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CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2010 SESSION

SUBJECT CODE	: FKB 23302
SUBJECT TITLE	: ENGINEERING MATHEMATICS 3
LEVEL	: BACHELOR
TIME / DURATION	: 8.00pm – 10.00pm (2 HOURS)
DATE	: 06 MAY 2010

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This questions paper consists of FIVE (5) questions. Answer FOUR (4) questions only.
6. Answer ALL questions in English.

THERE ARE 2 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

(Total : 60 marks)

INSTRUCTION : Answer FOUR (4) questions only.

Please use the answer booklet provided.

Question 1

(a) The first term of a geometric progression is 8 and the sum to infinity is 40. Determine the

(i) common ratio (3 marks)

(ii) least value of n for which this sum exceeds 39. (7 marks)

(b) Use the standard results for natural number series to evaluate

$$\sum_{r=1}^{80} r^2\left(1-\frac{5}{r}+\frac{8}{r^2}\right)$$

(5 marks)

Question 2

(a) Expand $\left(1+\frac{x}{3}\right)^{-\frac{1}{2}}$ as a series of ascending powers of x as far as the term in x^3.

(3 marks)

(b) Determine $e^{-x}\left(1+\frac{x}{3}\right)^{-\frac{1}{2}}$ as a series of ascending powers of x up to and including the terms in x^3. (4 marks)

(c) Express $\dfrac{2}{(r+2)(r+3)}$ in partial fractions and hence, use the method of differences to prove that $\displaystyle\sum_{r=1}^{n} \frac{2}{r^2+5r+6} = \frac{2n}{3(n+3)}$ (8 marks)

Question 3

Determine the particular solution to the differential equation

$$y'' - x^2 y = 0 \qquad \textit{given that} \quad y(0) = 1 \;, \quad y'(0) = 0$$

and write down the first three nonzero terms of the series solution. (15 marks)

Question 4

Determine the centre, radius and interval of convergence for

$$\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{3^{n+1}} (x-1)^n$$

Also, determine the behavior of the series at the endpoints of the interval. (15 marks)

Question 5

Solve the given difference equation using Z-Transform

$$3y_{n+2} - 7y_{n+1} + 2y_n = k \qquad \textit{if} \qquad y_0 = 0 \quad y_1 = 0$$

where ***k*** is a constant. (15 marks)

END OF QUESTION

APPENDIX 1

MACLAURIN SERIES FOR COMMON FUNCTIONS

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots \text{ for } -1 < x < 1$$

$$\mathrm{cn}(x, k) = 1 - \frac{1}{2} x^2 + \frac{1}{24}(1 + 4k^2) x^4 + \ldots$$

$$\cos x = 1 - \frac{1}{2} x^2 + \frac{1}{24} x^4 - \frac{1}{720} x^6 + \ldots \text{ for } -\infty < x < \infty$$

$$\cos^{-1} x = \frac{1}{2}\pi - x - \frac{1}{6} x^3 - \frac{3}{40} x^5 - \frac{5}{112} x^7 - \ldots \text{ for } -1 < x < 1$$

$$\cosh x = 1 + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{720} x^6 + \frac{1}{40{,}320} x^8 + \ldots$$

$$\cot^{-1} x = \frac{1}{2}\pi - x + \frac{1}{3} x^3 - \frac{1}{5} x^5 + \frac{1}{7} x^7 - \frac{1}{9} x^9 + \ldots$$

$$\mathrm{dn}(x, k) = 1 - \frac{1}{2} k^2 x^2 + \frac{1}{24} k^2 (4 + k^2) x^4 + \ldots$$

$$\mathrm{erf}(x) = \frac{1}{\sqrt{\pi}}\left(2x - \frac{2}{3} x^3 + \frac{1}{5} x^5 - \frac{1}{21} x^7 + \ldots\right)$$

$$e^x = 1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + \frac{1}{24} x^4 + \ldots \text{ for } -\infty < x < \infty$$

$${}_2F_1(\alpha, \beta; \gamma; x) = 1 + \frac{\alpha\beta}{1!\,\gamma} x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{2!\,\gamma(\gamma+1)} x^2 + \ldots$$

$$\ln(1 + x) = x - \frac{1}{2} x^2 + \frac{1}{3} x^3 - \frac{1}{4} x^4 + \ldots \text{ for } -1 < x < 1$$

$$\ln\left(\frac{1+x}{1-x}\right) = 2x + \frac{2}{3} x^3 + \frac{2}{5} x^5 + \frac{2}{7} x^7 + \ldots \text{ for } -1 < x < 1$$

$$\sec x = 1 + \frac{1}{2} x^2 + \frac{5}{24} x^4 + \frac{61}{720} x^6 + \frac{277}{8064} x^8 + \ldots$$

$$\mathrm{sech}\, x = 1 - \frac{1}{2} x^2 + \frac{5}{24} x^4 - \frac{61}{720} x^6 + \frac{277}{8064} x^8 + \ldots$$

$$\sin x = x - \frac{1}{6} x^3 + \frac{1}{120} x^5 - \frac{1}{5040} x^7 + \ldots \text{ for } -\infty < x < \infty$$

$$\sin^{-1} x = x + \frac{1}{6} x^3 + \frac{3}{40} x^5 + \frac{5}{112} x^7 + \frac{35}{1152} x^9 + \ldots$$

$$\sinh x = x + \frac{1}{6} x^3 + \frac{1}{120} x^5 + \frac{1}{5040} x^7 + \frac{1}{362880} x^9 + \ldots$$

