



UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2010 SESSION

SUBJECT CODE : FKB 24402 / FKB 33402
SUBJECT TITLE : ENGINEERING MATHEMATICS 4
LEVEL : BACHELOR
TIME / DURATION : 8.00pm – 10.00pm
(2 HOURS)
DATE : 07 MAY 2010

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper **CAREFULLY**.
 2. This question paper is printed on both sides of the paper.
 3. Please write your answers on the answer booklet provided.
 4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
 5. This question paper consists of **FIVE (5)** questions. Answer **FOUR (4)** questions only.
 6. Answer all questions in English.
 7. Two pages of formulas and three pages of statistical tables are appended.
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THERE ARE 5 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

(Total: 60 marks)

INSTRUCTION: There are five questions. **ANSWER** only **FOUR (4)** questions.

Please use the answer booklet provided.

Question 1 (15 Marks)

- (a) Enhanced protection against corrosion of steel sheet is a top priority of automakers. At Mazda Motor Corporation (Japan), there is a strong preference for thin, plated alloy coatings to improve protection against rust and adhesion. The accompanying table gives the breakdown of steel sheet usage in Mazda 626's exported to Malaysia. Suppose a single steel sheet is randomly selected from among those sheets used in the production of a Mazda 626 and we are interested in the type of steel sheet that is selected.

TYPE OF STEEL SHEET	PERCENTAGE USED
Cold rolled	27
Cold rolled, high strength	12
Cold rolled, plated	30
Cold rolled, high strength, plated	15
Hot rolled	8
Hot rolled, high strength	5
Hot rolled, plated	3
TOTAL	100

- (i) What is the probability that the steel sheet will be of the hot rolled, high strength type?

(1 mark)

- (ii) What is the probability that the steel sheet will not be plated?

(3 marks)

- (b) A manufacturer of gunpowder has developed a new powder, which was tested in eight shells. The resulting muzzle velocities, in feet per second, were as follows:

3005 2925 2935 2965
 2995 3005 2937 2905

Find a 95% confidence interval for the true average velocity μ for shells of this type. Assume that muzzle velocities are approximately normally distributed.

(5 marks)

(c) The achievement scores for a college entrance examination are normally distributed with mean 75 and standard deviation 10. What fraction of the scores lies between 80 and 90?

(4 marks)

(d) State whether each of these statements is TRUE or FALSE:

(i) The smaller the P-value, the more certain we can be that H_0 is false.

(1 mark)

(ii) The P-value is the probability that H_0 is true.

(1 mark)

Question 2 (15 Marks)

(a) A survey classified a large number of adults according to whether they were diagnosed as needing eyeglasses to correct their reading vision and whether they use eyeglasses when reading. The proportions falling into the four resulting categories are given in the following table.

Needs glasses	Uses Eyeglasses for Reading	
	Yes	No
Yes	0.44	0.14
No	0.02	0.40

If a single adult is selected from the large group, find the probabilities of the events defined below. The adult

(i) needs glasses

(2 marks)

(ii) needs glasses but does not use them

(2 marks)

- (b) Suppose that a random system of police patrol is devised so that a patrol officer may visit a given beat location $Y = 0, 1, 2, 3, \dots$ times per half-hour period, with each location being visited an average of once per time period. Assume that Y possesses, approximately, a Poisson probability distribution. Calculate the probability that the patrol officer will miss a given location during a half-hour period. What is the probability that it will be visited at least once?

(4 marks)

- (c) Use the method of least squares to fit a straight line to the $n = 5$ data points given in the following table. Calculate the value of $\hat{\beta}_1$, $\hat{\beta}_0$ and the fitted line.

x	y
-2	0
-1	0
0	1
1	1
2	3

(5 marks)

- (d) The random variable Y has a Poisson distribution and is such that $P(X \geq 0) = P(X = 1)$. What is $P(X = 2)$?

