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SET B

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2010 SESSION

SUBJECT CODE	:	FKB 13102/FKB 14102
SUBJECT TITLE	:	ENGINEERING MATHEMATICS 1
LEVEL	:	BACHELOR
TIME / DURATION	:	8.00pm – 10.00pm (2 HOURS)
DATE	:	05 MAY 2010

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of SIX (6) questions. Answer FOUR (4) questions only.
6. Answer ALL questions in English.
7. *Formula is appended.*

THERE ARE 4 PAGES OF QUESTIONS AND 7 PAGES OF APPENDIX, EXCLUDING THIS PAGE.

INSTRUCTION: Answer FOUR questions only (Total: 60 marks)
Please use the answer booklet provided.

Question 1

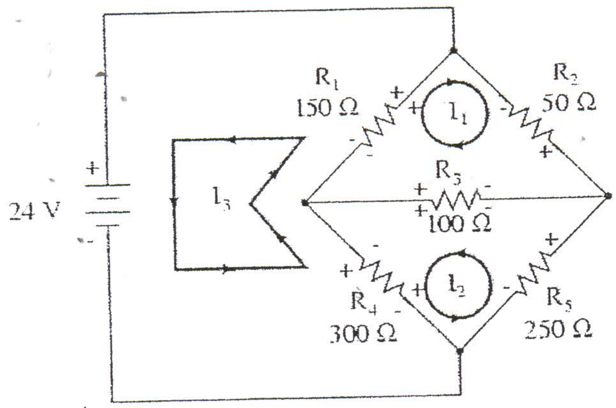
(a) The inverse of matrix $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is defined as $A^{-1} = \frac{1}{|A|} \times \text{adj} A$.

Given that the inverse of matrix $\begin{pmatrix} 4 & 5 \\ 3 & 2 \end{pmatrix}$ is $\begin{pmatrix} h & \frac{5}{7} \\ k & -\frac{4}{7} \end{pmatrix}$. Determine h and k.

(3 marks)

(b) A system of three linear equations relating currents in the circuit below, Figure (1) is formed as stated.

Figure (1)



$$300I_1 + 100I_2 = -150I_3$$

$$650I_2 + 100I_1 - 300I_3 = 0$$

$$-150I_1 - 450I_3 + 24 = -300I_2$$

Solve for I_1, I_2 and I_3 using the **Cramer's Rule**

(12 marks)

Question 2

The state matrix A of a system is given by $A = \begin{pmatrix} -5 & -6 & -3 \\ 4 & 6 & 2 \\ 10 & 8 & 6 \end{pmatrix}$. The system poles, λ of a system are the eigenvalues of the given matrix A.

(a) Determine the characteristic polynomial, $P(\lambda) = \det(A - \lambda I)$

(3 marks)

(b) Show that $\lambda_1 = 2$, is the eigenvalues of matrix A and determine the rest of the eigenvalues.

(4 marks)

(c) The eigenvector associated to the eigenvalue $\lambda_1 = 2$ is $\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$. Verify that $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ is the eigenvector of matrix A. Determine the eigenvector associated to the other eigenvalue.

(5 marks)

(e) Without computation, determine matrix P and matrix D.

(3 marks)

Question 3

(a) By assuming that $W = re^{j\theta}$, find all solutions to the equation $W^3 = -j27$.

Mark the solutions on an Argand Diagram.

(6 marks)

(b) A second degree equation with complex coefficients is defined as $Z^2 - (3 + j4)Z - 1 + j5 = 0$

(i) Calculate the discriminant, $\delta^2 = b^2 - 4ac$

(1 marks)

(ii) Assuming that $\delta = x + jy$, determine the two roots, δ_1 and δ_2

(4 marks)

(iii) Hence, determine the solutions of the second degree equation.

(4 marks)

Question 4

(a) A third degree polynomial with real coefficient is defined as $P(x) = x^3 + 2x^2 - 6x + 8$.

(i) Show that $x - (1 + j)$ is a factor of $P(x)$.

(2 marks)

(ii) Hence, write $P(x)$ as a product of linear factor.

