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CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION

JANUARY 2014 SESSION

SUBJECT CODE	:	**FEB 23023**
SUBJECT TITLE	:	**ELECTROMAGNETISM**
LEVEL	:	**BACHELOR**
TIME / DURATION	:	**3 HOURS**
DATE	:	

INSTRUCTIONS TO CANDIDATES

1. **Please read the instructions given in the question paper CAREFULLY.**
2. **This question paper is printed on both sides of the paper.**
3. **Please write your answers on the answer booklet provided.**
4. **Answers should be written in blue or black ink except for sketching, graphic and illustration.**
5. **This question paper consists of TWO (2) sections. Section A and B. Answer all questions in Section A. For Section B, answer three (3) questions only.**
6. **Answer all questions in English.**
7. **Do not open the question paper until instructed to do so**

THERE ARE 4 PAGES OF QUESTIONS, EXCLUDING THIS PAGE AND APPENDIX.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question1

In Cartesian coordinates, vector **A** is directed from origin to point P_1 (3, 2, 4) and vector **B** is directed from P_1 to point P_2 (-1, 3, 2). Determine:

(a) Vector **A** (2 marks)

(b) Vector **B** (2 marks)

(c) **A · B** (3 marks)

(d) **A** x **B**. (3 marks)

Question 2

Given point P_1 (2, π/4, 2) and vector $\mathbf{A} = \hat{\boldsymbol{r}} \cos \emptyset - \hat{\boldsymbol{\emptyset}} \sin \emptyset + \hat{\mathbf{z}} \cos \emptyset \, sin\emptyset$ defined in Cylindrical coordinates. Express P_1 and vector **A** in Spherical coordinates and evaluate **A** at P_1 (10 marks)

Question 3

A section of a sphere is described by $0 \leq R \leq 3,\ 0° \leq \theta \leq 90°,$ and $60° \leq \emptyset \leq 90°.$
Find the:

(a) Surface area of the spherical section (5 marks)

(b) Enclosed volume (5 marks)

Question 4

For the scalar function $V = xy - z^2$, determine its directional derivative along the direction of vector $\mathbf{A} = \hat{\boldsymbol{x}} - \hat{\boldsymbol{y}} z$ and then evaluate it at P (3, - 1, 2). (10 marks)

SECTION B (Total: 60 marks)

INSTRUCTION: Answer THREE (3) questions only.
Please use the answer booklet provided.

Question 5

(a) Given vectors $\mathbf{A} = -\hat{\mathbf{x}}5 - \hat{\mathbf{y}}2 + \hat{\mathbf{z}}3$, $\mathbf{B} = \hat{\mathbf{x}}2 - \hat{\mathbf{y}}6 + \hat{\mathbf{z}}4$ and $\mathbf{C} = \hat{\mathbf{x}}3 + \hat{\mathbf{y}} + \hat{\mathbf{z}}2$. Find:

(i) The magnitude, *A* and unit vector $\hat{\boldsymbol{a}}$ (4 marks)

(ii) The angle between vector **A** and **B**, θ_{AB} (4 marks)

(iii) A vector **D** whose magnitude is 9 and whose direction is perpendicular to both **B** and **C** (6 marks)

(b) Determine the divergence of $\mathbf{E} = \hat{\mathbf{x}}3x^2 + \hat{\mathbf{y}}2z + \hat{\mathbf{z}}x^2z$ at (2, -2, 0). (6 marks)

Question 6

(a) A square plate in the x-y plane is situated in the space defined by $0 \le x \le 2$ m and $0 \le y \le 3$ m. Calculate the total charge on the plate if the surface charge density is given by $\rho_s = 4xy^2$ µC/m^2 (5 marks)

(b) A square with sides 2 m each has a charge of $Q_1 = Q_2 = Q_3 = Q_4 = 20$ µC at each of its four corners as shown in **Figure 1** below. Determine the electric field, **E** at a point P, 5 m above the center of the square. (15 marks)

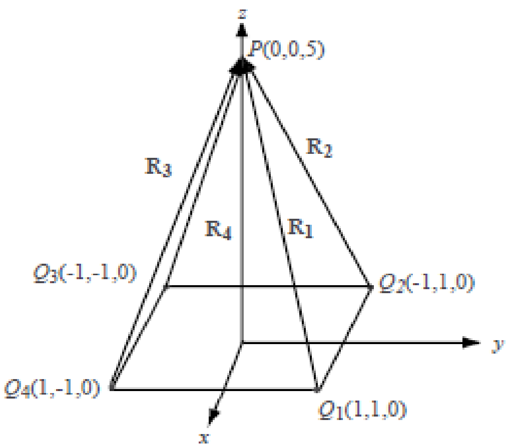

Figure 1

Question 7

(a) A proton with charge $q = 1.6 \times 10^{-19}$ C moving with a speed of 2.5×10^{6} m/s through a magnetic field with magnetic flux density of 2.5 T experiences a magnetic force of magnitude 3×10^{-13} N. Calculate the angle between the magnetic field and the proton's velocity. (5 marks)

(b) A 6 cm x 12 cm rectangular loop of wire is situated in the x-y plane with the center of the loop at the origin and its long sides parallel to the x-axis. The loop has a current of 20 A flowing anti-clockwise direction when viewed from above. Determine the magnetic field, **H** and magnetic flux density, **B** at the center of the loop. Take $\mu_0 = 4\pi \times 10^{-7}$

(15 marks)

