Document No : UniKL MFI_SD_AC41
Revision No: 02
Effective Date: 01 December 2008

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SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2010 SESSION

SUBJECT CODE	:	FKB 13202
SUBJECT TITLE	:	ENGINEERING MATHEMATICS 2
LEVEL	:	BACHELOR
TIME / DURATION	:	8.00pm – 10.00pm (2 HOURS)
DATE	:	07 MAY 2010

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This questions paper consists of FIVE (5) questions. Answer FOUR (4) questions only.
6. Answer ALL questions in English.

THERE ARE 2 PAGES OF QUESTIONS AND 6 PAGES OF FORMULA, EXCLUDING THIS PAGE.

INSTRUCTION: Answer FOUR (4) questions only.

Question 1

a) Determine the general solution of the given homogeneous differential equation $\frac{x+y}{y-x}=\frac{dy}{dx}$. (10 marks)

b) The Laplace transform of $f(t)$ is given by $F(s)=\frac{2s+3}{s^2+s-1}$, determine the Laplace transform of $\frac{f(t)}{e^{2t}}$. Simplify your answer. (5 marks)

Question 2

a) Using variation of parameters method, solve $\frac{d^2y}{dx^2}-9y=e^{5x}$. (12 marks)

b) Given $x''+2x'-3x=\sin 3t$, determine the following

i. Write down the auxiliary of equation (1 mark)

ii. State the type of the roots of the auxiliary equation (1 mark)

iii. Write down the suggested trial solution of the particular integral (1 mark)

Note: Do not solve the differential equation.

Question 3

a) Using Laplace transform, determine the particular solution of $\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=e^{3t}$ subject to $y(0)=1, \quad y'(0)=2$ (10 marks)

b) Show that the general solution of the separable variable differential equation $e^y(1+x^2)\frac{dy}{dx}-2x(1+e^y)=0$ is given by $\frac{1+e^y}{1+x^2}=K$, where K is a constant. (5 marks)

Question 4

a) Determine the general term of b_n of periodic function $f(t)=\begin{cases}-3 & -\pi \le t \le 0 \\ 3 & 0 \le t \le \pi\end{cases}$ with period 2π. State the first two terms of b_n

(7 marks)

b) Determine the half range sine series representation for the function $f(x)=2x$ in the range $x=0$ to $x=\pi$. (8 marks)

Question 5

a) Given $f(t)=\begin{cases}0 & -\pi < t < -\frac{\pi}{2} \\ 4 & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < t < \pi\end{cases}$ with period 2π,

i. Sketch $f(t)$ for two cycles (1 mark)

ii. State whether the function is even, odd or neither. (1 mark)

iii. Hence, determine the Fourier coefficients of a_0 and a_n (8marks)

b) Determine the inverse Laplace transform of $\dfrac{5s+2}{(s+1)(s+2)}$. (5 marks)

MATHEMATICS FORMULA

TRIGONOMETRIC IDENTITIES

FUNDAMENTAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

FORMULAS FOR NEGATIVES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

ADDITION FORMULAS

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

SUBTRACTION FORMULAS

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

HALF-ANGLE FORMULAS

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

DOUBLE-ANGLE FORMULAS

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\ldots\ldots\ldots = 1 - 2\sin^2\theta$$

$$\ldots\ldots\ldots = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

PRODUCT-TO-SUM FORMULAS

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

SUM-TO-PRODUCT FORMULAS

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

FOURIER SERIES

Fourier series for function with period 2π

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

where

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \qquad (n = 1,2,3\ldots)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \qquad (n = 1,2,3\ldots)$$

