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CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR

MALAYSIA FRANCE INSTITUTE

FINAL EXAMINATION

JANUARY 2014 SESSION

SUBJECT CODE	:	**FAB20703, FAB30703**
SUBJECT TITLE	:	**ROBOTICS**
LEVEL	:	**BACHELOR**
DURATION	:	**3 HOURS**
DATE / TIME	:	

INSTRUCTIONS TO CANDIDATES

1. **Please read the instructions given in the question paper CAREFULLY.**
2. **This question paper is printed on both sides of the paper.**
3. **Please write your answers on the answer booklet provided.**
4. **Answer should be written in blue or black ink except for sketching, graphic and illustration.**
5. **This question paper consists of TWO (2) sections. Section A and B. Answer all questions in Section A. For Section B, answer three (3) question only.**
6. **Answer all questions in English.**
7. **Fomula is appended.**

THERE ARE 8 PRINTED PAGES OF QUESTIONS, AND 5 PAGES OF FOMULA EXCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question 1

a) State the **four** (4) basic components of a robot manipulator. (5 marks)

b) How many Degrees-Of-Freedoms are required to position and orientate a robot end-effector in world space ?. (2 marks)

c) Define the term “robot’s reach”. (2 marks)

d) Give **two** (2) examples to demonstrate your understanding for a robot’s reach.

(4 marks)

e) State five (5) technical criteria commonly used in selecting a suitable robot for an industrial application. (7 marks)

Question 2

a) A point $P = [2 \quad 3 \quad 4]^T$ is attached to a rotating frame. The frame rotates 90^0 about the X-axis of the reference frame. Find the coordinates of the point relative to the reference frame after the rotation. (10 marks)

b) A frame represented by the matrix $\begin{bmatrix} 0.527 & -0.574 & 0.628 & 6 \\ 0.369 & 0.819 & 0.439 & 4 \\ -0.766 & 0 & 0.643 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ has been moved 6 units along the x-axis and 4 units along the z-axis. Find the representation of the new frame after the two movements. (10 marks)

SECTION B (Total: 60 marks)

INSTRUCTION: Answer THREE (3) questions only.
Please use the answer booklet provided.

Question 3

a) A 3 D-O-F articulated arm is shown in **Figure 1**. Determine the overall transformation matrix for the end point of the arm and find the home position forward kinematics model.

(10 marks)

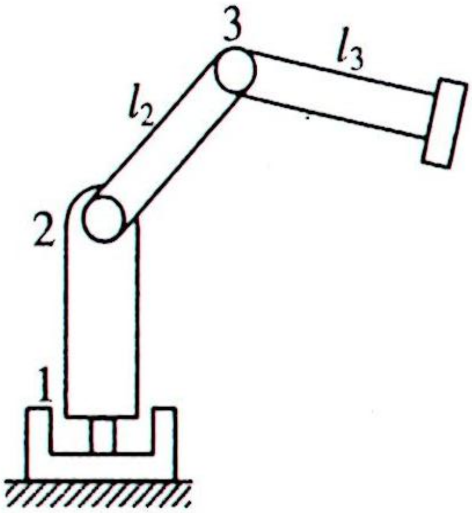

Figure 1 : Articulated arm subassembly.

b) A spatial three-degree of freedom spraying robot has been designed as shown in **Figure 2**.

i) Find the D-H parameter table for the robot. (5 marks)

ii) Obtain the final transformation matrices by using the direct kinematic analysis. (5 marks)

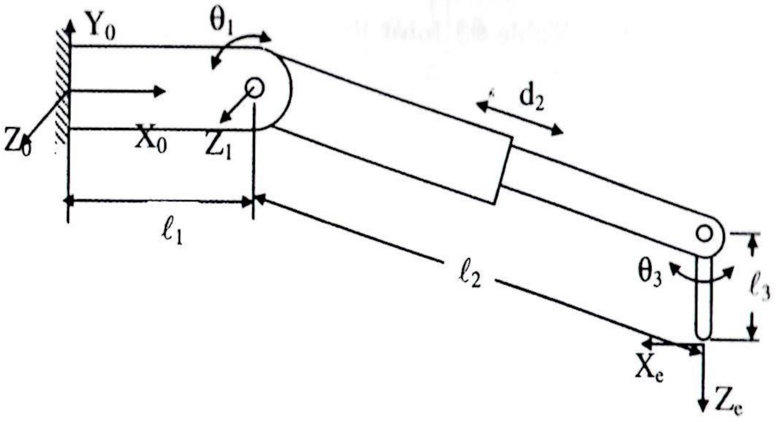

Figure 2 : A 3 D-O-F spraying robot.

