Document No : UniKL MFI_SD_AC41
Revision No: 02
Effective Date: 01 December 2008

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SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2010 SESSION

SUBJECT CODE	:	FIB 46503
SUBJECT TITLE	:	COMPUTER INTEGRATED MANUFACTURING
LEVEL	:	BACHELOR
TIME / DURATION	:	04.00 pm – 06.00 pm (2 HOURS)
DATE	:	29 APRIL 2010

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of TWO (2) sections. Section A and B. Answer all questions in Section A. For Section B, answer two (2) questions only.
6. Answer all questions in English.
7. Graph paper is appended.

THERE ARE 4 PAGES OF QUESTIONS AND 11 PAGES OF APPENDICES, EXCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question 1

Define cellular manufacturing, and describe 3 principle objectives when implementing cellular manufacturing.

(5 marks)

Question 2

Describe 5 benefits that can be expected from a successful Flexible Manufacturing System (FMS) installation?

(5 marks)

Question 3

Extended bottle neck model of FMS

A flexible machining system is used to produce two types of parts. The FMS consists of load/unload station, two automated processing stations, and an automated conveyor system.

The number of servers at each station is given in Table 1. The product mix and process times for the parts are presented in Table 2. The operation frequency f_{ijk} = 1 for all operations, and the move time between stations is 4 minutes

(a) Calculate workloads of each workstation

(10 marks)

(b) Determine the bottleneck station and calculate the FMS production rate.

(10 marks)

(c) Use the extended bottleneck model to determine the production rate and waiting time for the case when work in process (N) = 8.

(10 marks)

Table 1: Description of each station and its number of server

Station	Description	Number of servers
1	Load and unload	2 workers
2	Process X	3 servers
3	Process Y	4 servers
4	Transport (conveyor)	8 carriers

Table 2: Product mix fractions and station process times

Product j	Product mix p_j	Station i	Process time t_{ijk} (min)
A	0.6	1	5
		2	15
		3	25
		1	4
B	0.40	1	5
		2	10
		3	30
		1	4

SECTION B (Total: 60 marks)

INSTRUCTION: Answer TWO (2) questions only.
Please use the answer booklet provided.

Question 4

Cellular manufacturing/ Group technology problem

Apply the rank order clustering technique to the part-machine incidence matrix in Table 3 to identify logical part families and machine groups. Parts are identified by letters, and machines are identified numerically.

Table 3 : Part - machine incidence matrix

Machines	Parts								
	A	B	C	D	E	F	G	H	I
1	1								1
2		1					1		
3			1		1			1	
4		1				1	1		
5			1					1	
6						1	1		
7	1			1					
8			1		1				

(30 marks)

Question 5

Line balancing problem

A large manufacturer of pencil sharpeners is planning to add anew line of sharpeners, and you have been asked to balance the process, given the following task times and precedence relationship as shown in Table 4. The line is being designed to produce 252 units/day, with operating hours of 7 hours/day. Assume line efficiency = 0.90 and repositioning time = 12 seconds.

(a) Compute hourly production rate.

(5 marks)

(b) Use Kilbridge and Wester method to balance the line.

(15 marks)

(c) Determine the percentage of idle time (balance delay).

(10 marks)

Table 4: The work elements, time and predecessor

Element	Time T_e	Preceded by:	Element	Time T_e	Preceded by:
A	0.2	-	E	0.1	-
B	0.4	A	F	0.8	E
C	0.3	-	G	0.3	D,F
D	1.3	B,C	H	1.2	G

Question 6

Production line with storage buffers

A proposed synchronous transfer line will have 20 stations and will operate with an ideal cycle time of 0.5 min. All stations are expected to have an equal probability of breakdown, p = 0.01. The average downtime per breakdown is expected to be 5.0. An option under consideration is to divide the line into two stages, each stage having 10 stations, with a buffer storage zone between the stages. It has been decided that the storage capacity should be 20 units. The cost to operate the line is $96.00/hr. Installing the storage buffer would increase the line operating cost by $12.00/hr. Ignoring material and tooling costs, Determine:

(a) Line efficiency, production rate, and unit cost for the one-stage configuration,

(10 marks)

(b) Line efficiency, production rate, and unit cost for the optional two-stage configuration.

