



**UNIVERSITI KUALA LUMPUR
Malaysia France Institute**

**FINAL EXAMINATION
SEPTEMBER 2013 SESSION**

SUBJECT CODE : FAB 40803
SUBJECT TITLE : CONTROL SYSTEM 2
LEVEL : BACHELOR
TIME / DURATION : 3 HOURS
DATE :

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
 2. This question paper is printed on both sides of the paper.
 3. Please write your answers on the answer booklet provided.
 4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
 5. This question paper consists of five (5) questions. Answer four (4) questions only.
 6. Answer all questions in English.
 7. The semi-log paper, graph paper and formula are appended.
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THERE ARE 6 PAGES OF QUESTIONS AND 3 PAGES OF APPENDICES, EXCLUDING THIS PAGE.

Question 1

- (a) **Figure 1** shows the Root locus of a system, identify the number of finite open-loop poles and locate their location in s-plane. Hence, comment on the system stability. (4 marks)

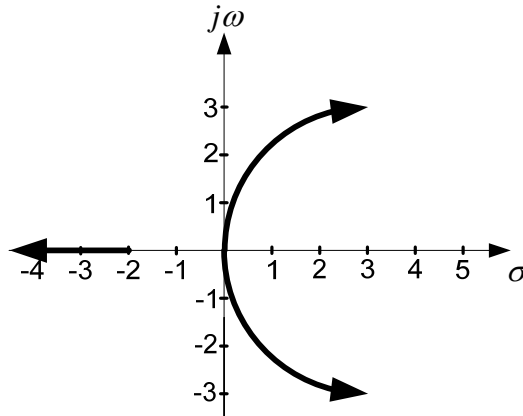


Figure 1: The root locus of a system.

- (b) The unity feedback system with feed-forward transfer function of

$$G(s) = \frac{K(s+4)(s+5)}{s(s+1)}$$

is to be designed for the desired closed loop poles located at $-1 \pm j1.2$.

- i. Sketch the root locus of the closed loop system as K varies from zero to infinity, in the s-plane and indicate the asymptotes, break points on the real axis, cross-over points at the imaginary axis, angle of departure or arrival, where appropriate. (8 marks)
- ii. Find the value of gain, K (for desired closed loop poles at $-1 \pm j1.2$), percent overshoot (%OS), settling time (T_s at 2% criteria) and peak time (T_p). (5 marks)
- iii. Design a suitable compensator to improve the settling time (T_s) by a factor of 2 with the same damping ratio. (8 marks)

Question 2

Block diagram of a unity feedback system is shown in **Figure 2**.

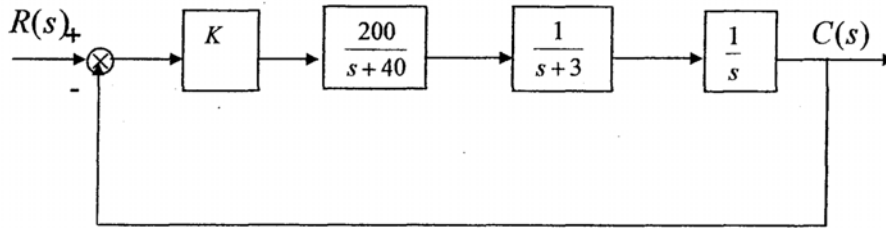


Figure 2: Block diagram of a unity feedback system

- (a) Determine the amplifier gain, K to yield a step response with 10% overshoot. (2 marks)
- (b) Draw the Bode diagram for magnitude and phase plots of the uncompensated system. (5 marks)
- (c) Find the gain margin, phase margin, gain crossover frequency and phase crossover frequency for the uncompensated system. (2 marks)
- (d) With the aid of the gain-adjusted magnitude plot, design a lead compensator, $G_c(s)$ to improve the transient response by increasing the percentage overshoot to 15% when the static velocity error constant is 31.6sec^{-1} . Apply the correction factor of 10° in your design. (12 marks)
- (e) Determine the open-loop transfer function of the compensated system with appropriate gain KK_c . (4 marks)