(5 marks)

(b) A rational function, $\dfrac{x+2}{x^3 + 2x^2 - 6x + 8}$ can be decomposed into three partial fractions,

$\dfrac{A}{(x - (1 + j))} + \dfrac{B}{(x - (1 - j))} + \dfrac{C}{(x + 4)}$. Determine the constants A, B and C by using the Heaviside expansion.

(8 marks)

Question 5

(a) A function is defined as $u = \dfrac{b+x}{1-bx}$, where b is a constant.

(i) Determine $\dfrac{du}{dx}$ **(3 marks)**

(ii) Hence, differentiate $y = \tan^{-1}\left(\dfrac{b+x}{1-bx}\right)$ giving your answer in the simplest form. **(3 marks)**

(b) Use the **Logarithmic Differentiation** method to find $\dfrac{dy}{dx}$ given that $y = \dfrac{e^{4x}}{x^3 \cosh 2x}$ **(5 marks)**

(c) If $y = x^2 \ln(\sinh x) + \tanh(5x+2)$, determine its derivative $\dfrac{dy}{dx}$. **(4 marks)**

Question 6

(a) (i) $\dfrac{x^3}{x^2-2x+4}$ is an improper fraction.

Show that $\dfrac{x^3}{x^2-2x+4} = Q(x) + \dfrac{R}{x^2-2x+4}$ where $Q(x)$ and R are to be determined. **(3 marks)**

(ii) By completing the square, prove that $x^2-2x+4 = u^2+3$. Determine u. **(1 marks)**

(iii) Hence, $\displaystyle\int \frac{x^3}{x^2-2x+4}\,dx$ **(6 marks)**

(b) If $I_n = \int x^n e^{2x}\,dx$, show that $I_n = \dfrac{x^n e^{2x}}{2} - \dfrac{n}{2} I_{n-1}$ and hence evaluate $\int x^3 e^{2x}\,dx$ **(5 marks)**

END OF QUESTION

APPENDIX 1

Table of Differentiation

Trigonometric Functions - GENERAL FORM
$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$
$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$
$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$

Exponential Function - GENERAL FORM
$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)\,e^{f(x)}$

Logarithmic Function - GENERAL FORM
$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)}\,f'(x)$

APPENDIX 2

Derivatives of Inverse Trigonometric Functions
$\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$, $\lvert u \rvert < 1$
$\frac{d}{dx}\left(\cos^{-1} u\right) = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$, $\lvert u \rvert < 1$
$\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{1}{1+u^2}\frac{du}{dx}$
$\frac{d}{dx}\left(\csc^{-1} u\right) = \frac{-1}{\lvert u \rvert\sqrt{u^2-1}}\frac{du}{dx}$, $\lvert u \rvert > 1$
$\frac{d}{dx}\left(\sec^{-1} u\right) = \frac{1}{\lvert u \rvert\sqrt{u^2-1}}\frac{du}{dx}$, $\lvert u \rvert > 1$
$\frac{d}{dx}\left(\cot^{-1} u\right) = \frac{-1}{1+u^2}\frac{du}{dx}$

Derivatives of Hyperbolic Functions
$\frac{d}{dx}\left(\sinh u\right) = \cosh u \frac{du}{dx}$
$\frac{d}{dx}\left(\cosh u\right) = \sinh u \frac{du}{dx}$
$\frac{d}{dx}\left(\tanh u\right) = \text{sech}^2 u \frac{du}{dx}$
$\frac{d}{dx}\left(\text{csch}\, u\right) = -\text{csch}\, u \coth u \frac{du}{dx}$
$\frac{d}{dx}\left(\text{sech}\, u\right) = -\text{sech}\, u \tanh u \frac{du}{dx}$
$\frac{d}{dx}\left(\coth u\right) = -\text{csch}^2 u \frac{du}{dx}$