(20 marks)

END OF QUESTION

FIB 46503 - COMPUTER INTEGRATED MANUFACTURING

APPENDICES

APPENDIX 1: SINGLE STATION MANUFACTURING CELL

$$UT = \sum_{j=1}^{n_p} T_{cj} \quad (14.1)$$

where UT = unattended time of operation of the manufacturing cell, min; T_{cj} = cycle time for part j that is held in the parts storage subsystem, for $j = 1, 2, \ldots, n_p$, where n_p = parts storage capacity of the storage subsystem, pc.

If T_m = machine processing time and T_s = worker service time (to perform unloading and loading or other tending duties), then the overall cycle time of the single station with no storage is

$$T_c = T_m + T_s \quad (14.3)$$

$$WL = QT_c \quad (14.5)$$

where WL = workload scheduled for a given period, hr of work/hr or hr of work/wk; Q = quantity to be produced during the period, pc/hr or pc/wk, etc.; and T_c = cycle time.

$$n = \frac{WL}{AT} \quad (14.7)$$

where n = number of workstations; and AT = available time on one station in the period, hr/period. Let us illustrate the use of these equations with a simple example, and then consider some of the complications.

$$AT = H_{sh} AU \quad (14.8)$$

where AT = available time, hr; H_{sh} = shift hours during the period, hr; A = availability; and U = utilization.

$$Q = Q_o(1 - q) \quad (14.9)$$

where Q = quantity of good units made in the process; Q_o = original or starting quantity; and q = fraction defect rate.

The combined effect of worker efficiency and fraction defect rate is given in the following equation, which amends the workload formula, Eq. (14.5):

$$WL = \frac{QT_c}{(1 - q)} \quad (14.11)$$

$$n = \frac{T_m + T_s}{T_s + T_r} \quad (14.12)$$

where n = number of machines; T_m = machine semi-automatic cycle time, min; T_s = worker service time per machine, min; T_r = worker repositioning time between machines, min.

APPENDIX 2: MANUAL ASSEMBLY LINES

ANALYSIS OF SINGLE MODEL ASSEMBLY LINES

$$R_p = \frac{D_a}{50 S_w H_{sh}} \tag{15.3}$$

where R_p = average hourly production rate, units/hr; D_a = annual demand for the single product to be made on the line, units/yr; S_w = number of shifts/wk; and H_{sh} = hr/shift.

$$T_c = \frac{60E}{R_p} \tag{15.4}$$

where T_c = cycle time of the line, min/cycle; R_p = required production rate, as determined from Eq. (15.3), units/hr; the constant 60 converts the hourly production rate to a cycle time in minutes; and E = line efficiency. Typical values of E for a manual assembly line are in the range 0.90 to 0.98. The cycle time T_c establishes the ideal cycle rate for the line

$$R_c = \frac{60}{T_c} \tag{15.5}$$

$$w = \frac{WL}{AT} \tag{15.7}$$

where w = number of workers on the line; WL = workload to be accomplished in a given time period; and AT = available time in the period. The time period of interest will be 60 min. The *workload* in that period is the hourly production rate multiplied by the work content time of the product, that is,

$$WL = R_p T_{wc} \tag{15.8}$$

where R_p = production rate, pc/hr; and T_{wc} = work content time, min/pc.

$$w^* = \text{Minimum Integer} \geq \frac{T_{wc}}{T_c} \tag{15.9}$$

where w^* = theoretical minimum number of workers. If we assume one worker per station ($M_i = 1$ for all i, $i = 1, 2, \ldots, n$; and the number utility workers $w_u = 0$), then this ratio also gives the theoretical minimum number of workstations on the line.

$$T_s = \text{Max}\{T_{si}\} \leq T_c - T_r \tag{15.11}$$

$$E_r = \frac{T_s}{T_c} = \frac{T_c - T_r}{T_c} \tag{15.12}$$

where E_r = *repositioning efficiency* and the other terms are defined above.