Question 4

a) Define 'Inverse Kinematics'. (1 marks)

b) **Figure 3** shows a cylindrical arm with two prismatic joints and a rotary joint. The corresponding arm parameters and the forward kinematics are given in **Table 1** and **2** respectively. Find the inverse kinematic equations of the robot.

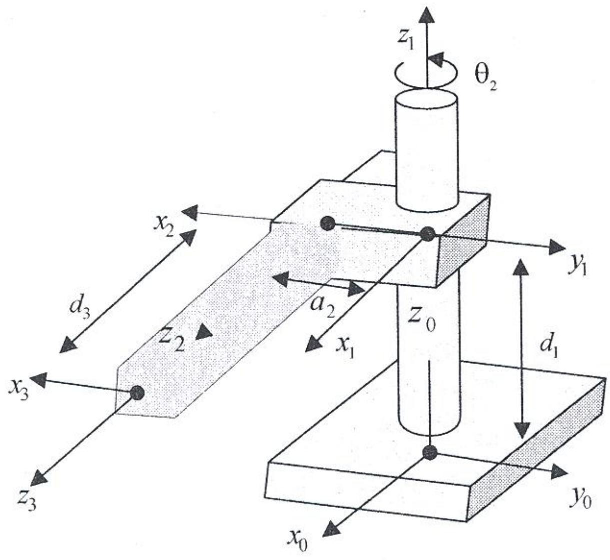

Figure 3. : Cylindrical Robot (RRP)

Table 1 : Arm Parameter Table

Link	θ_i	a_i	α_i	d_i
1	0°	0	0°	d_1
2	θ_2	a_2	-90°	0
3	0°	0	0°	d_3

Table 2 : Forward kinematics matrix

$$H_0^3 = \begin{bmatrix} c_2 & 0 & -s_2 & -d_3 s_2 + a_2 c_2 \\ s_2 & 0 & -c_2 & d_3 c_2 + a_2 s_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(7 marks)

c) A spherical wrist with three rotary joints is shown in **Figure 4**, where the joint axes z_3, z_4 and z_5, intersect at one point. The arm parameters are given in **Table 3**. Compute the Jacobian matrix J.

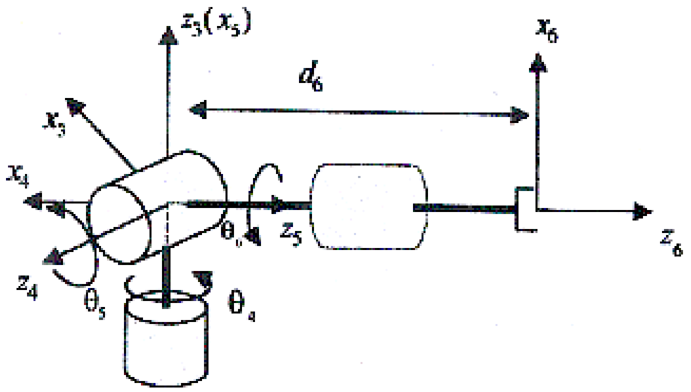

Figure 4. : Spherical wrist of a robot (RRR)

Table 3 : Arm parameter table

Link	θ_i	a_i	α_i	d_i
4	θ_4	0	-90°	0
5	θ_5	0	90°	0
6	θ_6	0	0°	d_6

(12 marks)

Question 5

The **Figure 5** below shows a relationship between 3 coordinate systems operating in the world coordinate frame, WC. In this robotics workcell the Robot Gripper is trying to track the Object Part as seen by the Camera.

