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CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2010 SESSION

SUBJECT CODE	:	FKB 15103
SUBJECT TITLE	:	ENGINEERING MATHEMATICS 1
LEVEL	:	BACHELOR
TIME / DURATION	:	4.00pm – 6.00pm (2 HOURS)
DATE	:	27 APRIL 2010

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This questions paper consists of SIX (6) questions. Answer FOUR (4) questions only.
6. Answer ALL questions in English.

THERE ARE 4 PAGES OF QUESTIONS AND 7 PAGES OF APPENDIX, EXCLUDING THIS PAGE.

(Total: 80 marks)

INSTRUCTION: Answer only FOUR questions.
Please use the answer booklet provided.

Question 1

The two points (-2,4) and (1,3) lie on a circle with equation in form $x^2 + y^2 + ax + by + c = 0$ where a, b and c are constants.

(a) Find two equations in a, b and c and solve the system of equations by using elementary row operations.

(10 marks)

(b) If (2,2) is also lie on the circle, determine the value of a, b and c by using Cramer's rule. Hence, find the equation of the circle.

(10 marks)

Question 2

Given that $(4 + j)$ is a root of the equation $x^3 - 6x^2 + x + 34 = 0$,

(a) Factorize the cubic expression $x^3 - 6x^2 + x + 34$ completely in complex domain.

(8 marks)

(b) Hence, decompose in complex domain the following fraction,

$$\frac{1}{x^3 - 6x^2 + x + 34}.$$

(12 marks)

Question 3

Given transfer function of a simple amplifier when negative feedback, β, is applied is given by

$$T = \frac{A_0/(1+j\omega T_1)}{1+\dfrac{A_0}{1+j\omega T_1}\beta(1+j\omega T_2)}$$

where A_0 is the low frequency gain, ω is the angular frequency, T_1 is the amplifier time constant, $T =$ new transfer function and $T_2 =$ feedback time constant.

(a) Show that $T = \dfrac{A_0}{1+A_0\beta + j\omega(T_1 + A_0\beta T_2)}$

(3 marks)

(b) Find the gain (modulus of T) and phase (argument of T) of the T at an angular frequency $\omega = 2\times10^3$ rad/s, $A_0 = 1000$, $\beta = 0.1$, $T_1 = 0.5\times10^{-3}\,s$ and $T_2 = 1\times10^{-4}\,s$.

(7 marks)

(c) A cable has the following constants : $R = 10\Omega$, $L = 0.1\times10^{-3}\,H$, $G = 1\times10^{-6}$ *siemen* and $C = 1\times10^{-9}\,F$. For $\omega = 10{,}000$ rad/s, determine the characteristic impedance, Z_0 where $Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$.

(10 marks)

Question 4

Given a point A with coordinates (3, -1, 5) and a line ***l*** with the equation

$$\underset{\sim}{r} = 8\underset{\sim}{i} - \underset{\sim}{k} + \lambda\left(-6\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}\right).$$

(a) Find the coordinates of a point B on line ***l*** if AB is perpendicular to line ***l***.

(7 marks)

Given a plane π_1 with the equation $\underset{\sim}{r} \cdot \left(\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}\right) = 15.$

(b) Find the coordinates of a point C on *l* if the line *l* intersects the plane π_1.

(5 marks)

(c) Determine the equation of a plane π_2 which contains the points A, B and C.

(8 marks)

Question 5

(a) If $y = \ln(\sin px + \cos px)$, where *p* is a constant, show that

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + p^2 = 0.$$

(13 marks)

(b) Differentiate $y = \tan^{-1}\left(\frac{1-x}{1+x}\right)$ with respect to *x*.

(7 marks)

Question 6

(a) Evaluate $\int_1^2 (x+1)\sqrt{2-x}\,dx$ by using a suitable substitution or otherwise.

(7 marks)

(b) (i) Express $f(x) = \dfrac{x^2+7x+2}{(1+x^2)(2-x)}$ in terms of partial fractions.

(6 marks)

(ii) Hence, prove that $\int_0^1 f(x)\,dx = \dfrac{11}{2}\ln 2 - \dfrac{\pi}{4}$.

(7 marks)

END OF QUESTION

APPENDIX 1

Table of Differentiation

Trigonometric Functions - GENERAL FORM
$\frac{d}{dx}(\sin f(x)) = f'(x) \cos f(x)$
$\frac{d}{dx}(\cos f(x)) = -f'(x) \sin f(x)$
$\frac{d}{dx}(\tan f(x)) = f'(x) \sec^2 f(x)$
$\frac{d}{dx}(\csc f(x)) = -f'(x) \csc f(x) \cot f(x)$
$\frac{d}{dx}(\sec f(x)) = f'(x) \sec f(x) \tan f(x)$
$\frac{d}{dx}(\cot f(x)) = -f'(x) \csc^2 f(x)$

Exponential Function - GENERAL FORM
$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)\, e^{f(x)}$

Logarithmic Function - GENERAL FORM
$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)} f'(x)$

