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SET A

# UNIVERSITI KUALA LUMPUR
## Malaysia France Institute

# FINAL EXAMINATION
## JANUARY 2010 SESSION

| | | |
|---|---|---|
| SUBJECT CODE | : | FAB 40803 |
| SUBJECT TITLE | : | CONTROL SYSTEM 2 |
| LEVEL | : | BACHELOR |
| TIME / DURATION | : | 9.00am – 12.00pm<br>3 HOURS |
| DATE | : | 30 APRIL 2010 |

## INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of five (5) questions. Answer four (4) questions only.
6. Answer all questions in English.
7. The semi-log paper, graph paper and formula are appended.

THERE ARE 5 PAGES OF QUESTIONS AND 3 PAGES OF APPENDICES, EXCLUDING THIS PAGE.

**Question 1**

**Figure 1** shows the control systems of a mobile robot.

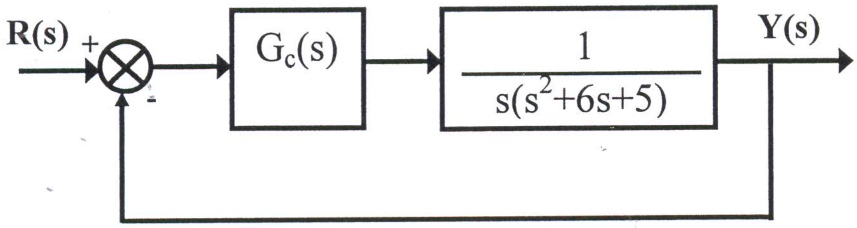


**Figure 1**: Control systems of a mobile robot.

a) Let the controller $G_c(s)$ be a simple proportional controller with gain **K**. Sketch the Root Locus of the systems as **K** varies from zero to infinities. Indicate the asymptotes, break points on the real axis, cross-over points at the imaginary axis, angle of departure or arrival, where appropriate.

(10 marks)

b) Find the value of **K** such that the systems operate at 22% overshoot and calculate the settling time, $T_s$ for a unit step input for this value of **K**.

(5 marks)

c) Design a suitable controller using Root Locus technique so that the transient response will behave according to a second order system with settling time reduced to 2.5 seconds at the same overshoot.

(10 marks)

## Question 2

A simplified representation of a vehicle's front wheel system is given by the following transfer function:-

$$G(s) = \frac{25K}{s^3 + 175s^2 + 850s}$$

a) Use asymptotic approximation frequency response technique to find the value of **K** to yield a closed-loop step response with 12% overshoot. Indicate zero dB frequency, -180° frequency, Phase Margin and Gain Margin in your plot.

(10 marks)

b) Design a suitable controller to yield 12% overshoot, peak time of 0.2 second and static velocity error constant, $K_v = 35$.

(12 Marks)

c) Evaluate the bandwidth and settling time of the compensated systems.

(3 marks)

## Question 3

The conceptual block diagram of a gas-fired heater is shown in **Figure 2**. The commanded fuel pressure is proportional to the desired temperature. The difference between the commanded fuel pressure and measured pressure related to the output temperature is used to actuate a valve and release fuel for the heater. The rate of the fuel flow determines the temperature. When the output temperature equals the equivalent commanded temperature as determined by the commanded fuel pressure, the fuel flow stops and the heater shuts off.

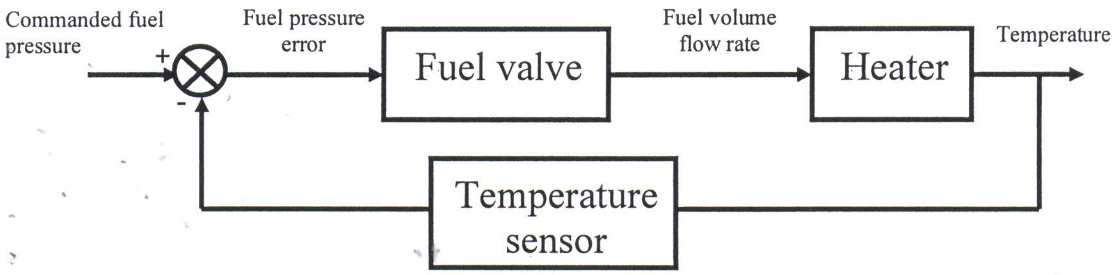


**Figure 2**: The conceptual block diagram of a gas-fired heater

The transfer function of the heater is $G_H(s) = \dfrac{1}{(s+0.8)}$, the transfer function of the temperature sensor is $G_F(s) = 1$ and the transfer function of the fuel valve is $G_V(s) = \dfrac{5}{(s+5)}$.

a) Design a state variable controller that yields a 5% overshoot and settling of 10 minutes using direct substitution approach.