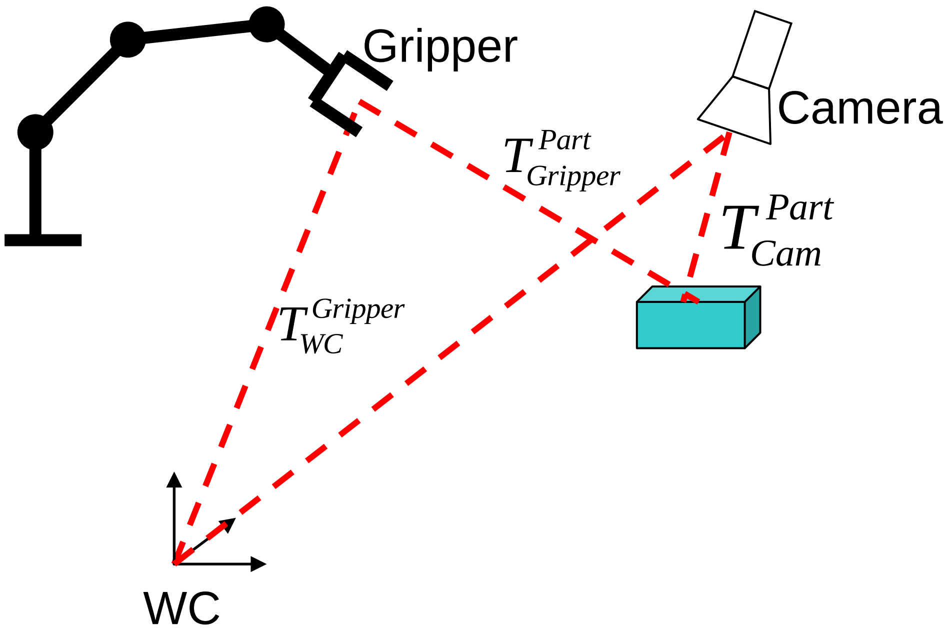

Figure 5: Coordinate Frame relationship between robot gripper, camera and object part operating in the world coordinate space.

a) Draw the "Motion Mapping" for the transformation of the Gripper frame to the Part frame.

(5 marks)

b) Derive the Del Operator equation matrix for $^{Gripper}\nabla$, the motion of the gripper about its own space.

(15 marks)

Question 6

a) The Lagrangian function, L, of a dynamical system is given as follows:

$$L = K(\dot{x}) - P(x)$$

Explain the meaning of the above equation. (5 marks)

b) Consider the point masses m_1 and m_2 at the distal end of links of the following θ - r robot manipulator with a rotary joint and a prismatic joint as shown in **Figure 6** below. Derive the differential equations of motion of the θ - r manipulator using the Lagrangian equation.

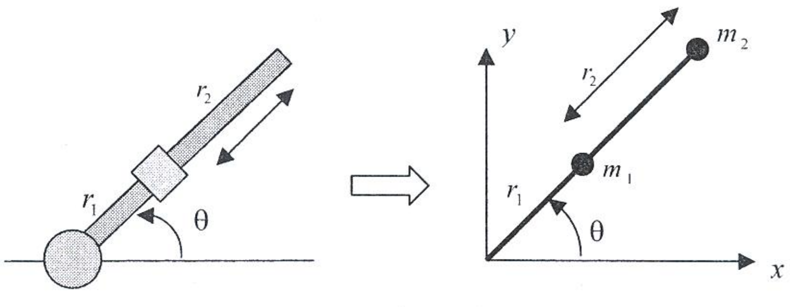

Figure 6. θ - r robot manipulator

(15 marks)

END OF QUESTION

APPENDIX

Matrix Functions

Rotation transformation:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad ; \quad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad ; \quad R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

The D-H transformation matrix from the frame $x_{i-1}y_{i-1}z_{i-1}$ to the frame $x_i y_i z_i$ is:

$$H_{i-1}^{i} = H(\theta_i) Tran(d_i) Tran(a_i) H(\alpha_i)$$

$$= \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The forward kinematics solution for an n-linked robot can be expressed as:

$$H_0^n = \begin{bmatrix} \mathbf{n} & \mathbf{o} & \mathbf{a} & \mathbf{P} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The inverse of a homogeneous transformation matrix H can be expressed as:

$$H^{-1} = \begin{bmatrix} R^T & -R^T P \\ 000 & 1 \end{bmatrix} = \begin{bmatrix} n_x & n_y & n_z & -n^T P \\ o_x & o_y & o_z & -o^T P \\ a_x & a_y & a_z & -a^T P \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Trigonometry Functions