APPENDIX 2

Derivatives of Inverse Trigonometric Functions
$\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \quad , \quad \lvert u \rvert < 1$
$\frac{d}{dx}\left(\cos^{-1} u\right) = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx} \quad , \quad \lvert u \rvert < 1$
$\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{1}{1+u^2}\frac{du}{dx}$
$\frac{d}{dx}\left(\csc^{-1} u\right) = \frac{-1}{\lvert u \rvert\sqrt{u^2-1}}\frac{du}{dx} \quad , \quad \lvert u \rvert > 1$
$\frac{d}{dx}\left(\sec^{-1} u\right) = \frac{1}{\lvert u \rvert\sqrt{u^2-1}}\frac{du}{dx} \quad , \quad \lvert u \rvert > 1$
$\frac{d}{dx}\left(\cot^{-1} u\right) = \frac{-1}{1+u^2}\frac{du}{dx}$

Derivatives of Hyperbolic Functions
$\frac{d}{dx}\left(\sinh u\right) = \cosh u \frac{du}{dx}$
$\frac{d}{dx}\left(\cosh u\right) = \sinh u \frac{du}{dx}$
$\frac{d}{dx}\left(\tanh u\right) = \text{sech}^2 u \frac{du}{dx}$
$\frac{d}{dx}\left(\text{csch}\, u\right) = -\text{csch}\, u \coth u \frac{du}{dx}$
$\frac{d}{dx}\left(\text{sech}\, u\right) = -\text{sech}\, u \tanh u \frac{du}{dx}$
$\frac{d}{dx}\left(\coth u\right) = -\text{csch}^2 u \frac{du}{dx}$

APPENDIX 3

Derivatives of Inverse Hyperbolic Functions
$\frac{d}{dx}\left(\sinh^{-1} u\right) = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$
$\frac{d}{dx}\left(\cosh^{-1} u\right) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx} \quad , \quad u > 1$
$\frac{d}{dx}\left(\tanh^{-1} u\right) = \frac{1}{1-u^2}\frac{du}{dx} \quad , \quad \lvert u \rvert < 1$
$\frac{d}{dx}\left(\text{csch}^{-1} u\right) = \frac{-1}{\lvert u \rvert\sqrt{1+u^2}}\frac{du}{dx} \quad , \quad u \neq 0$
$\frac{d}{dx}\left(\text{sech}^{-1} u\right) = \frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx} \quad , \quad 0 < u < 1$
$\frac{d}{dx}\left(\coth^{-1} u\right) = \frac{1}{1-u^2}\frac{du}{dx} \quad , \quad \lvert u \rvert > 1$

APPENDIX 4

Trigonometric Identities and Formulas

FUNDAMENTAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

FORMULAS FOR NEGATIVES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

ADDITION FORMULAS

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

SUBTRACTION FORMULAS

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

HALF-ANGLE FORMULAS

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

DOUBLE-ANGLE FORMULAS

$$\sin 2\theta = 2\sin\theta\sin\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\ldots\ldots\ldots = 1 - 2\sin^2\theta$$

$$\ldots\ldots\ldots = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

PRODUCT-TO-SUM FORMULAS

$$\sin\alpha\text{in}\alpha\text{c} = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\text{os}\alpha\text{s} = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

$$\cos\alpha\text{os}\alpha\text{c} = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

$$\sin\alpha\text{in}\alpha\text{s} = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

SUM-TO-PRODUCT FORMULAS

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

APPENDIX 5

Table of Integration

Trigonometric Functions - GENERALFORM Where : $f(x)=ax+b$
$\int \cos f(x)dx = \dfrac{\sin f(x)}{f'(x)} + C$
$\int \sin f(x)dx = \dfrac{-\cos f(x)}{f'(x)} + C$
$\int \sec^2 f(x)dx = \dfrac{\tan f(x)}{f'(x)} + C$
$\int \sec f(x)\tan f(x)dx = \dfrac{\sec f(x)}{f'(x)} + C$
$\int \csc f(x)\cot f(x)dx = \dfrac{-\csc f(x)}{f'(x)} + C$
$\int \csc^2 f(x)dx = \dfrac{-\cot f(x)}{f'(x)} + C$

Exponential Function - GENERALFORM Where : $f(x)=ax+b$
$\int e^{f(x)}\,dx = \dfrac{e^{f(x)}}{f'(x)} + C$

Logarithmic Function - GENERALFORM Where : $f(x)=ax+b$
$\int \dfrac{1}{f(x)}dx = \dfrac{\ln\lvert f(x)\rvert}{f'(x)} + C$

APPENDIX 6

Integration of Inverse Trigonometric Functions
$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{-1}{\lvert x \rvert \sqrt{x^2 - a^2}} dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \frac{1}{\lvert x \rvert \sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert \geq a$
$\int \frac{-1}{a^2 + x^2} dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$

Integration of Hyperbolic Functions Where $f(x) = ax + b$ and $f'(x) = a$
$\int \cosh f(x) dx = \frac{\sinh f(x)}{f'(x)} + C$
$\int \sinh f(x) dx = \frac{\cosh f(x)}{f'(x)} + C$
$\int \text{sech}^2 f(x) dx = \frac{\tanh f(x)}{f'(x)} + C$
$\int \text{csch}^2 f(x) dx = \frac{-\coth f(x)}{f'(x)} + C$
$\int \text{sech}\, f(x) \tanh f(x) dx = \frac{-\text{sech}\, f(x)}{f'(x)} + C$
$\int \text{csch}\, f(x) \coth f(x) dx = \frac{-\text{csch}\, f(x)}{f'(x)} + C$