The Line Balancing Problem

$$T_{wc} = \sum_{k=1}^{n_e} T_{ek} \tag{15.13}$$

where T_{ek} = time to perform work element k, min; and n_e = number of work elements into which the work content is divided, that is, $k = 1, 2, \ldots, n_e$.

$$E_b = \frac{T_{wc}}{wT_s} \tag{15.16}$$

where E_b = balance efficiency, often expressed as a percent; T_s = the maximum available service time on the line $(\text{Max}\{T_{si}\})$, min/cycle; and w = number of workers.

$$d = \frac{(wT_s - T_{wc})}{wT_s} \tag{15.17}$$

where d = balance delay and the other terms have the same meaning as before. A balance delay of zero indicates a perfect balance. Note that $E_b + d = 1$.

.1 Largest Candidate Rule

In this method, work elements are arranged in descending order according to their T_{ek} values, as in Table 15.5. Given this list, the algorithm consists of the following steps: (1) assign elements to the worker at the first workstation by starting at the top of the list and selecting the first element that satisfies precedence requirements and does not cause the total sum of T_{ek} at that station to exceed the allowable T_s; when an element is selected for assignment to the station, start back at the top of the list for subsequent assignments; (2) when no more elements can be assigned without exceeding T_s, then proceed to the next station; (3) repeat steps 1 and 2 for as many additional stations as necessary until all elements have been assigned.

.2 Kilbridge and Wester Method

In the Kilbridge and Wester method, work elements in the precedence diagram are arranged into columns, as shown in Figure 15.7. The elements can then be organized into a list according to their columns, with the elements in the first column listed first.

.3 Ranked Positional Weights Method

The ranked positional weights method was introduced by Helgeson and Birnie [13]. In this method, a ranked positional weight value (call it *RPW* for short) is computed for each element. The *RPW* takes into account both the T_{ek} value and its position in the precedence diagram. Specifically, RPW_k is calculated by summing T_{ek} and all other times for elements that follow T_{ek} in the arrow chain of the precedence diagram. Elements are compiled into a list according to their *RPW* value, and the algorithm proceeds using the same three steps as before.

APPENDIX 3: AUTOMATED PRODUCTION LINES

Transfer Lines with No Internal Parts Storage

$$T_c = \text{Max}\{T_{si}\} + T_r \qquad (16.5)$$

where T_c = ideal cycle time on the line, min; T_{si} = the processing time at station i, min; and T_r = repositioning time, called the transfer time here, min. We use the Max{ [illegible]

We can formulate the following expression for the actual average production time T_p:

$$T_p = T_c + FT_d \qquad (16.6)$$

where F = downtime frequency, line stops/cycle; and T_d = average downtime per line stop, min.

$$F = \sum_{i=1}^{n} p_i \qquad (16.7)$$

where F = expected frequency of line stops per cycle, first encountered in Eq. (16.6); p_i = frequency of station breakdown per cycle, causing a line stop; and n = number of workstations on the line.

$$R_p = \frac{1}{T_p} \qquad (16.9)$$

where R_p = actual average production rate, pc/min; and T_p is the actual average production time from Eq. (16.6), min. It is of interest to compare this rate with the ideal production rate given by

$$R_c = \frac{1}{T_c} \qquad (16.10)$$

where R_c = Ideal production rate, pc/min. It is customary to express production rates on automated production lines as hourly rates, so we must multiply the rates in Eqs. (16.9) and (16.10) by 60.

$$E = \frac{T_c}{T_p} = \frac{T_c}{T_c + FT_d} \qquad (16.11)$$

where E = the proportion of uptime on the production line, and the other terms are as previously defined.

$$D = \frac{FT_d}{T_p} = \frac{FT_d}{T_c + FT_d} \qquad (16.12)$$

where D = proportion of downtime on the line. It is obvious that

$$E + D = 1.0 \qquad (16.13)$$

$$C_{pc} = C_m + C_o T_p + C_t \qquad (16.14)$$

where C_{pc} = cost per piece, \$/pc; C_m = cost of starting material, \$/pc; C_o = cost per minute to operate the line, \$/min; T_p = average production time per piece, min/pc; and C_t = cost of tooling per piece, \$/pc.