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\frac{\partial}{\partial \theta_A}\left(\cos(A+B)\right) = -\sin(A+B)$$

$$\frac{\partial}{\partial \theta_B}\left(\cos(A+B)\right) = -\sin(A+B)$$

$$\frac{\partial}{\partial \theta_A}\left(\sin(A+B)\right) = \cos(A+B)$$

$$\frac{\partial}{\partial \theta_B}\left(\sin(A+B)\right) = \cos(A+B)$$

Equation	Solution
(a) $\sin\theta = a$	$\theta = A\tan 2\left(a, \quad \pm\sqrt{1-a^2}\right)$
(b) $\cos\theta = b$	$\theta = A\tan 2\left(\pm\sqrt{1-b^2}, \quad b\right)$
(c) $\begin{cases} \sin\theta = a \\ \cos\theta = b \end{cases}$	$\theta = A\tan 2(a,b)$
(d) $a\cos\theta - b\sin\theta = 0$	$\theta^{(1)} = A\tan 2\left(a, \quad b\right)$ $\theta^{(2)} = A\tan 2\left(-a, \quad -b\right) = \pi + \theta^{(1)}$
(e) $a\cos\theta + b\sin\theta = c$	$\theta^{(1)} = A\tan 2\left(c, \quad \sqrt{a^2+b^2-c^2}\right) - A\tan 2\left(a, \quad b\right)$ $\theta^{(2)} = A\tan 2\left(c, \quad -\sqrt{a^2+b^2-c^2}\right) - A\tan 2\left(a, \quad b\right)$
(f) $\begin{cases} a\cos\theta - b\sin\theta = c \\ a\sin\theta + b\cos\theta = d \end{cases}$	$\theta = A\tan 2\left(ad-bc, \quad ac+bd\right)$
(g) $\begin{cases} \sin\alpha\sin\beta = a \\ \cos\alpha\sin\beta = b \\ \cos\beta = c \end{cases}$	$\begin{cases} \alpha^{(1)} = A\tan 2\left(a, \quad b\right) \\ \beta^{(1)} = A\tan 2\left(\sqrt{a^2+b^2}, \quad c\right) \end{cases}$ $\begin{cases} \alpha^{(2)} = A\tan 2\left(-a, \quad -b\right) = \pi + \alpha^{(1)} \\ \beta^{(2)} = A\tan 2\left(-\sqrt{a^2+b^2}, \quad c\right) \end{cases}$

The Atan2 function can be defined as follows:

$$A\tan 2(p_x, p_y) = \begin{cases} \arctan\left(\dfrac{p_y}{p_x}\right) & p_x > 0 \\ \arctan\left(\dfrac{p_y}{p_x}\right) + \pi & p_x < 0 \\ \dfrac{\pi}{2} & p_x = 0 \quad \& \quad p_y > 0 \\ \dfrac{-\pi}{2} & p_x = 0 \quad \& \quad p_y < 0 \end{cases}$$

Jacobian Function.

The Del Operator ∇ for small motion about a fixed world coordinate frame can be given as follows:-

$$\Delta T \approx \begin{bmatrix} 0 & -\delta_z & \delta_y & dx \\ \delta_z & 0 & -\delta_x & dy \\ -\delta_y & \delta_x & 0 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet T \approx \nabla \bullet T$$

where,

$$\nabla \approx \begin{bmatrix} 0 & -\delta_z & \delta_y & dx \\ \delta_z & 0 & -\delta_x & dy \\ -\delta_y & \delta_x & 0 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Del Operator for small motion with respect to its own frame ${}^T\nabla$ can be defined as follows:

$$^{T}\nabla = \begin{bmatrix} 0 & -{}^T\delta_z & {}^T\delta_y & {}^Tdx \\ {}^T\delta_z & 0 & -{}^T\delta_x & {}^Tdy \\ -{}^T\delta_y & {}^T\delta_x & 0 & {}^Tdz \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where,