Half range Fourier series

a) Half range Fourier cosine series

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx$$

where

$$a_0 = \frac{1}{\pi}\int_{0}^{\pi} f(x)\,dx$$

$$a_n = \frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx\,dx \qquad (n = 1,2,3\ldots)$$

b) Half range Fourier sine series

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

where

$$b_n = \frac{2}{\pi}\int_{0}^{\pi} f(x)\sin nx\,dx \qquad (n = 1,2,3\ldots)$$

Fourier series for function with period l

$$f(x)=a_0+\sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{2\pi nx}{l}\right)+b_n\sin\left(\frac{2\pi nx}{l}\right)\right)$$

where

$$a_0=\frac{2}{l}\int_{-\frac{l}{2}}^{\frac{l}{2}}f(x)\,dx$$

$$a_n=\frac{2}{l}\int_{-\frac{l}{2}}^{\frac{l}{2}}f(x)\cos\left(\frac{2\pi nx}{l}\right)dx \qquad (n=1,2,3\ldots)$$

$$b_n=\frac{2}{l}\int_{-\frac{l}{2}}^{\frac{l}{2}}f(x)\sin\left(\frac{2\pi nx}{l}\right)dx \qquad (n=1,2,3\ldots)$$

Half range Fourier series for function with period l

a) Half range Fourier cosine series

$$f(x)=a_0+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{l}\right)$$

where

$$a_0=\frac{1}{l}\int_0^l f(x)\,dx$$

$$a_n=\frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx \qquad (n=1,2,3\ldots)$$

b) Half range Fourier sine series

$$f(x)=\sum_{n=1}^{\infty}b_n\sin\left(\frac{n\pi x}{l}\right)$$

where $b_n=\dfrac{2}{l}\displaystyle\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx \qquad (n=1,2,3\ldots)$

TABLE OF LAPLACE TRANSFORMS

$f(t)=L^{-1}\{F(s)\}$	$F(s)=L\{f(t)\}$	$f(t)=L^{-1}\{F(s)\}$	$F(s)=L\{f(t)\}$
1	$\frac{1}{s}$	e^{at}	$\frac{1}{s-a}$
$t^n,\quad n=1,2,3\ldots$	$\frac{n!}{s^{n+1}}$	$t^p,\ p>-1$	$\frac{n!}{s^{n+1}}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	$t^{n-\frac{1}{2}}$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$\cos(at)$	$\frac{s}{s^2+a^2}$
$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)^2}{(s^2+a^2)^2}$	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
$u_c(t)=u(t-c)$	$\frac{e^{-cs}}{s}$	$\delta(t-c)$	e^{-cs}
$e^{at}f(t)$	$F(s-a)$	$t^n f(t)$	$(-1)F^{(n)}(s)$
$\frac{1}{t}f(t)$	$\int_0^\infty F(u)\,du$	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	$F(t+\alpha)=f(t)$	$\frac{\int_0^\alpha e^{-st}f(t)\,dt}{1-e^{-s\alpha}}$
$f'(t)$	$sF(s)-f(0)$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$

$$L\{f^{(n)}(t)\}=s^nF(s)-s^{(n-1)}f(0)-s^{(n-2)}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$$

MATHEMATICS FORMULA

DIFFERENTIATION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$

MATHEMATICS FORMULA

INTEGRATION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int \cos x\,dx=\sin x+c$	$\int \cos f(x)\,dx=\dfrac{\sin f(x)}{f'(x)}+c$
$\int \sin x\,dx=-\cos x+c$	$\int \sin f(x)\,dx=\dfrac{-\cos f(x)}{f'(x)}+c$
$\int \sec^2 x\,dx=\tan x+c$	$\int \sec^2 f(x)\,dx=\dfrac{\tan f(x)}{f'(x)}+c$
$\int \sec x\tan x\,dx=\sec x+c$	$\int \sec f(x)\tan f(x)\,dx=\dfrac{\sec f(x)}{f'(x)}+c$
$\int \csc x\cot x\,dx=-\csc x+c$	$\int \csc f(x)\cot f(x)\,dx=\dfrac{-\csc f(x)}{f'(x)}+c$
$\int \csc^2 x\,dx=-\cot x+c$	$\int \csc^2 f(x)\,dx=\dfrac{-\cot f(x)}{f'(x)}+c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int e^x\,dx=e^x+c$	$\int e^{f(x)}\,dx=\dfrac{e^{f(x)}}{f'(x)}+c$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int \dfrac{1}{x}\,dx=\ln\lvert x\rvert+c$	$\int \dfrac{1}{f(x)}\,dx=\dfrac{\ln\lvert f(x)\rvert}{f'(x)}+c$