Transfer Lines with Internal Storage Buffers

$$E_k = \frac{T_c}{T_c + F_k T_{dk}} \qquad (16.16)$$

where the subscript k is used to identify the stage. According to our argument above, the overall line efficiency would be given by

$$E_\infty = \text{Minimum}\{E_k\} \quad \text{for } k = 1, 2, \ldots, K \qquad (16.17)$$

where the subscript ∞ identifies E_∞ as the efficiency of a line whose storage buffers all have infinite capacity.

Analysis of a Two-Stage Transfer Line.

$$E_b = E_0 + D'_1 h(b) E_2 \qquad (16.19)$$

where E_b = overall line efficiency for a two-stage line with buffer capacity b;

$$E_0 = \frac{T_c}{T_c + (F_1 + F_2)T_d} \qquad (16.20)$$

The term D'_1 can be thought of as the proportion of total time that stage 1 is down, defined as follows:

$$D'_1 = \frac{F_1 T_d}{T_c + (F_1 + F_2)T_d} \qquad (16.21)$$

TABLE Formulas for Computing $h(b)$ in Eq. (16.19) for a Two-Stage Automated Production Line Under Several Downtime Distributions

Assumptions and definitions: Assume that the two stages have equal downtime distributions ($T_{d1} = T_{d2} = T_d$) and equal cycle times ($T_{c1} = T_{c2} = T_c$). Let F_1 = downtime frequency for stage 1 and F_2 = downtime frequency for stage 2. Define r to be the ratio of breakdown frequencies as follows:

$$r = \frac{F_1}{F_2} \qquad (16.22)$$

Equations for h(b): With these definitions and assumptions, we can express the relationships for $h(b)$ for two theoretical downtime distributions as derived by Buzacott [2]:

Constant downtime: Each downtime occurrence is assumed to be of constant duration T_d. This is a case of no downtime variation. Given buffer capacity b, define B and L as

$$b = B\frac{T_d}{T_c} + L \qquad (16.23)$$

where B = Maximum Integer $\leq b\frac{T_c}{T_d}$ and L represents the leftover units, the amount by which b exceeds $B\frac{T_d}{T_c}$.

There are two cases:

$$\text{Case 1: } r = 1.0.\ h(b) = \frac{B}{B+1} + L\frac{T_c}{T_d}\frac{1}{(B+1)(B+2)} \qquad (16.24)$$

$$\text{Case 2: } r \neq 1.0.\ h(b) = r\frac{1-r^B}{1-r^{B+1}} + L\frac{T_c}{T_d}\frac{r^{B+1}(1-r)^2}{(1-r^{B+1})(1-r^{B+2})} \qquad (16.25)$$

Geometric downtime distribution: In this downtime distribution, the probability that repairs are completed during any cycle duration T_c is independent of the time since repairs began. This is a case of maximum downtime variation. There are two cases:

$$\text{Case 1: } r = 1.0.\ h(b) = \frac{b\frac{T_c}{T_d}}{2 + (b-1)\frac{T_c}{T_d}} \qquad (16.26)$$

$$\text{Case 2: } r \neq 1.0.\ \text{Define } K = \frac{1 + r - \frac{T_c}{T_d}}{1 + r - r\frac{T_c}{T_d}} \quad \text{then } h(b) = \frac{r(1-K^b)}{1-rK^b} \qquad (16.27)$$