Question 3

A compact disc player for a portable use requires good rejection of disturbances and accurate position of the optical reader sensor. The position control system uses unity feedback with a plant transfer function as follows:

$$G(s) = \frac{2s + 3}{s^2 + 8s + 9}$$

- (a) Represent the system in controllable canonical form
(4 marks)
- (b) Determine whether the system is observable, controllable or both observable and controllable.
(4 marks)
- (c) Design a controller using Ackermann's formula such that the percent overshoot (%OS) and settling time (T_s (5% criteria)) are less than 5% and 1.5 sec respectively.
(8 marks)
- (d) Design an observer using the same Ackermann's formula such that the settling time improves to 0.3 sec.
(7 marks)

Question 4

Consider a block diagram of a sampled-data system as shown **Figure 5**.

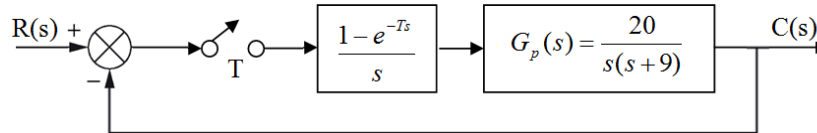


Figure 5: Block diagram of a closed-loop sampled system.

- (a) Find the open-loop transfer function, $G(z) = \frac{C(z)}{R(z)}$ of the system if sampling interval, $T = 0.5$ second. (8 marks)
- (b) Find the closed-loop transfer function $T(z)$ for a unity feedback system (3 marks)
- (c) Determine the range of sampling interval, T that make the system stable. (4 marks)
- (d) Find the sampled time function for the following transfer function using partial fraction. (5 marks)

$$F(z) = \frac{(z + 0.2)(z + 0.4)}{(z - 0.1)(z - 0.5)(z - 0.9)}$$

- (e) Find $f(kT)$ for the following transfer function (5 marks)

$$F(z) = \frac{z(z + 3)(z + 5)}{(z - 0.4)(z - 0.6)(z - 0.8)}$$

Question 5

(a) With reference to the functional block diagram and equations of an antenna azimuth position control system are shown in **Figure 6** and **Figure 7** respectively. Given that $K_1=100$, $K_m=2$, $K_g=0.1$ and $K_{pot}=2$.

i. Use the frequency response technique to determine the value of the pre-amplifier gain(K) for the closed-loop response to have 9% overshoot for a step input.

(12 marks)

ii. Estimate the settling time, T_s .

[Hint: Find the frequency, ω_{BW} at -7dB for the closed-loop bandwidth]

(3 marks)

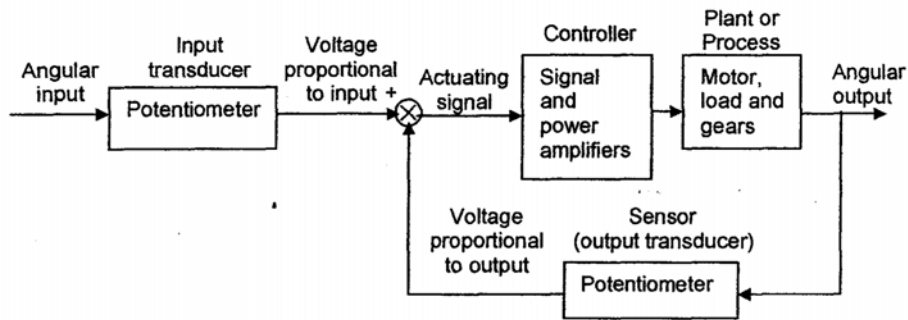


Figure 6: The functional block diagram of an antenna azimuth position control system