(14 marks)

b) Design an observer that will respond 10 times faster than the system but with the same percent overshoot using Ackermann's Formula.

(11 marks)

## Question 4

a) Give three advantages of replacing analog components with a digital computer in feedback control application.

(3 marks)

b) Find the z-transform of a sampled unit ramp.

(7 marks)

c) The functional block diagram and equations of an antenna azimuth position control system are shown in **Figure 3** and **Figure 4** respectively. Use the frequency response technique to calculate the value of the pre-amplifier gain (K) for the closed-loop response to have a 12% overshoot for a step input. Then, estimate the settling time. Given that $K_1$=100, $K_m$=0.1 and $K_{pot}$=2. (Hint: Find the frequency, $\omega$ at -7dB for the closed-loop bandwidth)

(15 marks)

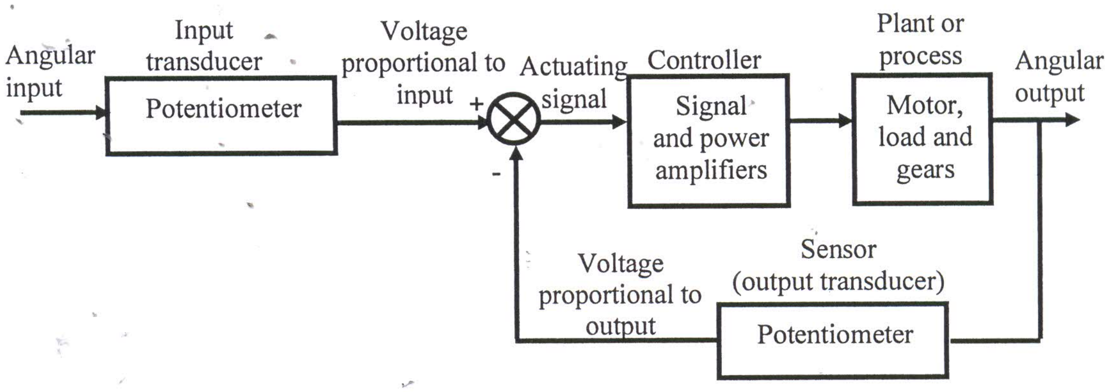


**Figure 3**: The functional block diagram of an antenna azimuth position control system

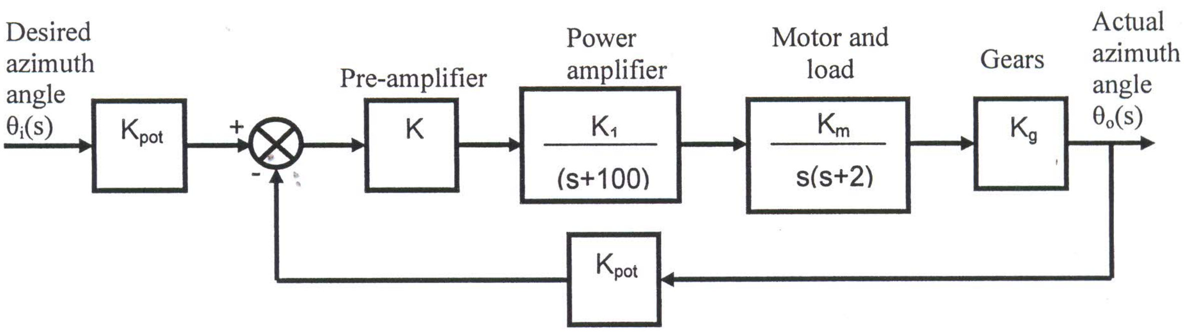


**Figure 4**: The block diagram with the respective equations of an antenna azimuth position control system

## Question 5

a) Find the sampled-data transfer function $G(z) = \dfrac{Y(z)}{R(z)}$ for the block diagram shown in **Figure 5**.

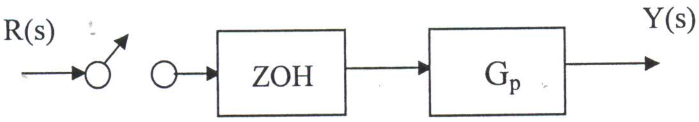


**Figure 5**: Block diagram of sample-data systems.

Given

$$G_p(s) = \frac{(s+2)}{(s+1)}$$

and

$$T = 0.5 \text{ s}$$

(10 marks)

b) Hydraulic power actuators were used to drive the dinosaurs of the Jurassic Park movie. The motion of the large monster required high-power actuators requiring 1200 watts. One specific limb motion has dynamic represented by

$$G(s) = \frac{1}{s(s+2)}$$

It is desired to place the closed-loop poles at $s = -2 \pm j2$. Determine the required state variable feedback using Ackermann's formula. Assume that the position and velocity of the output motion are available for measurement.