APPENDIX 4: AUTOMATED ASSEMBLY SYSTEMS

Multi-Station Assembly Machines

$$P_{ap} = \prod_{i=1}^{n}(1 - q_i + m_i q_i) \quad (17.5)$$

where P_{ap} can be thought of as the *yield* of good assemblies produced by the assembly machine. If P_{ap} = the proportion of good assemblies, then the proportion of assemblies containing at least one defective component P_{qp} is given by

$$P_{qp} = 1 - P_{ap} = 1 - \prod_{i=1}^{n}(1 - q_i + m_i q_i) \quad (17.6)$$

the frequency of downtime occurrences per cycle F; that is,

$$F = \sum_{i=1}^{n} p_i = \sum_{i=1}^{n} m_i q_i \quad (17.9)$$

The average actual production time per assembly is given by

$$T_p = T_c + \sum_{i=1}^{n} m_i q_i T_d \quad (17.11)$$

$$R_{ap} = P_{ap} R_p = \frac{P_{ap}}{T_p} = \frac{\prod_{i=1}^{n}(1 - q_i + m_i q_i)}{T_p} \quad (17.14)$$

$$C_{pc} = \frac{C_m + C_o T_p + C_t}{P_{ap}} \quad (17.17)$$

where C_{pc} = cost per good assembly, \$/pc; C_m = cost of materials, which includes the cost of the base part plus components added to it, \$/pc; C_o = operating cost of the assembly system, \$/min; T_p = average actual production time, min/pc; C_t = cost of disposable tooling, \$/pc; and P_{ap} = yield from Eq. (17.5).

Single-Station Assembly Machines

We can express this ideal cycle time as

$$T_c = T_h + \sum_{j=1}^{n_e} T_{ej} \quad (17.18)$$

where T_h = handling time, min.

Many of the assembly elements involve the addition of a component to the existing subassembly. As in our analysis of multiple station assembly, each component type has a certain fraction defect rate, q_j, and there is a certain probability that a defective component will jam the workstation, m_j. When a jam occurs, the assembly machine stops, and it takes an average T_d to clear the jam and restart the system. The inclusion of downtime resulting from jams in the machine cycle time gives

$$T_p = T_c + \sum_{j=1}^{n_e} q_j m_j T_d \quad (17.19)$$

APPENDIX 5: CELLULAR MANUFACTURING

	Digit 1	Digit 2	Digit 3	Digit 4	Digit 5
	Part class	External shape, external shape elements	Internal shape, internal shape elements	Plane surface machining	Auxiliary holes and gear teeth
0	Rotational parts: L/D 0.5	Smooth, no shape elements	No hole, no breakthrough	No surface machining	No auxiliary hole
1	Rotational parts: 0.5 < L/D < 3	Stepped to one end or smooth: No shape elements	Smooth or stepped to one end: No shape elements	Surface plane and/or curved in one direction, external	No gear teeth: Axial, not on pitch circle diameter
2	Rotational parts: L/D 3	Stepped to one end or smooth: Thread	Smooth or stepped to one end: Thread	External plane surface related by graduation around the circle	No gear teeth: Axial on pitch circle diameter
3	Nonrotational parts	Stepped to one end or smooth: Functional groove	Smooth or stepped to one end: Functional groove	External groove and/or slot	No gear teeth: Radial, not on pitch circle diameter
4	Nonrotational parts	Stepped to both ends: No shape elements	Stepped to both ends: No shape elements	External spline (polygon)	No gear teeth: Axial and/or radial and/or other direction
5	Nonrotational parts	Stepped to both ends: Thread	Stepped to both ends: Thread	External plane surface and/or slot, external spline	No gear teeth: Axial and/or radial on PCD and/or other directions
6	Nonrotational parts	Stepped to both ends: Functional groove	Stepped to both ends: Functional groove	Internal plane surface and/or slot	With gear teeth: Spur gear teeth
7	Nonrotational parts	Functional cone	Functional cone	Internal spline (polygon)	With gear teeth: Bevel gear teeth
8	Nonrotational parts	Operating thread	Operating thread	Internal and external polygon, groove and/or slot	With gear teeth: Other gear teeth
9	Nonrotational parts	All others	All others	All others	All others

Figure 18.6 Form code (digits 1 through 5) for rotational parts in the Opitz coding system. The first digit of the code is limited to values of 0, 1, or 2.