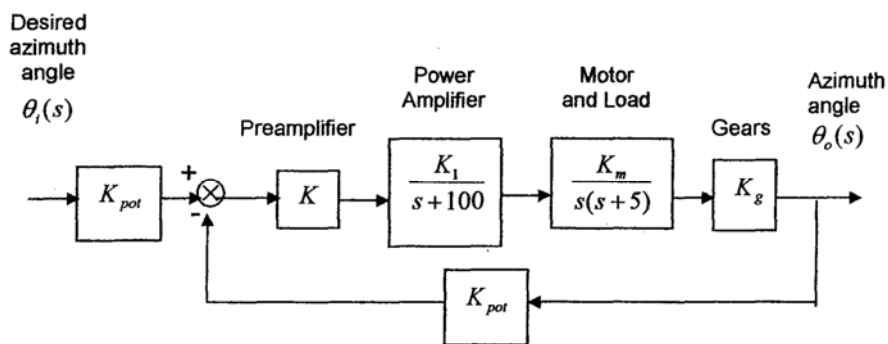


Figure 7: The equations of an antenna azimuth position control system

- (b) Consider the following transfer function of a system,

$$\frac{C(s)}{R(s)} = \frac{(s + 0.8)}{s^2 + 1.3s + 0.4}$$

- i. Obtain the state space representation of this system in the controllable canonical form and the observable canonical form
(6 marks)
- ii. Show that the state space representation obtain in (i) is state controllable but not observable.
(4 marks)

END OF QUESTION

APPENDIX

Time Response:

Underdamped Second Order Systems

Settling Time (within 2% of steady state value)	$T_s = \frac{4}{\zeta w_n}$
Peak Time	$T_p = \frac{\pi}{w_d}$
Damped Frequency	$w_d = w_n \sqrt{1 - \zeta^2}$
Damping Ratio	$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$
Closed-Loop Transfer Function	$G(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$
Percent overshoot, %OS	$e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\%$

Frequency Response:

Closed-Loop Bandwidth	$w_{BW} = \frac{4}{T_s \zeta} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$ $= \frac{\pi}{T_p \sqrt{1 - \zeta^2}} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$ $= w_n \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$
Phase Margin	$\phi_m = \tan^{-1} \frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}}$
Maximum Phase Shift of Compensator	$\phi_{max} = \sin^{-1} \left(\frac{1 - \beta}{1 + \beta} \right)$
Magnitude at w_{max}	$M_{max} = \frac{1}{\sqrt{\beta}}$

The State Model

For linear time-invariant system:

$$\dot{X} = AX(t) + BU(t)$$

$$Y(t) = CX(t) + DU(t)$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times n_i}$, $C \in \mathbb{R}^{n_o \times n}$, $D \in \mathbb{R}^{n_o \times n_i}$.

The solution $X(t)$	$e^{At} X(0) + \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau$
The matrix exponential e^{At}	$e^{At} = I + \frac{At}{1!} + \frac{A^2 t^2}{2!} + \dots$ $e^{At} = \mathcal{L}^{-1}((sI - A)^{-1})$
Eigenvalues	$\det(\lambda I - A) = 0$
Transfer function	$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1} B + D$
Controllability matrix	$C = [B \quad AB \quad A^2 B \quad \dots \quad A^{n-1} B]$
Observability matrix	$O = [C \quad CA \quad CA^2 \quad \dots \quad CA^{n-1}]^T$
Ackermann's formula	Controller: $K = [0 \quad 0 \quad \dots \quad 0 \quad 1][B \quad AB \quad \dots \quad A^{n-2} B \quad A^{n-1} B]^{-1} \alpha_c(A)$ Estimator: $G = \alpha_e(A)([C \quad CA \quad \dots \quad CA^{n-1}]^T)^{-1} [0 \quad 0 \quad \dots \quad 0 \quad 1]^T$

Table 13.1 Partial table of z- and s-transforms

	f(t)	F(s)	F(z)	f(kT)
1.	$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$	$u(kT)$
2.	t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$	kT
3.	t^n	$\frac{n!}{s^{n+1}}$	$\lim_{a \rightarrow 0} (-1)^n \frac{d^n}{da^n} \left[\frac{z}{z - e^{-aT}} \right]$	$(kT)^n$
4.	e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z - e^{-aT}}$	e^{-akT}
5.	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$(-1)^n \frac{d^n}{da^n} \left[\frac{z}{z - e^{-aT}} \right]$	$(kT)^n e^{-akT}$
6.	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$\frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$	$\sin \omega kT$
7.	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	$\frac{z(z - \cos \omega T)}{z^2 - 2z \cos \omega T + 1}$	$\cos \omega kT$
8.	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$	$\frac{ze^{-aT} \sin \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$	$e^{-akT} \sin \omega kT$
9.	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$	$\frac{z^2 - ze^{-aT} \cos \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$	$e^{-akT} \cos \omega kT$