(2 marks)

Question 3 (15 Marks)

- (a) Determine whether the given values are from a discrete or continuous data set.
- (i) The amount of nicotine in a Marlboro cigarette is 1.2 mg. discrete (1 mark)
- (ii) A car is randomly selected at a traffic safety checkpoint, and the car has 6 cylinders. (1 mark)
- (b) Two brands of refrigerators, denoted A and B, are each guaranteed for 1 year. In a random sample of 50 refrigerators of brand A, 12 were observed to fail before the guarantee period ended. An independent random sample of 60 brand B refrigerators also revealed 12 failures during the guarantee period. Estimate the true difference between proportions of failures during the guarantee period, with confidence level 98%.

(6 marks)

(c) A vice president in charge of sales for a large corporation claims that salespeople are averaging no more than 15 sales contacts per week. (He would like to increase this figure.) As a check on his claim, 36 salespeople are selected at random, and the number of contacts made by each is recorded for a single randomly selected week. The mean and variance of the 36 measurements were 17 and 9, respectively. Does the evidence contradict the vice president's claim? Use a test with level 0.05.

(4 marks)

(d) Determine whether each of these statements is TRUE or FALSE:

(i) The correlation coefficient remains unchanged if we interchange the values of x and y .

(1 mark)

(ii) The correlation coefficient has the same units as the sample data.

(1 mark)

(iii) Whenever $r \neq 0$, x and y are said to be correlated.

(1 mark)

Question 4 (15 Marks)

(a) Listed below are the nicotine amounts (in mg per cigarette) for samples of filtered and nonfiltered cigarettes. Do filters appear to be effective in reducing the amount of nicotine?

Nonfiltered:

1.1	1.7	1.7	1.1	1.1	1.4	1.1	1.4	1.0	1.2	1.1	1.1	1.1
1.1	1.1	1.8	1.6	1.1	1.2	1.5	1.3	1.1	1.3	1.1	1.1	

Filtered:

0.4	1.0	1.2	0.8	0.8	1.0	1.1	1.1	1.1	0.8	0.8	0.8	0.8
1.	0.2	1.1	1.0	0.8	1.0	0.9	1.1	1.1	0.6	1.3	1.1	

(4 marks)

- (b) The service times for customers coming through a checkout counter in a retail store are independent random variables with mean 1.5 minutes and variance 1.0. Approximate the probability that 100 customers can be served in less than 2 hours of total service time. (5 marks)
- (c) Answer the following questions.
- (i) What is the standard deviation? (2 marks)
- (ii) Can the value of the standard deviation be zero? Explain. (2 marks)
- (iii) What is the unit of the standard deviation? (2 marks)

Question 5 (15 Marks)

- (a) Suppose a lot of 5000 electrical fuses contain 5% defectives. If a sample of 5 fuses is tested, find the probability of observing at least one defective. (4 marks)
- (b) A machine in a factory must be repaired if it produces more than 10% defectives among the large lot of items that it produces in a day. A random sample of 100 items from the day's production contains 15 defectives, and the supervisor says that the machine must be repaired. Does the sample evidence support his decision? Use a test with level 0.01. (7 marks)
- (c) Given $f(y) = cy^2$, $0 \leq y \leq 2$, and $f(y) = 0$ elsewhere, find the value of c for which $f(y)$ is a valid density function. (4 marks)