Grouping Parts and Machines by Rank Order Clustering

1. In each row of the matrix, read the series of 1's and 0's (blank entries = 0's) from left to right as a binary number. Rank the rows in order of decreasing value. In case of a tie, rank the rows in the same order as they appear in the current matrix.
2. Numbering from top to bottom, is the current order of rows the same as the rank order determined in the previous step? If yes, go to step 7. If no, go to the following step.
3. Re-order the rows in the part-machine incidence matrix by listing them in decreasing rank order, starting from the top.
4. In each column of the matrix, read the series of 1's and 0's (blank entries = 0's) from top to bottom as a binary number. Rank the columns in order of decreasing value. In case of a tie, rank the columns in the same order as they appear in the current matrix.

5. Numbering from left to right, is the current order of columns the same as the rank order determined in the previous step? If yes, go to step 7. If no, go to the following step.
6. Re-order the columns in the part-machine incidence matrix by listing them in decreasing rank order, starting with the left column. Go to step 1.
7. Stop.

Arranging Machines in a GT Cell

1. *Develop the From-To chart.* The data contained in the chart indicate numbers of part moves between the machines (or workstations) in the cell. Moves into and out of the cell are not included in the chart.
2. *Determine the "From/To ratio" for each machine.* This is accomplished by summing all of the "From" trips and "To" trips for each machine (or operation). The "From" sum for a machine is determined by adding the entries in the corresponding row, and the "To" sum is determined by adding the entries in the corresponding column. For each machine, the "From/To ratio" is calculated by taking the "From" sum for each machine and dividing by the respective "To" sum.
3. *Arrange machines in order of decreasing From/To ratio.* Machines with a high From/To ratio distribute more work to other machines in the cell but receive less work from other machines. Conversely, machines with a low From/To ratio receive more work than they distribute. Therefore, machines are arranged in order of descending From/To ratio; that is, machines with high ratios are placed at the beginning of the work flow, and machines with low ratios are placed at the end of the work flow. In case of a tie, the machine with the higher "From" value is placed ahead of the machine with a lower value.

APPENDIX 6: FLEXIBLE MANUFACTURING SYSTEM (FMS)

Part mix. The mix of the various part or product styles produced by the system is defined by p_j, where p_j = the fraction of the total system output that is of style j. The subscript $j = 1, 2, \ldots P$, where P = the total number of different part styles made in the FMS during the time period of interest. The values of p_j must sum to unity, that is,

$$\sum_{j=1}^{P} p_j = 1.0 \qquad (19.1)$$

$$WL_i = \sum_j \sum_k t_{ijk} f_{ijk} p_j \qquad (19.2)$$

$$n_t = \sum_i \sum_j \sum_k f_{ijk} p_j - 1 \qquad (19.3)$$

where n_t = mean number of transports and the other terms are defined above.

$$R_p^* = \frac{s^*}{WL^*} \qquad (19.5)$$

where R_p^* = maximum production rate of all part styles produced by the system, which is determined by the capacity of the bottleneck station, pc/min; s^* = number of servers at the bottleneck station, and WL^* = workload at the bottleneck station, min/pc.

$$R_{pj}^* = p_j(R_p^*) = p_j \frac{s^*}{WL^*} \qquad (19.6)$$

where R_{pj}^* = maximum production rate of part style j, pc/min; and p_j = part mix fraction for part style j.

$$U_i = \frac{WL_i}{s_i}(R_p^*) = \frac{WL_i}{s_i}\frac{s^*}{WL^*} \qquad (19.7)$$

where U_i = utilization of station i; WL_i = workload of station i, min/pc; s_i = number of servers at station i; and R_p^* = overall production rate, pc/min. The utilization of the bottleneck station is 100% at R_p^*.

$$\overline{U} = \frac{\sum_{i=1}^{n+1} U_i}{n+1} \qquad (19.8)$$

where $\overline{U}$ is an unweighted average of the workstation utilizations.

$$\overline{U}_s = \frac{\sum_{i=1}^{n} s_i U_i}{\sum_{i=1}^{n} s_i} \qquad (19.9)$$

where $\overline{U}_s$ = overall FMS utilization, s_i = number of servers at station i, and U_i = utilization of station i.

$$BS_i = WL_i(R_p^*) = WL_i \frac{s^*}{WL^*} \qquad (19.10)$$

where BS_i = number of busy servers on average at station i and WL_i = workload at station i.