APPENDIX 3

Derivatives of Inverse Hyperbolic Functions
$\frac{d}{dx}\left(\sinh^{-1} u\right)=\frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$
$\frac{d}{dx}\left(\cosh^{-1} u\right)=\frac{1}{\sqrt{u^2-1}}\frac{du}{dx}\quad,\quad u>1$
$\frac{d}{dx}\left(\tanh^{-1} u\right)=\frac{1}{1-u^2}\frac{du}{dx}\quad,\quad \lvert u\rvert<1$
$\frac{d}{dx}\left(\operatorname{csch}^{-1} u\right)=\frac{-1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx}\quad,\quad u\neq 0$
$\frac{d}{dx}\left(\operatorname{sech}^{-1} u\right)=\frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx}\quad,\quad 0<u<1$
$\frac{d}{dx}\left(\coth^{-1} u\right)=\frac{1}{1-u^2}\frac{du}{dx}\quad,\quad \lvert u\rvert>1$

APPENDIX 4

Trigonometric Identities and Formulas

FUNDAMENTAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\,\theta = \frac{\sin\,\theta}{\cos\,\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

FORMULAS FOR NEGATIVES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\mathrm{ta\ n}\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

ADDITION FORMULAS

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

SUBTRACTION FORMULAS

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A - B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

HALF-ANGLE FORMULAS

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

DOUBLE-ANGLE FORMULAS

$$\sin 2\theta = 2\sin\theta\sin\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\text{.........} = 1 - 2\sin^2\theta$$

$$\text{.........} = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

PRODUCT-TO-SUM FORMULAS

$$\mathrm{sin\alpha in\alpha c} = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\mathrm{cos\alpha os\alpha s} = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

$$\mathrm{cos\alpha os\alpha c} = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

$$\mathrm{sin\alpha in\alpha s} = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

SUM-TO-PRODUCT FORMULAS

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

APPENDIX 5

Integration of Inverse Trigonometric Functions
$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \frac{-1}{\sqrt{a^2 - x^2}}\, dx = \cos^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{-1}{\lvert x \rvert \sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \frac{1}{\lvert x \rvert \sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \frac{-1}{a^2 + x^2}\, dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$

Integration of Hyperbolic Functions Where $f(x) = ax + b$ and $f'(x) = a$
$\int \cosh f(x)\, dx = \frac{\sinh f(x)}{f'(x)} + C$
$\int \sinh f(x)\, dx = \frac{\cosh f(x)}{f'(x)} + C$
$\int \text{sech}^2 f(x)\, dx = \frac{\tanh f(x)}{f'(x)} + C$
$\int \text{csch}^2 f(x)\, dx = \frac{-\coth f(x)}{f'(x)} + C$
$\int \text{sech}\, f(x) \tanh f(x)\, dx = \frac{-\text{sech}\, f(x)}{f'(x)} + C$
$\int \text{csch}\, f(x) \coth f(x)\, dx = \frac{-\text{csch}\, f(x)}{f'(x)} + C$

APPENDIX 6

Integration of Inverse Hyperbolic Functions
$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right)+C \quad , \quad a>0$
$\int \frac{-1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\left(\frac{x}{a}\right)+C \quad , \quad x>a$
$\int \frac{1}{a^2-x^2}\,dx = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+C \quad , \quad \lvert x\rvert<a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right)+C \quad , \quad \lvert x\rvert>a \end{cases}$
$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{1}{a}\operatorname{csch}^{-1}\left(\frac{x}{a}\right)+C \quad , \quad 0<x<a$
$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right)+C \quad , \quad 0<x<a$

APPENDIX 7

Table of Integration

Where $f(x) = ax + b$ and $f'(x) = a$

Trigonometric Functions - GENERAL FORM
$\int \cos f(x)dx = \frac{\sin f(x)}{f'(x)} + C$
$\int \sin f(x)dx = -\frac{\cos f(x)}{f'(x)} + C$
$\int \sec^2 f(x)dx = \frac{\tan f(x)}{f'(x)} + C$
$\int \csc^2 f(x)dx = -\frac{\cot f(x)}{f'(x)} + C$
$\int \sec f(x)\tan f(x)dx = \frac{\sec f(x)}{f'(x)} + C$
$\int \csc f(x)\cot f(x)dx = -\frac{\csc f(x)}{f'(x)} + C$

Exponential Function - GENERAL FORM
$\int e^{f(x)}\, dx = \frac{e^{f(x)}}{f'(x)} + C$

Logarithmic Function - GENERAL FORM
$\int \frac{1}{f(x)} dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + C$