END OF QUESTION

FORMULA

Confidence Intervals for the Difference Between Two Means	Small-Sample Confidence Intervals for the Difference Between Two Means
$\bar{X} - \bar{Y} \pm z_{\alpha/2} \sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}$	$\bar{X} - \bar{Y} \pm t_{v, \alpha/2} \sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}$
Pooled standard deviation	Confidence Intervals for the Difference Between Two Proportions
$s_p = \sqrt{\frac{(n_X - 1)s_X^2 + (n_Y - 1)s_Y^2}{n_X + n_Y - 2}}$	$\tilde{p}_X - \tilde{p}_Y \pm z_{\alpha/2} \sqrt{\frac{\tilde{p}_X(1 - \tilde{p}_X)}{\tilde{n}_X} + \frac{\tilde{p}_Y(1 - \tilde{p}_Y)}{\tilde{n}_Y}}$
The number of degrees of freedom (small-sample confidence intervals)	Confidence Intervals for the Difference Between Two Proportions (adjustments)
$v = \frac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}\right)^2}{\frac{(s_X^2/n_X)^2}{n_X - 1} + \frac{(s_Y^2/n_Y)^2}{n_Y - 1}}$	$\tilde{n}_X = n_X + 2 \quad \tilde{n}_Y = n_Y + 2$ $\tilde{p}_X = (X + 1) / \tilde{n}_X \quad \tilde{p}_Y = (Y + 1) / \tilde{n}_Y$
Confidence Intervals for the Difference Between Two Means	Confidence Intervals for Proportions (adjustments)
$\bar{X} - \bar{Y} \pm t_{n_X + n_Y - 2, \alpha/2} \cdot s_p \sqrt{\frac{1}{n_X} + \frac{1}{n_Y}}$	$\tilde{n} = n + 4 \quad \tilde{p} = \frac{X + 2}{\tilde{n}}$
Large-Sample Confidence Intervals for a Population Mean	Small-Sample Confidence Intervals for a Population Mean
$\bar{X} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$	$\bar{X} \pm t_{n-1, \alpha/2} \frac{s}{\sqrt{n}}$
The Binomial Distribution	
$p(x) = P(X = x) = \begin{cases} \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} & x = 0, 1, \dots, n \\ 0 & \text{otherwise} \end{cases}$	
The Poisson Distribution	
$P(X = x) = \begin{cases} \frac{e^{-\lambda} \lambda^x}{x!} & \text{if } x \text{ is a non-negative integer} \\ 0 & \text{otherwise} \end{cases}$	
The Normal Distribution	Sample Variance
$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$s^2 = \frac{1}{n-1} \left(\sum_{i=1}^n X_i^2 - n\bar{X}^2 \right)$

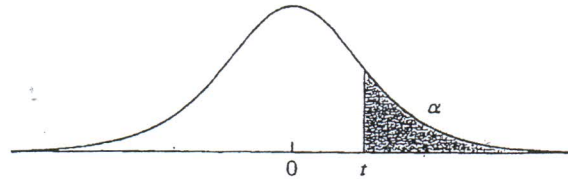
Test Statistic (Small-Sample)	Test Statistic (Small-Sample Tests)
$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$	$t = \frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{s_X^2/n_X + s_Y^2/n_Y}}$
Pooled Proportion (Test for the Difference Between Two Proportions)	Test Statistic (Two Proportions)
$\hat{p} = \frac{X+Y}{n_X+n_Y} \quad \hat{p}_X = \frac{X}{n_X} \quad \hat{p}_Y = \frac{Y}{n_Y}$	$z = \frac{\hat{p}_X - \hat{p}_Y}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_X} + \frac{1}{n_Y}\right)}}$
Standard Unit	z-score (CLT)
$z = \frac{X - \mu}{\sigma}$	$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$
Test Statistic (Small-Sample with Equal Variances)	Confidence Interval for Proportions
$t = \frac{(\bar{X} - \bar{Y}) - \Delta_0}{s_p \sqrt{1/n_X + 1/n_Y}}$	$\tilde{p} \pm z_{\alpha/2} \sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}}$
Test Statistic (One-proportion)	Intercept
$z^* = \frac{\bar{p} - p_0}{\sqrt{\hat{p}_0(1-p_0)/n}}$	$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$
Slope	Test Statistic (Large Sample Tests)
$\hat{\beta}_1 = \frac{\sum xy - n\bar{X}\bar{Y}}{\sum x^2 - n\bar{X}^2}$	$z = \frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{s_X^2/n_X + s_Y^2/n_Y}}$