Extended Bottleneck Model

WIP corresponds to N, and MLT is the sum of processing times at the workstations, transport times between stations, and any waiting time experienced by the parts in the system. We can express MLT as

$$MLT = \sum_{i=1}^{n} WL_i + WL_{n+1} + T_w \tag{19.11}$$

where $\sum_{i=1}^{n} WL_i$ = summation of average workloads over all stations in the FMS, min; WL_{n+1} = workload of the part handling system, min; and T_w = mean waiting time experienced by a part due to queues at the stations, min.

WIP (that is, N) and MLT are correlated. If N is small, then MLT will take on its smallest possible value because waiting time will be short or even zero. If N is large, then

$$N = R_p(MLT) \tag{19.12}$$

where N = number of parts in the system, pc; R_p = production rate of the system, pc/min; and MLT = manufacturing lead time (time spent in the system by a part), min. Now, let us examine the two cases:

Case 1: When N is small, production rate is less than in the bottleneck case because the bottleneck station is not fully utilized. In this case, the waiting time T_w of a unit is theoretically zero, and Eq. (19.11) reduces to

$$MLT_1 = \sum_{i=1}^{n} WL_i + WL_{n+1} \tag{19.13}$$

where the subscript in MLT_1 is used to identify case 1. Production rate can be estimated using Little's formula:

$$R_p = \frac{N}{MLT_1} \tag{19.14}$$

and production rates of the individual parts are given by

$$R_{pj} = p_j R_p \tag{19.15}$$

As indicated waiting time is assumed to be zero:

$$T_w = 0 \tag{19.16}$$

Case 2: When N is large, the estimate of maximum production rate provided by Eq. (19.5) should be valid: $R_p^* = s^*/WL^*$, where the asterisk (*) denotes that production rate is constrained by the bottleneck station in the system. The production rates of the individual products are given by

$$R_{pj}^* = p_j R_p^* \tag{19.17}$$

In this case, average manufacturing lead time is evaluated using Little's formula:

$$MLT_2 = \frac{N}{R_p^*} \tag{19.18}$$

TABLE 19.5 Equations and Guidelines for the Extended Bottleneck Model

Case 1: $N < N^* = R_p^*\left(\sum_{i=1}^{n} WL_i + WL_{n+1}\right)$	Case 2: $N \geq N^* = R_p^*\left(\sum_{i=1}^{n} WL_i + WL_{n+1}\right)$
$MLT_1 = \sum_{i=1}^{n} WL_i + WL_{n+1}$	$R_p^* = \frac{S^*}{WL^*}$
$R_p = \frac{N}{MLT_1}$	$R_{pj}^* = p_j R_p^*$
$R_{pj} = p_j R_p$	$MLT_2 = \frac{N}{R_p^*}$
$T_w = 0$	$T_w = MLT_2 - \left(\sum_{i=1}^{n} WL_i + WL_{n+1}\right)$

The mean waiting time a part spends in the system can be estimated by rearranging Eq. (19.11) to solve for T_w:

$$T_w = MLT_2 - \left(\sum_{i=1}^{n} WL_i + WL_{n+1}\right) \tag{19.19}$$

The decision on whether to use case 1 or case 2 depends on the value of N. The dividing line between cases 1 and 2 is determined by whether N is greater than or less than a critical value given by

$$N^* = R_p^*\left(\sum_{i=1}^{n} WL_i + WL_{n+1}\right) = R_p^*(MLT_1) \tag{19.20}$$

where N^* = critical value of N, the dividing line between the bottleneck and nonbottleneck cases. If $N < N^*$, then case 1 applies. If $N \geq N^*$, then case 2 applies. The applicable equations for the two cases are summarized in Table 19.5.