(15 marks)

**END OF QUESTION**

## APPENDIX

**Time Response:**

Underdamped Second Order Systems

| | |
|---|---|
| Settling Time (within 2% of steady state value | $T_s = \frac{4}{\zeta w_n}$ |
| Peak Time | $T_p = \frac{\pi}{w_d}$ |
| Damped Frequency | $w_d = w_n\sqrt{1-\zeta^2}$ |
| Damping Ratio | $\varsigma = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$ |
| Closed-Loop Transfer Function | $G(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$ |
| Percent overshoot, %OS | $e^{\frac{-\varsigma\pi}{\sqrt{1-\varsigma^2}}} \times 100\%$ |

**Frequency Response:**

| | |
|---|---|
| Closed-Loop Bandwidth | $w_{BW} = \frac{4}{T_S\zeta}\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}}$ <br> $= \frac{\pi}{T_p\sqrt{1-\zeta^2}}\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}}$ <br> $= w_n\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}}$ |
| Phase Margin | $\phi_m = \tan^{-1}\frac{2\varsigma}{\sqrt{-2\varsigma^2+\sqrt{1+4\varsigma^4}}}$ |
| Maximum Phase Shift of Compensator | $\phi_{max} = \sin^{-1}\left(\frac{1-\beta}{1+\beta}\right)$ |
| Magnitude at $w_{max}$ | $M_{max} = \frac{1}{\sqrt{\beta}}$ |

**The State Model**

For linear time-invariant system:

$$\dot{X} = AX(t) + BU(t)$$
$$Y(t) = CX(t) + DU(t)$$

where $A \in \Re^{nXn}$, $B \in \Re^{nXn_i}$, $C \in \Re^{n_oXn}$, $D \in \Re^{nXn_i}$.

| | |
|---|---|
| The solution $X(t)$ | $e^{At}X(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$ |
| The matrix exponential $e^{At}$ | $e^{At} = I + \frac{At}{1!} + \frac{A^2t^2}{2!} + \cdots$<br>$e^{At} = \mathcal{L}^{-1}((sI - A)^{-1})$ |
| Eigenvalues | $\det(\lambda I - A) = 0$ |
| Transfer function | $G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$ |
| Controllability matrix | $C = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B]$ |
| Observability matrix | $O = [C \quad CA \quad CA^2 \quad \cdots \quad CA^{n-1}]^T$ |
| Ackermann's formula | Controller:<br>$K = [0 \quad 0 \quad \cdots \quad 0 \quad 1][B \quad AB \quad \cdots \quad A^{n-2}B \quad A^{n-1}B]^{-1}\alpha_c(A)$<br>Estimator:<br>$G = \alpha_e(A)([C \quad CA \quad \cdots \quad CA^{n-1}]^T)^{-1}[0 \quad 0 \quad \cdots \quad 0 \quad 1]^T$ |

**Table 13.1** Partial table of z- and s-transforms

| | f(t) | F(s) | F(z) | f(kT) |
|---|---|---|---|---|
| 1. | $u(t)$ | $\frac{1}{s}$ | $\frac{z}{z-1}$ | $u(kT)$ |
| 2. | $t$ | $\frac{1}{s^2}$ | $\frac{Tz}{(z-1)^2}$ | $kT$ |
| 3. | $t^n$ | $\frac{n!}{s^{n+1}}$ | $\lim_{a\to 0}(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$ | $(kT)^n$ |
| 4. | $e^{-at}$ | $\frac{1}{s+a}$ | $\frac{z}{z-e^{-aT}}$ | $e^{-akT}$ |
| 5. | $t^n e^{-at}$ | $\frac{n!}{(s+a)^{n+1}}$ | $(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$ | $(kT)^n e^{-akT}$ |
| 6. | $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ | $\frac{z \sin \omega T}{z^2 - 2z\cos \omega T + 1}$ | $\sin \omega kT$ |
| 7. | $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ | $\frac{z(z-\cos \omega T)}{z^2 - 2z\cos \omega T + 1}$ | $\cos \omega kT$ |
| 8. | $e^{-at}\sin \omega t$ | $\frac{\omega}{(s+a)^2+\omega^2}$ | $\frac{ze^{-aT}\sin \omega T}{z^2 - 2ze^{-aT}\cos \omega T + e^{-2aT}}$ | $e^{-akT}\sin \omega kT$ |
| 9. | $e^{-at}\cos \omega t$ | $\frac{s+a}{(s+a)^2+\omega^2}$ | $\frac{z^2 - ze^{-aT}\cos \omega T}{z^2 - 2ze^{-aT}\cos \omega T + e^{-2aT}}$ | $e^{-akT}\cos \omega kT$ |