Areas Under the Normal Curve

Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p
0.00	0.5000	0.5000	0.40	0.6554	0.3446	0.80	0.7881	0.2119	1.20	0.8849	0.1151	1.60	0.9452	0.0548
0.01	0.5040	0.4960	0.41	0.6591	0.3409	0.81	0.7910	0.2090	1.21	0.8869	0.1131	1.61	0.9463	0.0537
0.02	0.5080	0.4920	0.42	0.6628	0.3372	0.82	0.7939	0.2061	1.22	0.8888	0.1112	1.62	0.9474	0.0526
0.03	0.5120	0.4880	0.43	0.6664	0.3336	0.83	0.7967	0.2033	1.23	0.8907	0.1093	1.63	0.9484	0.0516
0.04	0.5160	0.4840	0.44	0.6700	0.3300	0.84	0.7995	0.2005	1.24	0.8925	0.1075	1.64	0.9495	0.0505
0.05	0.5199	0.4801	0.45	0.6736	0.3264	0.85	0.8023	0.1977	1.25	0.8944	0.1056	1.65	0.9505	0.0495
0.06	0.5239	0.4761	0.46	0.6772	0.3228	0.86	0.8051	0.1949	1.26	0.8962	0.1038	1.66	0.9515	0.0485
0.07	0.5279	0.4721	0.47	0.6808	0.3192	0.87	0.8078	0.1922	1.27	0.8980	0.1020	1.67	0.9525	0.0475
0.08	0.5319	0.4681	0.48	0.6844	0.3156	0.88	0.8106	0.1894	1.28	0.8997	0.1003	1.68	0.9535	0.0465
0.09	0.5359	0.4641	0.49	0.6879	0.3121	0.89	0.8133	0.1867	1.29	0.9015	0.0985	1.69	0.9545	0.0455
0.10	0.5398	0.4602	0.50	0.6915	0.3085	0.90	0.8159	0.1841	1.30	0.9032	0.0968	1.70	0.9554	0.0446
0.11	0.5438	0.4562	0.51	0.6950	0.3050	0.91	0.8186	0.1814	1.31	0.9049	0.0951	1.71	0.9564	0.0436
0.12	0.5478	0.4522	0.52	0.6985	0.3015	0.92	0.8212	0.1788	1.32	0.9066	0.0934	1.72	0.9573	0.0427
0.13	0.5517	0.4483	0.53	0.7019	0.2981	0.93	0.8238	0.1762	1.33	0.9082	0.0918	1.73	0.9582	0.0418
0.14	0.5557	0.4443	0.54	0.7054	0.2946	0.94	0.8264	0.1736	1.34	0.9099	0.0901	1.74	0.9591	0.0409
0.15	0.5596	0.4404	0.55	0.7088	0.2912	0.95	0.8289	0.1711	1.35	0.9115	0.0885	1.75	0.9599	0.0401
0.16	0.5636	0.4364	0.56	0.7123	0.2877	0.96	0.8315	0.1685	1.36	0.9131	0.0869	1.76	0.9608	0.0392
0.17	0.5675	0.4325	0.57	0.7157	0.2843	0.97	0.8340	0.1660	1.37	0.9147	0.0853	1.77	0.9616	0.0384
0.18	0.5714	0.4286	0.58	0.7190	0.2810	0.98	0.8365	0.1635	1.38	0.9162	0.0838	1.78	0.9625	0.0375
0.19	0.5753	0.4247	0.59	0.7224	0.2776	0.99	0.8389	0.1611	1.39	0.9177	0.0823	1.79	0.9633	0.0367
