

(5 marks)

c) The unit step function $u(t)$ is defined as

$$u(t)=\begin{cases} t, & 0\le t<1 \\ 1, & t\ge 1 \end{cases}$$

Show from the definition of Laplace transform that

$$L\{u(t)\}=\frac{1}{s^2}\left(1-e^{-s}\right)$$

Hint: Use $L\{u(t)\}=\int_0^{\infty} e^{-st}u(t)\ dt$

(5 marks)

d) Determine the following inverse Laplace transform by using partial fractions:

$$L^{-1}\left[\frac{(s+2)}{(s+2)^2+3^2}\right]$$

(5 marks)

Question 2

Consider the following initial-value problem (IVP):

$$y''-2y'+5y=10\sin x, \qquad y(0)=2, \quad y'(0)=1$$

a) By solving the auxiliary equation, find the complementary function, y_c.

(5 marks)

b) Hence, find the particular solution of the IVP, i.e. $y(x)=y_c+y_p$ by using the suitable particular integral, y_p, for the given ODE.

(15 marks)

Question 3

Consider the following non-homogeneous 2nd order linear differential equation,

$$\begin{cases} \dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}+y=3te^{-t} \\ y(0)=4 \\ y'(0)=2 \end{cases}$$

a) Taking the Laplace transform of both sides of the ODE, find an equation that $Y(s)$ satisfies.

(15 marks)

b) Hence, determine $y(t)$ by computing the inverse Laplace transform of $Y(s)$.

(5 marks)

Question 4

Find the solution to the heat flow problem

(20 marks)

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2} \qquad 0 < x < \pi \qquad t > 0$$

$$u(0,t) = u(\pi,t) = 0 \qquad t > 0$$
$$u(x,0) = \sin x - 6\sin 4x \qquad 0 < x < \pi$$

Question 5

The periodic function $f(x)$ is defined as

$$f(x) = \begin{cases} -x, & -\pi < x \le 0 \\ x, & 0 < x \le \pi \end{cases}$$

a) Sketch $f(x)$ in the range $-4\pi \le x \le 4\pi$.

(2 marks)

b) Obtain the Fourier series for the function $f(x)$.

(15 marks)

c) Deduce a series for $-\frac{\pi^2}{8}$ when $x = \pi$.

(3 marks)

Question 6

a) The periodic function is defined as

$$f(t)=\begin{cases} t+2, & -2<t<-1 \\ 1, & -1<t<1 \\ 2-t, & 1<t<2 \end{cases}$$

(i) Sketch the periodic extensions of the function for 3 periods.

(3 marks)

(ii) From the graph of $f(t)$, determine whether the function $f(t)$ is even function, odd function or neither odd nor even function. State your reasoning.

(2 marks)

b) Determine the period of the function $f(t)$ and hence find the Fourier coefficients a_0 , a_n and b_n of the function $f(t)$.

(15 marks)

END OF QUESTION

APPENDIX

DIFFERENTIATION RULES			
1.	$\frac{d}{dx}(k) = 0$	5.	$\frac{d}{dx}\cos x = -\sin x$
2.	$\frac{d}{dx}(x^n) = nx^{n-1}$	6.	$\frac{d}{dx}\ln x = \frac{1}{x}$
3.	$\frac{d}{dx}e^x = e^x$	7.	Product Rule $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
4.	$\frac{d}{dx}\sin x = \cos x$	8.	Quotient Rule $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

INTEGRATION RULES			
1.	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$	4.	$\int \sin x\, dx = -\cos x + c$
2.	$\int e^x dx = e^x + c$	5.	$\int \cos x\, dx = \sin x + c$
3.	$\int \frac{1}{x} dx = \ln x + c$	6.	Integration by parts $\int u dv = uv - \int v du$

FIRST ORDER LINEAR DIFFERENTIAL EQUATION		
1.	General Form of 1st order ODE nonhomogeneous	$\frac{dy}{dx} + P(x)y = Q(x)$
2.	Integrating Factor	$V(x) = e^{\int P(x)\,dx}$
3.	General Solution	$V(x)y = \int V(x)Q(x)\,dx$

SECOND ORDER LINEAR DIFFERENTIAL EQUATION			
1.	General Form of 2nd order ODE nonhomogeneous	$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = Q(x)$	
2.	Complementary Function	$y_c = Ae^{\alpha x} + Be^{\beta x}$	2 distinct real roots
		$y_c = (A + Bx)e^{\alpha x}$	2 equal (repeated) real roots
		$y_c = e^{\alpha x}(A\sin\beta x + B\cos\beta x)$	Complex roots
3.	Particular Solution,	$Q(x)$	y_p
		x^n ax^n $ax^n + bx^{n-1} + \cdots$	$A + Bx + Cx^2 + \cdots + kx^n$
		ae^{bx}	Ae^{bx}
		axe^{bx}	$Ae^{bx} + Bxe^{bx}$
		$a\cos bx$ $a\sin bx$	$A\sin bx + B\cos bx$
		$ax\cos bx$ $ax\sin bx$	$A\sin bx + B\cos bx + Cx\cos bx + Ex\sin bx$
4.	General Solution	$y = y_c + y_p$	

TABLE OF LAPLACE TRANSFORMS

	$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$		$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p, p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1.3.5\ldots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	e^{-cs}
27.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	28.	$u_c(t)g(t)$	$e^{-cs}\mathcal{L}\{g(t+c)\}$
29.	$e^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)du$	32.	$\int_0^t f(v)dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)dt}{1-e^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\ldots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

FOURIER SERIES

$$f(x)=\frac{a_o}{2}+\sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi x}{L}+b_n\sin\frac{n\pi x}{L}\right],\ -L<x<L,\ T=2L.$$

where the Fourier coefficients, a_o, a_n, b_n, as following:

$$a_o=\frac{1}{L}\int_{-L}^{L}f(x)dx$$

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}dx$$

FOURIER COSINE SERIES

The Fourier series of an **even** function on the interval $-L<x<L$ is the **cosine series**

$$f(x)=\frac{a_o}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}$$

where the Fourier coefficients, a_o, a_n, b_n, as following:

$$a_o=\frac{2}{L}\int_0^L f(x)dx$$

$$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n=0$$

FOURIER SINE SERIES

The Fourier series of an **odd** function on the interval $-L<x<L$ is the **sine series**

$$f(x)=\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$$

where the Fourier coefficients, a_o, a_n, b_n, as following:

$$a_o=0$$

$$a_n=0$$

$$b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$$

Heat Equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \qquad 0 < x < L \qquad t > 0$$

$$u(0,t) = u(L,t) = 0 \qquad t > 0$$
$$u(x,0) = f(x) \qquad 0 < x < L$$
$$Solution : u(x,t) = \sum_{n=1}^{\infty} c_n e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi x}{L}\right)$$

Wave Equation

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \qquad 0 < x < L \qquad t > 0$$

$$u(0,t) = u(L,t) = 0 \qquad t > 0$$
$$u(x,0) = f(x) \qquad \frac{\partial u}{\partial t}(x,0) = g(x)$$
$$Solution : u(x,t) = \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi\alpha}{L}t + b_n \sin\frac{n\pi\alpha}{L}t\right]\sin\left(\frac{n\pi x}{L}\right)$$