$$\sinh^{-1} x = x - \frac{1}{6} x^3 + \frac{3}{40} x^5 - \frac{5}{112} x^7 + \frac{35}{1152} x^9 - \ldots$$

$$\mathrm{sn}(x, k) = x - \frac{1}{6}(1 + k^2) x^3 + \frac{1}{120}(1 + 14k^2 + k^4) x^5 + \ldots$$

$$\tan x = x + \frac{1}{3} x^3 + \frac{2}{15} x^5 + \frac{17}{315} x^7 + \frac{62}{2835} x^9 + \ldots$$

$$\tan^{-1} x = x - \frac{1}{3} x^3 + \frac{1}{5} x^5 - \frac{1}{7} x^7 + \ldots \text{ for } -1 < x < 1$$

$$\tanh x = x - \frac{1}{3} x^3 + \frac{2}{15} x^5 - \frac{17}{315} x^7 + \frac{62}{2835} x^9 + \ldots$$

$$\tanh^{-1} x = x + \frac{1}{3} x^3 + \frac{1}{5} x^5 + \frac{1}{7} x^7 + \frac{1}{9} x^9 + \ldots$$

APPENDIX 2

TABLE OF Z-TRANSORM

	$X(z)$	Region of existence
1	$\frac{z}{z-1}$	$All\ z$
$(-1)^n$	$\frac{z}{z+1}$	$\|z\| > 1$
$a^n u(n)$	$\frac{z}{z-a}$	$\|z\| > \|a\|$
$u(n-m)$	$z^{-m}.\frac{z}{z-1}$	$\|z\| > 1$
n	$\frac{z}{(z-1)^2}$	$\|z\| > 1$
n^2	$\frac{z^2+z}{(z-1)^3}$	$\|z\| > 1$
$n(n-1)$	$\frac{2z}{(z-1)^2}$	$\|z\| > 1$
n^k	$\frac{k!z}{(z-1)^{k+1}}$	$\|z\| > 1$
$u(n-1)$	$\frac{1}{z-1}$	$\|z\| > 1$
na^n	$\frac{az}{(z-a)^2}$	$\|z\| > \|a\|$
$u(n)\cos n\theta$	$\frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$	$\|z\| > 1$
$u(n)\sin n\theta$	$\frac{z\sin\theta}{z^2-2z\cos\theta+1}$	$\|z\| > 1$
$r^n \cos n\theta$	$\frac{z(z-r\cos\theta)}{z^2-2zr\cos\theta+r^2}$	$\|z\| > \|r\|$

$r^n \sin n\theta$	$\dfrac{zr\sin\theta}{z^2 - 2zr\cos\theta + r^2}$	$\lvert z\rvert > \lvert r\rvert$
$a^n x(n)$	$X\left(\dfrac{z}{a}\right)$	$\lvert z\rvert > \lvert a\rvert$
$nx(n)$	$\dfrac{1}{z}\dfrac{dX(z)}{dz^{-1}}$	
$x(n)$ or $f(t)$	$X(z)$ or $F(z)$	
$\dfrac{1}{n}$	$\log\left(\dfrac{z}{z-1}\right)$	$\lvert z\rvert > 1$
$\delta(n)$	1	
$\delta(n-k)$	$\dfrac{1}{z^k}$	
$a^n u(n)$	$\dfrac{z}{z-a}$	
$a^n \cos n\theta \,.\, u(n)$	$\dfrac{z(z-a\cos\theta)}{z^2 - 2az\cos\theta + a^2}$	
$a^n \sin n\theta \,.\, u(n)$	$\dfrac{az\sin\theta}{z^2 - 2az\cos\theta + a^2}$	
$na^n \,.\, u(n)$	$\dfrac{az}{(z-a)^2}$	
$(n+1)a^n \,.\, u(n)$	$\dfrac{z^2}{(z-a)^2}$	
$n(n-1)a^n \,.\, u(n)$	$\dfrac{2a^2 z}{(z-a)^3}$	
$u(n)$	$\dfrac{z}{z-1}$	$\lvert z\rvert > 1$
$Z(t)$	$\dfrac{Tz}{(z-1)^2}$	

$Z(t^2)$	$\dfrac{T^2 z(z+1)}{(z-1)^3}$	
$Z(t^3)$	$\dfrac{T^3 z(1+4z-z^2)}{(z-1)^4}$	
$Z(t^k)$	$-Tz\dfrac{d}{dz}\left[Z(t^{k-1})\right]$	
$a^n \cos\dfrac{n\pi}{2}$	$\dfrac{z^2}{z^2+a^2}$	
$a^n \sin\dfrac{n\pi}{2}$	$\dfrac{az}{z^2+a^2}$	
$a^n f(t)$	$F\left(\dfrac{z}{a}\right)$	
$nf(nT)=nf(t)$	$-z\dfrac{d}{dz}\left[F(z)\right]$	
k	$\dfrac{kz}{z-1}$	$\lvert z\rvert > 1$
e^{-at}	$\dfrac{z}{z-e^{-aT}}$	$\lvert z\rvert > \lvert e^{-aT}\rvert$
e^{at}	$\dfrac{z}{z-e^{aT}}$	$\lvert z\rvert > \lvert e^{aT}\rvert$
$\cos\omega t$	$\dfrac{z(z-\cos\varpi T)}{z^2-2z\cos\omega T+1}$	$\lvert z\rvert > 1$
$\sin\omega t$	$\dfrac{z\sin\omega T}{z^2-2z\cos\omega T+1}$	$\lvert z\rvert > 1$