0.20	0.5793	0.4207	0.60	0.7257	0.2743	1.00	0.8413	0.1587	1.40	0.9192	0.0808	1.80	0.9641	0.0359
0.21	0.5832	0.4168	0.61	0.7291	0.2709	1.01	0.8438	0.1562	1.41	0.9207	0.0793	1.81	0.9649	0.0351
0.22	0.5871	0.4129	0.62	0.7324	0.2676	1.02	0.8461	0.1539	1.42	0.9222	0.0778	1.82	0.9656	0.0344
0.23	0.5910	0.4090	0.63	0.7357	0.2643	1.03	0.8485	0.1515	1.43	0.9236	0.0764	1.83	0.9664	0.0336
0.24	0.5948	0.4052	0.64	0.7389	0.2611	1.04	0.8508	0.1492	1.44	0.9251	0.0749	1.84	0.9671	0.0329
0.25	0.5987	0.4013	0.65	0.7422	0.2578	1.05	0.8531	0.1469	1.45	0.9265	0.0735	1.85	0.9678	0.0322
0.26	0.6026	0.3974	0.66	0.7454	0.2546	1.06	0.8554	0.1446	1.46	0.9279	0.0721	1.86	0.9686	0.0314
0.27	0.6064	0.3936	0.67	0.7486	0.2514	1.07	0.8577	0.1423	1.47	0.9292	0.0708	1.87	0.9693	0.0307
0.28	0.6103	0.3897	0.68	0.7517	0.2483	1.08	0.8599	0.1401	1.48	0.9306	0.0694	1.88	0.9699	0.0301
0.29	0.6141	0.3859	0.69	0.7549	0.2451	1.09	0.8621	0.1379	1.49	0.9319	0.0681	1.89	0.9706	0.0294
0.30	0.6179	0.3821	0.70	0.7580	0.2420	1.10	0.8643	0.1357	1.50	0.9332	0.0668	1.90	0.9713	0.0287
0.31	0.6217	0.3783	0.71	0.7611	0.2389	1.11	0.8665	0.1335	1.51	0.9345	0.0655	1.91	0.9719	0.0281
0.32	0.6255	0.3745	0.72	0.7642	0.2358	1.12	0.8686	0.1314	1.52	0.9357	0.0643	1.92	0.9726	0.0274
0.33	0.6293	0.3707	0.73	0.7673	0.2327	1.13	0.8708	0.1292	1.53	0.9370	0.0630	1.93	0.9732	0.0268
0.34	0.6331	0.3669	0.74	0.7704	0.2296	1.14	0.8729	0.1271	1.54	0.9382	0.0618	1.94	0.9738	0.0262
0.35	0.6368	0.3632	0.75	0.7734	0.2266	1.15	0.8749	0.1251	1.55	0.9394	0.0606	1.95	0.9744	0.0256
0.36	0.6406	0.3594	0.76	0.7764	0.2236	1.16	0.8770	0.1230	1.56	0.9406	0.0594	1.96	0.9750	0.0250
0.37	0.6443	0.3557	0.77	0.7794	0.2206	1.17	0.8790	0.1210	1.57	0.9418	0.0582	1.97	0.9756	0.0244
0.38	0.6480	0.3520	0.78	0.7823	0.2177	1.18	0.8810	0.1190	1.58	0.9429	0.0571	1.98	0.9761	0.0239
0.39	0.6517	0.3483	0.79	0.7852	0.2148	1.19	0.8830	0.1170	1.59	0.9441	0.0559	1.99	0.9767	0.0233

Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p
2.00	0.9772	0.0228	2.40	0.9918	0.0082	2.80	0.9974	0.0026	3.20	0.9993	0.0007
2.01	0.9778	0.0222	2.41	0.9920	0.0080	2.81	0.9975	0.0025	3.21	0.9993	0.0007
2.02	0.9783	0.0217	2.42	0.9922	0.0078	2.82	0.9976	0.0024	3.22	0.9994	0.0006
2.03	0.9788	0.0212	2.43	0.9925	0.0075	2.83	0.9977	0.0023	3.23	0.9994	0.0006
2.04	0.9793	0.0207	2.44	0.9927	0.0073	2.84	0.9977	0.0023	3.24	0.9994	0.0006
2.05	0.9798	0.0202	2.45	0.9929	0.0071	2.85	0.9978	0.0022	3.25	0.9994	0.0006
2.06	0.9803	0.0197	2.46	0.9931	0.0069	2.86	0.9979	0.0021	3.26	0.9994	0.0006
2.07	0.9808	0.0192	2.47	0.9932	0.0068	2.87	0.9979	0.0021	3.27	0.9995	0.0005
2.08	0.9812	0.0188	2.48	0.9934	0.0066	2.88	0.9980	0.0020	3.28	0.9995	0.0005
2.09	0.9817	0.0183	2.49	0.9936	0.0064	2.89	0.9981	0.0019	3.29	0.9995	0.0005
2.10	0.9821	0.0179	2.50	0.9938	0.0062	2.90	0.9981	0.0019	3.30	0.9995	0.0005
2.11	0.9826	0.0174	2.51	0.9940	0.0060	2.91	0.9982	0.0018	3.31	0.9995	0.0005
2.12	0.9830	0.0170	2.52	0.9941	0.0059	2.92	0.9982	0.0018	3.32	0.9995	0.0005
2.13	0.9834	0.0166	2.53	0.9943	0.0057	2.93	0.9983	0.0017	3.33	0.9996	0.0004
2.14	0.9838	0.0162	2.54	0.9945	0.0055	2.94	0.9984	0.0016	3.34	0.9996	0.0004
2.15	0.9842	0.0158	2.55	0.9946	0.0054	2.95	0.9984	0.0016	3.35	0.9996	0.0004
2.16	0.9846	0.0154	2.56	0.9948	0.0052	2.96	0.9985	0.0015	3.36	0.9996	0.0004
2.17	0.9850	0.0150	2.57	0.9949	0.0051	2.97	0.9985	0.0015	3.37	0.9996	0.0004
2.18	0.9854	0.0146	2.58	0.9951	0.0049	2.98	0.9986	0.0014	3.38	0.9996	0.0004
2.19	0.9857	0.0143	2.59	0.9952	0.0048	2.99	0.9986	0.0014	3.39	0.9997	0.0003
2.20	0.9861	0.0139	2.60	0.9953	0.0047	3.00	0.9987	0.0013	3.40	0.9997	0.0003
2.21	0.9864	0.0136	2.61	0.9955	0.0045	3.01	0.9987	0.0013	3.41	0.9997	0.0003
2.22	0.9868	0.0132	2.62	0.9956	0.0044	3.02	0.9987	0.0013	3.42	0.9997	0.0003
2.23	0.9871	0.0129	2.63	0.9957	0.0043	3.03	0.9988	0.0012	3.43	0.9997	0.0003
2.24	0.9875	0.0125	2.64	0.9959	0.0041	3.04	0.9988	0.0012	3.44	0.9997	0.0003
2.25	0.9878	0.0122	2.65	0.9960	0.0040	3.05	0.9989	0.0011	3.45	0.9997	0.0003
2.26	0.9881	0.0119	2.66	0.9961	0.0039	3.06	0.9989	0.0011	3.46	0.9997	0.0003
2.27	0.9884	0.0116	2.67	0.9962	0.0038	3.07	0.9989	0.0011	3.47	0.9997	0.0003
2.28	0.9887	0.0113	2.68	0.9963	0.0037	3.08	0.9990	0.0010	3.48	0.9997	0.0003
2.29	0.9890	0.0110	2.69	0.9964	0.0036	3.09	0.9990	0.0010	3.49	0.9998	0.0002
2.30	0.9893	0.0107	2.70	0.9965	0.0035	3.10	0.9990	0.0010	3.50	0.9998	0.0002
2.31	0.9896	0.0104	2.71	0.9966	0.0034	3.11	0.9991	0.0009			
2.32	0.9898	0.0102	2.72	0.9967	0.0033	3.12	0.9991	0.0009	3.60	0.9998	0.0002
2.33	0.9901	0.0099	2.73	0.9968	0.0032	3.13	0.9991	0.0009	3.70	0.9999	0.0001
2.34	0.9904	0.0096	2.74	0.9969	0.0031	3.14	0.9992	0.0008	3.80	0.9999	0.0001
2.35	0.9906	0.0094	2.75	0.9970	0.0030	3.15	0.9992	0.0008	3.90	1.0000	0.0000
2.36	0.9909	0.0091	2.76	0.9971	0.0029	3.16	0.9992	0.0008			
2.37	0.9911	0.0089	2.77	0.9972	0.0028	3.17	0.9992	0.0008			
2.38	0.9913	0.0087	2.78	0.9973	0.0027	3.18	0.9993	0.0007			
2.39	0.9916	0.0084	2.79	0.9974	0.0026	3.19	0.9993	0.0007			

TABLE Upper percentage points for the Student's t distribution



v	α								
	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
35	0.255	0.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291