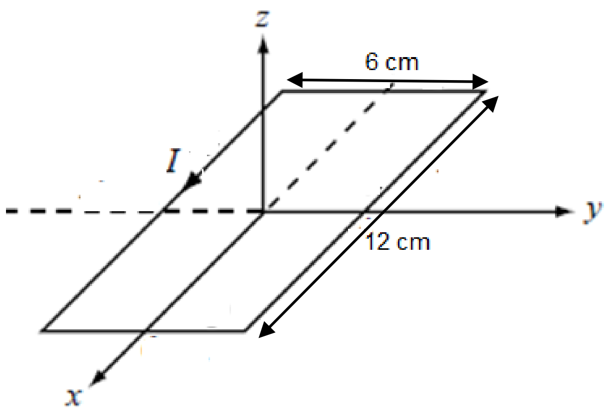

Figure 2

Question 8

(a) Determine voltages V_1 and V_2 across the 2 Ω and 4 Ω resistors shown in **Figure 3** below. The loop is located in the x-y plane, and its area is $4m^2$. The magnetic flux density is $\mathbf{B} = -\hat{z}0.3t$ (T) and the internal resistance of the wire may be ignored.

(10 marks)

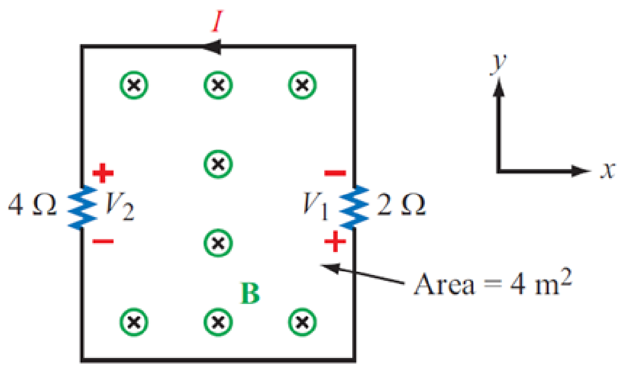

Figure 3

(b) The wire shown in Figure 4 below carries a current I = 10 A. A 30 cm long metal rod moves with a constant velocity $\mathbf{u} = \hat{z}5$ m/s. Find V_{12}. (10 marks)

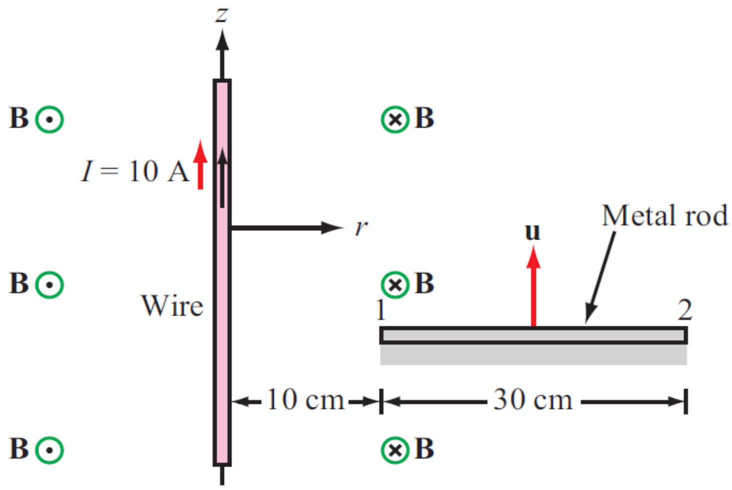

Figure 4

END OF QUESTION PAPER

APPENDIX

Table 1: Summary of vector relations

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Table 2: Coordinate transformation relations

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

FORMULA

Vector analysis

Gradient of a scalar T:

$$\nabla T = \text{grad } T = \hat{\mathbf{x}}\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\frac{\partial T}{\partial z}$$

Directional derivative of T along $\hat{\mathbf{a}}_l$:

$$\frac{dT}{dl} = \nabla T \cdot \hat{\mathbf{a}}_l .$$

Divergence of a vector **E**:

$$\nabla \cdot \mathbf{E} = \text{div } \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

Laplacian of a scalar *V:*

$$\nabla^2 V = \nabla \cdot (\nabla V) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} .$$

Electrostatics

Charge distributions:

$$Q = \int_l \rho_l \, dl$$

$$Q = \int_S \rho_s \, ds$$

$$Q = \int_V \rho_v \, dV \qquad \text{(C)}$$

Electric field due to multiple charges:

$$\mathbf{E} = \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N} \frac{q_i (\mathbf{R} - \mathbf{R}_i)}{|\mathbf{R} - \mathbf{R}_i|^3} \qquad \text{(V/m)}.$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \simeq (1/36\pi) \times 10^{-9} \qquad \text{(F/m)}$$

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Magnetostatics

Magnetic force:

$$\mathbf{F}_{\mathrm{m}} = q\mathbf{u} \times \mathbf{B} \qquad \text{(N)}$$

$F_m = quB \sin \theta$

Electromagnetic (Lorentz) force:

$$\mathbf{F} = \mathbf{F}_{\mathrm{e}} + \mathbf{F}_{\mathrm{m}} = q\mathbf{E} + q\mathbf{u} \times \mathbf{B} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B}).$$

Magnetic flux density:

$$\mathbf{B} = \mu_0 \mathbf{H} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I l}{2\pi r \sqrt{4r^2 + l^2}} \qquad \text{(T)}.$$

For an infinitely long wire with $l \gg r$,

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r} \qquad \text{(infinitely long wire)}.$$

Maxwell's equation for time-varying field

Magnetic flux:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} \qquad \text{(Wb)}$$

Transformer emf:

$$V_{\mathrm{emf}}^{\mathrm{tr}} = -N \frac{d\Phi}{dt} \qquad \text{(V)}$$

Motional emf:

$$V_{\mathrm{emf}}^{\mathrm{m}} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} \qquad \text{(V)}$$

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