$$^{T}dx = \delta \bullet (\overrightarrow{d \times n}) + \vec{d}_p \bullet \vec{n} \qquad {}^T\delta x = \delta \bullet \vec{n}$$

$$^{T}dy = \delta \bullet (\overrightarrow{d \times o}) + \vec{d}_p \bullet \vec{o} \qquad {}^T\delta y = \delta \bullet \vec{o}$$

$$^{T}dz = \delta \bullet (\overrightarrow{d \times a}) + \vec{d}_p \bullet \vec{a} \qquad {}^T\delta x = \delta \bullet \vec{a}$$

Note:
d, **n**, **o** & **a** vectors are extracts from the *T* Matrix
d_p is the translation vector in $\forall$
δ is the rotational effects in $\forall$

The Jacobian matrix for an n-linked robot with position vector $\mathbf{P}$ at the end of the robot arm and joint variables $\mathbf{q}$ where $P = \begin{bmatrix} p_x & p_y & p_z \end{bmatrix}^T$ and $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & \cdots & q_n \end{bmatrix}^T$, can be defined as follows:

$$p_x = f_1(q_1, q_2, ..., q_n)$$
$$p_y = f_2(q_1, q_2, ..., q_n)$$
$$p_z = f_3(q_1, q_2, ..., q_n)$$

$$v_x = \frac{dp_x}{dt} = \frac{\partial f_1}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial f_1}{\partial q_2}\frac{dq_2}{dt} + \cdots + \frac{\partial f_1}{\partial q_n}\frac{dq_n}{dt}$$
$$v_y = \frac{dp_y}{dt} = \frac{\partial f_2}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial f_2}{\partial q_2}\frac{dq_2}{dt} + \cdots + \frac{\partial f_2}{\partial q_n}\frac{dq_n}{dt}$$
$$v_z = \frac{dp_z}{dt} = \frac{\partial f_3}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial f_3}{\partial q_2}\frac{dq_2}{dt} + \cdots + \frac{\partial f_3}{\partial q_n}\frac{dq_n}{dt}$$

where,

$$\frac{dq_i}{dt} = \dot{q}_i$$

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \cdots & \frac{\partial f_1}{\partial q_n} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \cdots & \frac{\partial f_2}{\partial q_n} \\ \frac{\partial f_3}{\partial q_1} & \frac{\partial f_3}{\partial q_2} & \cdots & \frac{\partial f_3}{\partial q_n} \\ \eta_1 R^0_{0(3col)} & \eta_2 R^1_{0(3col)} & \cdots & \eta_n R^{n-1}_{0(3col)} \end{bmatrix}$$

where,
$\eta_1 = 1$ for revolute joint
$\eta_1 = 0$ for prismatic joint

In vector form, the Jacobian matrix can also be defined as follows:

i) For Revolute Joint:

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} \overrightarrow{Z_{i-1}} \times \left(\overrightarrow{O_n} - \overrightarrow{O_{i-1}} \right) \\ \overrightarrow{Z_{i-1}} \end{bmatrix}$$

ii) For Prismatic Joint:

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} \overrightarrow{Z_{i-1}} \\ \vec{0} \end{bmatrix}$$

where, Z_{i-1}'s and O_{i-1}'s are the frame coordinates for the *i-1*th robot joint given by:

- Z_{i-1} is the 3rd column of the T_0^{i-1} (=A_1*… *A_{i-1})
- O_{i-1} is 4th column of the T_0^{i-1} (=A_1*… *A_{i-1})
- O_n is 4th column Of T_0^n (the FKS!)
- NOTE: when we extract the columns we only need the first 3 rows !!!.

Vector Function.

Cross Product of two vectors **A** and **B**. The end product is a vector **C**.

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i + \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

$$= \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix} = (a_2 b_3 - a_3 b_2) i + (a_3 b_1 - a_1 b_3) j + (a_1 b_2 - a_2 b_1) k$$

Dot Product of two vectors A and B. The end product is a scalar c.

$$c = A \bullet B = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = (a_1 b_1 + a_2 b_2 + a_3 b_3)$$